

with MRC and the scheduled STBC systems, respectively. Using the pdfs, we derived the approximated closed-form expressions of the average postprocessing SNR and ergodic capacity for the scheduled CR systems with MIMO transmission. Through the analysis, we provide that the scheduled MIMO systems have both SNR gains and ergodic capacity gain compared with a nonscheduled system by MUD. Through numerical results, we found that the performance improvement of scheduled TAS with MRC by MUD is higher than that of the scheduled STBC. In addition, both scheduled systems under imperfect CSI degrade the ergodic capacity over the perfect CSI.

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A New Beamforming Design for Multicast Systems

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Abstract—In this paper, we study a transmit beamforming technique for multiple-input–multiple-output (MIMO) downlink multicast systems. Assuming that channel state information (CSI) is available at a transmitter, we try to maximize the lowest transmit data rate among users for a given transmit power level. Since finding the optimal beamforming vector requires prohibitively high computational complexity, we propose a reduced complexity scheme by applying greedy vector search and a simple power-allocation (PA) algorithm. Simulation results show that the proposed scheme outperforms the conventional semidefinite relaxation (SDR) technique and provides a performance quite close to the optimal performance with much reduced complexity.

Index Terms—Downlink beamforming, multicast channels, multicasting.

I. INTRODUCTION

Multicasting is one of the important applications in wireless systems, particularly for multimedia services. A traditional approach in the multicasting is radiating transmission power isotropically [1] or using open-loop techniques, such as space–time coding [2], so that all users in a cell can be covered. Unlike broadcasting, which distributes data to all users in a cell, multicasting transmits a common signal only to a group of intended users. Thus, we do not necessarily design a transmission scheme to support all users in a cell. In the meantime, we can utilize channel state information (CSI) for designing the transmit beamformer to improve the system capacity or coverage in closed-loop multiple-input–multiple-output (MIMO) systems [3]–[5].

There have been two major approaches for determining the transmit beamforming vector in MIMO downlink multicasting systems. One is based on finding the minimum transmit power level, which guarantees the transmit-data-rate requirement for all users. The other is maximizing the lowest transmit data rate among users for a given transmit power level. Between these approaches, we focus on the latter case in this paper since the solution of the former problem can be derived by the solution of the latter problem with slight modification [6].

It is known that optimizing a transmit beamformer in the multicast scenario is an NP-hard problem [6]. Therefore, various suboptimal precoding techniques have been studied in the literature [6]–[8]. To solve the optimization problem, Sidiropoulos *et al.* [6] relaxed the problem by employing semidefinite relaxation (SDR) techniques. Another method was proposed in [7], which slowly changes the transmit beamforming vector so that the worst user data rate can be improved. In addition, a beamforming scheme was introduced in [8], which employs Lagrangian formulation to solve the problem for each

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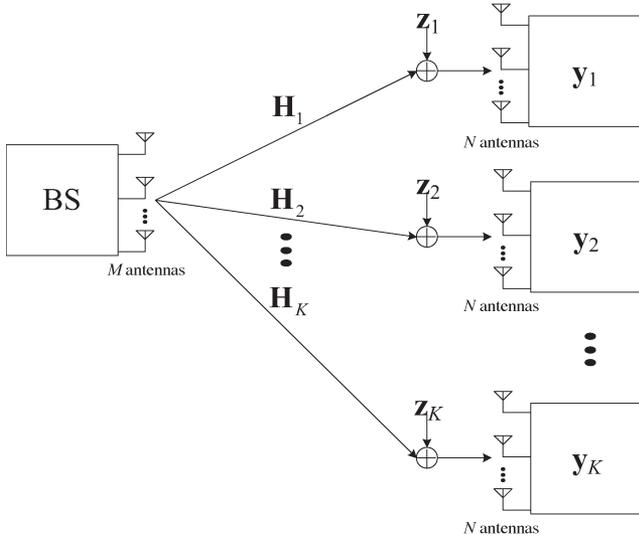


Fig. 1. Wireless multicasting systems with transmit beamforming.

permuted user channel vector. In this case, $K!$ iterations are required to search all possible user permutations, where K is the number of users.

In this paper, we propose a simple algorithm that achieves the near-optimal performance of MIMO multicasting systems. To develop an efficient algorithm, we first consider a beamforming vector, which consists of basis vectors and transmit power, and then present a new algorithm by applying basis vector set selection using a greedy method and power allocation (PA) for the given vector set. It is confirmed from simulations that the proposed scheme can achieve near-optimal performance with much reduced complexity, and it outperforms the conventional SDR scheme.

The following notations are used throughout this paper. We employ uppercase boldface letters for matrices and lowercase boldface for vectors. For any general matrix \mathbf{A} , \mathbf{A}^H and \mathbf{A}^T denote Hermitian and transpose, respectively. In addition, for any complex scalar variable a , $\Re(a)$ represents the real part of a . $\mathbb{C}^{M \times N}$ and $\mathbb{R}_+^{M \times N}$ indicate a set of complex and positive real matrices of size M by N , respectively.

The rest of this paper is organized as follows. In Section II, we introduce MIMO multicast systems and formulate the optimization problem of interest. The proposed scheme is shown in Section III. Section IV presents the performance results of the proposed scheme. Section V concludes this paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a wireless multicasting system that consists of a single base station with M transmit antennas and K users equipped with N receive antennas,¹ as shown in Fig. 1. Under frequency-flat fading channels, the received signal vector \mathbf{y}_k of length N at user k ($k = 1, \dots, K$) is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{w} x + \mathbf{z}_k \quad (1)$$

where $\mathbf{H}_k \in \mathbb{C}^{N \times M}$ stands for the complex channel response matrix from the base station to the k th user, $\mathbf{w} \in \mathbb{C}^{M \times 1}$ represents the transmit beamforming vector with transmit power constraint $\mathbf{w}^H \mathbf{w} \leq P$, x is the transmit data symbol with unit variance, and $\mathbf{z}_k \in \mathbb{C}^{N \times 1}$ indicates the complex additive white Gaussian noise vector at the k th

user with an identity covariance matrix. In this case, the received SNR at the k th user, i.e., g_k ($k = 1, \dots, K$), can be written as

$$g_k = \|\mathbf{H}_k \mathbf{w}\|^2. \quad (2)$$

We assume that perfect CSI for all users are available at the transmitter. The goal of our multicasting problem is maximizing the lowest transmit data rate, i.e., $\max_{\mathbf{w}} \min_{k=1, \dots, K} \log(1 + g_k)$ or, equivalently, maximizing the minimum SNR, which can be formulated as

$$\max_{\mathbf{w}} \min_{k=1, \dots, K} g_k \text{ s.t. } \mathbf{w}^H \mathbf{w} \leq P. \quad (3)$$

Since any complex vector of length M can be represented by a linear combination of M basis vectors, a solution of the problem (3) can be written as

$$\mathbf{w} = \mathbf{V} \mathbf{c} \quad (4)$$

where we have basis matrix $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_M] \in \mathbb{C}^{M \times M}$ and coefficient vector $\mathbf{c} = [c_1, c_2, \dots, c_M]^T \in \mathbb{C}^{M \times 1}$. Here, $\mathbf{v}_i \in \mathbb{C}^{M \times 1}$ denotes the i th basis vector, and c_i represents the complex coefficient of the i th basis vector. Note that any vectors $\{\mathbf{v}_i\}$ can be employed as long as \mathbf{w} spans $\mathbb{C}^{M \times 1}$ space, but to make (3) tractable, we choose orthonormal basis vectors, i.e., $\mathbf{v}_m^H \mathbf{v}_n = \delta_{mn}$. Now, let us define $c_i = \sqrt{P_i} e^{j\theta_i}$, where θ_i and P_i indicate the phase and the transmit power level for the i th basis vector, respectively, and $\sum_{i=1}^M P_i \leq P$ to meet the power constraint in (3).

One method to solve problem (3) is exhaustive search by examining all possible continuous values of $\{P_1, \dots, P_M\}$ and $\{\theta_1, \dots, \theta_M\}$, which is too complicated. Instead, we implement an exhaustive search method by identifying a solution over discrete values. Note that the beamforming vector $e^{j\theta} \mathbf{w}$ produces the same performance as \mathbf{w} for any θ . In addition, all P_i are positive values, and one of the power value is determined automatically to meet the power constraint. Thus, the complexity of the exhaustive search becomes $\mathcal{O}((K/(M-1)!) D_1^{M-1} D_2^{M-1})$, where D_1 and D_2 represent the number of the discrete values within the search range of $\{P_1, \dots, P_M\}$ and $\{\theta_1, \dots, \theta_M\}$, respectively. It is clear that the complexity becomes prohibitive and exponentially increases with respect to M .

III. PROPOSED MULTICAST BEAMFORMING SCHEME

Here, we propose a low-complexity multicast beamforming scheme. We first revisit the optimal beamforming vector in (4). It is easy to show that the solution \mathbf{w} of the optimization problem (3) is spanned by the stacked channel matrix $\mathbf{H} = [\mathbf{h}_{1,1}^H, \dots, \mathbf{h}_{1,N}^H, \dots, \mathbf{h}_{K,1}^H, \dots, \mathbf{h}_{K,N}^H] \in \mathbb{C}^{M \times NK}$, where $\mathbf{h}_{k,l}$ is the l th row vector of the k th user's channel matrix \mathbf{H}_k . This can be explained by contraction as follows. Suppose that there exists a certain vector \mathbf{v}_n orthogonal to \mathbf{H}^H with $c_n \neq 0$. Then, since $\mathbf{v}_n^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{v}_n = 0$ for any k , it follows that $g_k = \|\mathbf{H}_k \mathbf{w}\|^2 = \|\mathbf{H}_k \tilde{\mathbf{w}}\|^2$, where $\tilde{\mathbf{w}} = \sum_{i=1, i \neq n}^M c_i \mathbf{v}_i$. This is equivalent to solving (3) with constraint $\tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P'$, where P' is given as $P' = \sum_{i=1, i \neq n}^M |c_i|^2$. However, the objective function of (3) increases linearly with the maximum power P . Thus, P' should be equal to P to maximize the objective function. In other words, c_n needs to be 0, which contradicts to the assumption. This means that the basis vector of \mathbf{w} corresponding to a nonzero coefficient should be spanned by \mathbf{H} . Since \mathbf{w} is a linear combination of the basis vectors, it is also spanned by \mathbf{H} . Furthermore, the optimum value of (3) is obtained when $\mathbf{w}^H \mathbf{w} = P$.

Therefore, we can rewrite (4) as

$$\mathbf{w} = \tilde{\mathbf{V}} \tilde{\mathbf{c}} \quad (5)$$

where $\tilde{\mathbf{V}} = [\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_R] \in \mathbb{C}^{M \times R}$, and $\tilde{\mathbf{c}} = [\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_R]^T \in \mathbb{C}^{R \times 1}$. Here, $\tilde{\mathbf{c}}^H \tilde{\mathbf{c}} = P$, and R ($R \leq M$) is the rank of the stacked

¹Although we assume that all users are equipped with the same number of receive antennas, all work in this paper can be simply extended to the case of a different number of antennas at each users.

channel matrix \mathbf{H} . Note that the space spanned by $\tilde{\mathbf{V}}$ is the same as that spanned by \mathbf{H} . Since $\tilde{\mathbf{V}}$ consists of orthonormal basis vectors, we can obtain $\tilde{\mathbf{V}}$ by Gram–Schmidt orthogonalization from the column vectors of the stacked channel matrix \mathbf{H} .

Solving problem (3) using the beamforming vector in (5) is still difficult, since there exist too many parameters to optimize. To overcome this issue, we restrict c_i to be real valued, i.e., $\theta_i = 0$. From this relaxation, we can formulate the solution of problem (3) as

$$\mathbf{w} = \tilde{\mathbf{V}} \tilde{\mathbf{p}} \quad (6)$$

where $\tilde{\mathbf{p}} = [\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_R}]^T \in \mathbb{R}_+^{R \times 1}$, and $\tilde{\mathbf{p}}^T \tilde{\mathbf{p}} = P$.

In the following, we describe a method to select the basis vector set of $\tilde{\mathbf{V}}$, and then we present how to determine the PA vector $\tilde{\mathbf{p}}$ for the given basis matrix $\tilde{\mathbf{V}}$.

A. Selection of the Basis Vector Set

As we restrict the coefficient of basis vectors to be real valued in (6), the space spanned by the vectors does not fully cover the whole space of the original structure in (4). In this case, a different ordering of column vectors in \mathbf{H} for Gram–Schmidt orthogonalization produces different subspaces, which affects performance. However, examining all possible orderings of channel vectors requires $R! \binom{N}{R}$ operations, which becomes prohibitive as N , K , or R grows. Thus, we propose a greedy user selection algorithm similar to [9].

Denoting (k, l) as the index of the l th receive antenna at the k th user, we can represent the set of all channel vector indices as $U = \{(1, 1), \dots, (1, N), \dots, (K, 1), \dots, (K, N)\}$. Let us define $\tilde{\mathbf{V}}^{(R)} = [\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_R]$, $\tilde{\mathbf{p}}^{(R)} = [\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_R}]^T$, and $S_R = \{s_1, \dots, s_R\} \subset U$ with $|S_R| = R$ as the basis matrix with rank R , the PA vector of length R , and the set of selected channel vector indices, respectively. Here, $\tilde{\mathbf{V}}^{(R)}$ can be computed by performing the Gram–Schmidt orthogonalization with $[\mathbf{h}_{s_1}^H, \mathbf{h}_{s_2}^H, \dots, \mathbf{h}_{s_R}^H]$.

Now, we explain our algorithm starting with $R = 1$. First, we find the best vector $\mathbf{h}_{k,l}^H / \|\mathbf{h}_{k,l}\|$ among $(k, l) \in U$, which provides the best performance of the objective function of (3). We set $\tilde{\mathbf{V}}^{(1)} = [\tilde{\mathbf{v}}_1]$, and $S_1 = \{s_1\}$, where s_1 is the index corresponding to the best vector, and $\tilde{\mathbf{v}}_1 = \mathbf{h}_{s_1}^H / \|\mathbf{h}_{s_1}\|$. Next, we increase R by 1. Then, all the vectors $\tilde{\mathbf{v}}(k, l)$ are examined, which can generate the best performance of the objective function with $\tilde{\mathbf{V}}_{k,l}^{(R)} \triangleq [\tilde{\mathbf{V}}^{(R-1)} \tilde{\mathbf{v}}(k, l)] = [\tilde{\mathbf{v}}_1, \dots, \tilde{\mathbf{v}}_{R-1}, \tilde{\mathbf{v}}(k, l)]$ and the corresponding PA vector $\tilde{\mathbf{p}}_{k,l}^{(R)}$. Note that $\tilde{\mathbf{v}}(k, l)$ can be calculated using the Gram–Schmidt orthogonalization with $[\tilde{\mathbf{V}}^{(R-1)} \mathbf{h}_{k,l}^H]$, where index (k, l) is determined as $U \setminus S_{R-1}$. Moreover, for the given $\tilde{\mathbf{V}}_{k,l}^{(R)}$, $\tilde{\mathbf{p}}_{k,l}^{(R)}$ can be determined by the PA algorithm for a given vector set which will be described in Section III-B. We repeat this until no additional performance gain is obtained by adding one more basis vector. We summarize the proposed algorithm as follows.

Proposed Scheme

Set $R = 0$, $\tilde{\mathbf{V}}^{(0)} = []$, and $S_0 = \emptyset$.

While $R < M$

Update $R \leftarrow R + 1$.

$s_R = \arg \max_{(k,l) \in U \setminus S_{R-1}} \min_{m=1, \dots, K} \|\mathbf{H}_m \tilde{\mathbf{V}}_{k,l}^{(R)} \tilde{\mathbf{p}}_{k,l}^{(R)}\|^2$.

Set $\tilde{\mathbf{V}}^{(R)} = [\tilde{\mathbf{V}}^{(R-1)} \mathbf{v}(s_R)]$ and $S_R = S_{R-1} \cup \{s_R\}$.

If $\min_m \|\mathbf{H}_m \tilde{\mathbf{V}}^{(R)} \tilde{\mathbf{p}}^{(R)}\|^2 \leq \min_m \|\mathbf{H}_m \tilde{\mathbf{V}}^{(R-1)} \tilde{\mathbf{p}}^{(R-1)}\|^2$

Set $R \leftarrow R - 1$ and break.

End

End

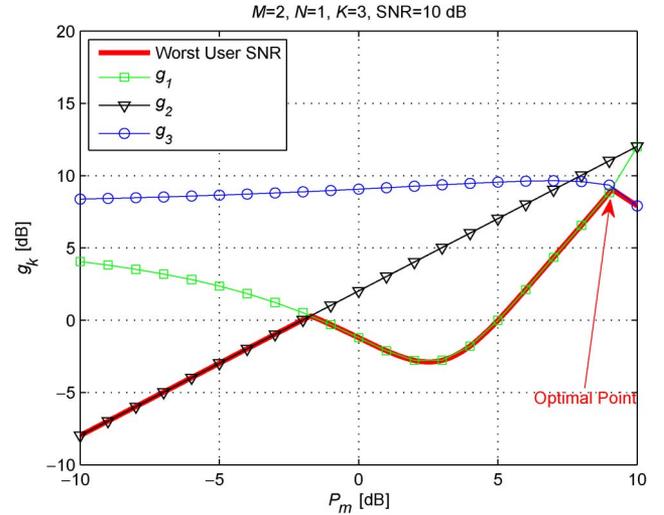


Fig. 2. Exemplary plot of g_k with respect to P_m .

B. PA Algorithm for a Given Vector Set

Based on the beamforming vector in (6), we can rewrite SNR at the k th user, i.e., g_k , as

$$\begin{aligned} g_k &= \|\mathbf{H}_k \tilde{\mathbf{V}} \tilde{\mathbf{p}}\|^2 \\ &= \|\mathbf{A}_k \tilde{\mathbf{p}}\|^2 \end{aligned} \quad (7)$$

where $\mathbf{A}_k = \mathbf{H}_k \tilde{\mathbf{V}} = [\mathbf{a}_{k,1}, \dots, \mathbf{a}_{k,R}] \in \mathbb{C}^{N \times R}$ and $\mathbf{a}_{k,i} = \mathbf{H}_k \mathbf{v}_i \in \mathbb{C}^{N \times 1}$. To find the optimal PA, we first check the crossover points $\tilde{\mathbf{p}}_c$, $\tilde{\mathbf{p}}_c \in \mathbb{R}_+^{R \times 1}$, which satisfy $g_k(\tilde{\mathbf{p}}_c) = g_l(\tilde{\mathbf{p}}_c)$ for $l \neq k$. The crossover points can be obtained by solving

$$\tilde{\mathbf{p}}_c^T \mathbf{B}_{kl} \tilde{\mathbf{p}}_c = 0 \text{ s.t. } \tilde{\mathbf{p}}_c^T \tilde{\mathbf{p}}_c = P$$

where $\mathbf{B}_{kl} = \mathbf{A}_k^H \mathbf{A}_k - \mathbf{A}_l^H \mathbf{A}_l$. Unfortunately, it is difficult to solve this quadratic equation in general. Instead, we employ an alternating optimization method that computes a local optimal solution. In this algorithm, we recursively determine a power level pair (P_m, P_n) ($m \neq n$) at a time.

To illustrate the key idea of our proposed PA method, we describe how to optimize P_m and P_n for fixed P_i ($i \neq m, n$). Let us denote \mathbf{p}_{mn} as a vector that is equal to $\tilde{\mathbf{p}}$, except that the m th and the n th elements are zero. Then, using the power constraint $\tilde{\mathbf{p}}^T \tilde{\mathbf{p}} = P$, P_n can be expressed as $P_n = P'_{mn} - P_m$, where $P'_{mn} = P - \mathbf{p}_{mn}^T \mathbf{p}_{mn}$. In this case, we can rewrite (7) as a function of P_m as

$$\begin{aligned} g_k(P_m) &= (\mathbf{a}_{k,m}^H \mathbf{a}_{k,m} - \mathbf{a}_{k,n}^H \mathbf{a}_{k,n}) P_m \\ &\quad + 2\Re(\mathbf{a}_{k,m}^H \mathbf{a}_{k,n}) \sqrt{P_m(P'_{mn} - P_m)} \\ &\quad + 2\Re(\mathbf{p}_{mn}^T \mathbf{A}_k^H \mathbf{a}_{k,m}) \sqrt{P_m} \\ &\quad + 2\Re(\mathbf{p}_{mn}^T \mathbf{A}_k^H \mathbf{a}_{k,n}) \sqrt{P'_{mn} - P_m} + C_k \end{aligned} \quad (8)$$

for $0 \leq P_m \leq P'_{mn}$, where C_k is a constant term independent of P_m . Fig. 2 depicts an exemplary plot of g_k with respect to P_m . As shown in Fig. 2, the worst user SNR for a given P_m is changed only after the crossover point, and clearly, a local optimal point is one of the boundary points ($P_m = 0$ or P'_{mn}), the crossover points $P_{m,c}$, where $g_k(P_{m,c}) = g_j(P_{m,c})$, or the inflection points $P_{m,i}$, where $g'_k(P_{m,i}) = 0$, with $g''_k(P_{m,i}) < 0$. From this observation, we can come up with the following algorithm.

First, for $R = 1$, it is clear that $\tilde{\mathbf{p}} = \sqrt{P}$. In the case of $R = 2$, the optimal PA is calculated as follows. We start from the minimum point

of P_m , i.e., $P_m = 0$. Let us assume that the range of P_m is $P_{m, \min} \leq P_m \leq P_{m, \max}$. Then, suppose that the n^* -th user has the minimum SNR $g_{n^*}(P_m)$ among K users at $P_m = P_{m, \min}$, and $P_{m, c}$ is the minimum crossover point between user n^* and j^* ($j^* \neq n^*$), which satisfies $g_{n^*}(P_{m, c}) = g_{j^*}(P_{m, c})$ and $P_{m, \min} \leq P_{m, c} \leq P_{m, \max}$. In this case, it is clear that $g_{n^*}(P_m)$ is the minimum among SNR of K users for $P_{m, \min} \leq P_m \leq P_{m, c}$. Among these minimum points, the maximum $g_{n^*}(P_m)$ can be obtained at either the boundary points ($P_m = P_{m, \min}$ or $P_{m, c}$) or the inflection points with $g''_{n^*}(P_{m, i}) < 0$. If $g'_{n^*}(P_{m, c}) > g'_{j^*}(P_{m, c})$, then the j^* -th user's SNR becomes the minimum at $P_{m, c}$ since $g_{j^*}(P_m) < g_{n^*}(P_m)$ for $P_{m, c} < P_m$. If $g'_{n^*}(P_{m, c}) < g'_{j^*}(P_{m, c})$, then the n^* -th user still has the minimum SNR at $P_{m, c}$. With the same logic, we repeat the given procedure until there is no crossover point between $P_{m, \min}$ and $P_{m, \max}$. Note that the crossover point or the inflection point is computed by solving a polynomial equation of at most fourth order, which has a closed-form solution.

To address the problem for $R > 2$, let us denote \mathcal{S} as a set of all possible combinations of (m, n) (m and $n = 1, \dots, R$). Then, we can find the PA vector as in the following example. In case of $R = 3$, we have $\mathcal{S} = \{(1, 2), (2, 3), (1, 3)\}$, and we first identify the optimal P_1 and P_2 for given P_3 using the algorithm described earlier for $R = 2$. Then, P_2 and P_3 are calculated for fixed P_1 , and P_1 and P_3 are computed for fixed P_2 . Since the formulated problem is nonconvex, the proposed alternating PA algorithm results in a local optimal point. To improve the local optimal point, we can run the algorithm with multiple random initial points $\tilde{\mathbf{p}}$ and choose the best point among solutions generated by these initial points. As will be shown in the simulation section, we only need about three initial points to achieve the near-optimal performance. Note that we need to run the algorithm for $R = 2|\mathcal{S}|$ times for each initial point. Denoting P_m^* and $f(P_m^*)$ are the optimal power level of P_m for a given condition and the corresponding minimum SNR among K users, we summarize the proposed PA algorithm for the given basis matrix $\tilde{\mathbf{V}}$ as follows.

PA Algorithm

```

Set max = 0 and initialize  $\tilde{\mathbf{p}}$  with a random vector.
For  $\mathcal{S} = \{(1, 2), (2, 3), \dots\}$ 
    Find  $(P_m^*, P_n^*)$ .
    If  $\max < f(P_m^*)$ 
        Update  $P_m = P_m^*, P_n = P_n^*$ , and  $\max \leftarrow f(P_m^*)$ .
    End
End
    
```

Since the PA algorithm searches for at most $\sum_{k=1}^{K-1} (K-k)$ crossover points and K inflection points for each stage, the complexity of the PA algorithm becomes less than $\mathcal{O}((N_i/4)(K^2 + K)(R^2 - R))$ for $R > 1$, where N_i is the number of initial points. In addition, the vector set selection algorithm finds $(NK - R + 1)$ indices for a given R . Thus, the complexity of the proposed scheme in the worst case is bounded by $\mathcal{O}((N_i/12)NK^3M^3)$. Simulations show that, with $M = 3$, $K = 10$, and SNR = 10 dB, the computational complexity of our scheme is five orders of magnitude lower than that of the near-optimal solution, which is determined with a 0.1-dB resolution and a 0.1-rad resolution.

IV. SIMULATION RESULTS

Here, we demonstrate the efficiency of our proposed algorithm compared with the conventional SDR scheme in [6] and the near-optimal solution introduced in Section II through Monte Carlo simulations. In our simulations, it is assumed that the channel coefficients

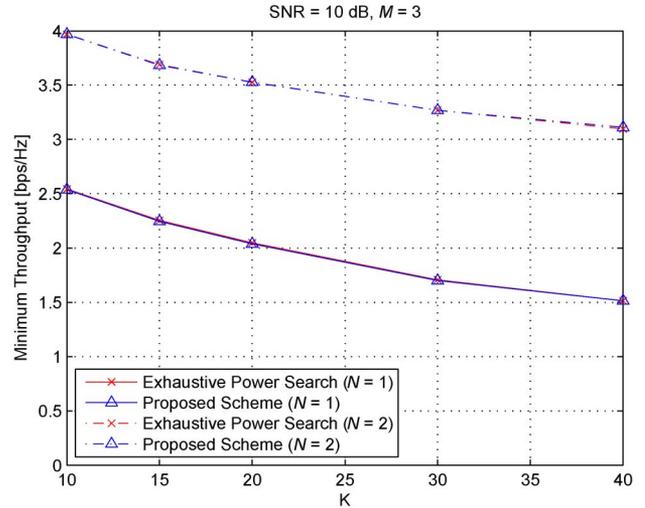


Fig. 3. Performance of the PAGBM algorithm.

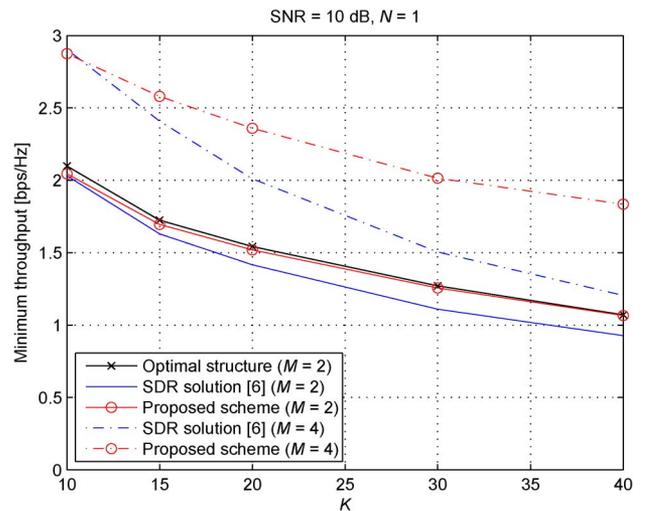


Fig. 4. Performance comparison for $N = 1$

are sampled from independent and identically distributed complex Gaussian random variables with zero mean and unit variance.

First of all, we evaluate the performance of the PA algorithm in Fig. 3, which plots the minimum user throughput performance with $N_i = 3$. We assume that all users are equipped with two receive antennas ($N = 2$), whereas a BS has three transmit antennas ($M = 3$). In addition, we include the performance of an exhaustive power search method that serves as an upper bound to validate our algorithm. The exhaustive search method searches for all possible power level combinations with a 0.1-dB resolution. Here, we compare both methods with the vector set selection proposed in Section III-A. From the plot, we confirm that the PA algorithm with three initial points provides similar performance to that of the exhaustive scheme with much reduced complexity.

In Figs. 4 and 5, we illustrate the average worst user rate of the proposed scheme and the conventional SDR scheme for the case of $N = 1$ and 2, respectively. We assume that two or four transmit antennas are equipped at the BS. For $M = 4$, we run the PA algorithm with three initial points. Note that we do not need random initial points in the case of $M = 2$ since $|\mathcal{S}|$ is always 1 in this case. We also include the performance of the optimal solution (4), which is obtained with a 0.1-dB resolution for the power level and a 0.1-rad resolution for

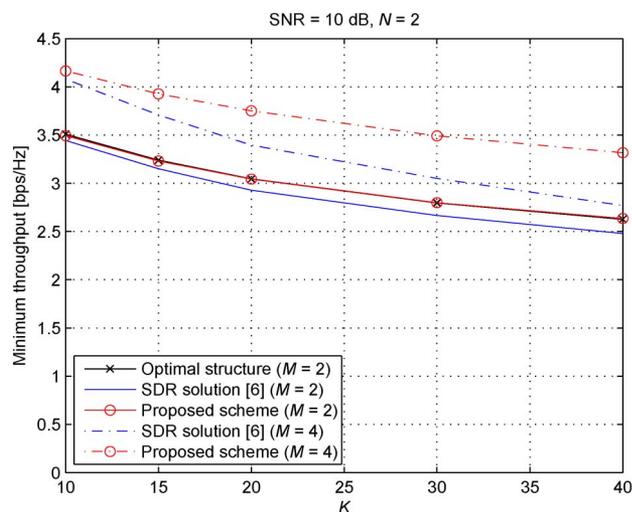


Fig. 5. Performance comparison for $N = 2$

the phase. Here, the optimal solution for $M = 4$ case is not included due to its extremely high complexity. One interesting point is that, as K grows, our proposed scheme approaches the performance of the optimal solution, whereas the performance of the conventional scheme degrades. This is due to the fact that the subspace covered by the vector set selection increases as K grows, whereas the performance of the conventional scheme deteriorates when K is large [10]. In addition, in the case of $N = 1$, about 15% and 52% gains at $K = 40$ can be obtained over the conventional scheme for $M = 2$ and 4, respectively. In addition, about 6% and 20% gains are observed in the case of $N = 2$ for $M = 2$ and 4, respectively. Clearly, the gain of the proposed scheme over the conventional scheme increases as M increases.

V. CONCLUSION

In this paper, we have studied a transmit beamforming technique for MIMO downlink multicast systems. We have focused on maximizing the lowest SNR among intended users, assuming that CSI is available at a transmitter. To solve the problem, we proposed a new beamforming method that reduces the complexity of the system. Simulation results show that the proposed scheme outperforms the conventional SDR-based scheme, and achieves near-optimal performance with much reduced complexity.

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Maximum-Likelihood Detector for Differential Amplify-and-Forward Cooperative Networks

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Abstract—The exact maximum-likelihood (ML) detector for amplify-and-forward (AF) cooperative networks employing M -ary differential phase-shift keying (DPSK) in Rayleigh fading is derived in a single-integral form, which serves as a benchmark for differential AF networks. Two algorithms are then developed to reduce the complexity of the ML detector. Specifically, the first algorithm can eliminate a number of candidates in the ML search, while causing no loss of optimality of ML detection. In high signal-to-noise ratios (SNRs), this algorithm almost surely identifies a *single* candidate that amounts to the ML estimate of the signal. For low to medium SNRs with multiple candidates determined, we then derive an accurate closed-form approximation for the integral involved in the likelihood function, which only requires a five-sample evaluation per symbol candidate. Finally, combining these algorithms, we propose a *closed-form* approximate ML detector, which achieves an almost identical bit-error-rate (BER) performance to the exact ML detector at practical complexity. In particular, it is shown that the proposed approximate ML detector is far less complex than the well-known diversity combiner in high SNRs, while achieving approximately 1.7-dB gain in the 10^{-5} BER when the relay is closer to the destination.

Index Terms—Amplify-and-forward (AF), cooperative networks, differential phase-shift keying (DPSK), maximum-likelihood (ML).

I. INTRODUCTION

Noncoherent signalings lend simplicity to receiver designs by eliminating the need for instantaneous channel state information (CSI), thus constituting an attractive approach particularly for fast-fading channels [1], [2]. Recently, *noncoherent* modulations have been studied for amplify-and-forward (AF) cooperative networks. In particular, Annavaajala *et al.* derived a maximum-likelihood (ML) detector for *binary* noncoherent frequency-shift keying (NCFSK) in Rayleigh fading [3]. However, the ML detector in [3] involves integrals that have no closed-form solutions and, hence, are very complex for implementation. In addition, suboptimum detectors with lower complexity but degraded performance have been developed for NCFSK [3]–[5]. Since FSK modulation is not a bandwidth-efficient noncoherent solution, it might not be suitable for bandwidth-limited applications. For such

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