Beamforming and Power Allocation Designs for Energy Efficiency Maximization in MISO Distributed Antenna Systems

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***Abstract*—In this paper, we present a beamforming and power allocation algorithm for a downlink multiple-input single-output distributed antenna system which maximizes energy efficiency (EE). To reduce the computational complexity of conventional joint optimization approaches relying on an iterative method, we propose a near optimal scheme based on a closed-form solution. Employing the decomposition property of the joint optimization problem, the EE problem is solved in two steps. First, we determine the beamforming strategy for the EE maximization exposing the structure of beamforming vectors. Then, the optimal power allocation is presented as a closed-form solution by solving Karush-Kuhn-Tucker conditions. Through numerical simulations, we confirm that the proposed solution shows the performance almost identical to the jointly optimum method with much reduced complexity.**

***Index Terms*—Energy efficiency, distributed antenna systems.**

I. INTRODUCTION

RECENTLY, green communication, which pursues high energy efficiency, has drawn increasing attentions for future wireless communications designs. Energy efficiency (EE) is defined as the ratio of the sum-rate to the total power consumption measured in bit/Hz/Joule, and various energy efficient methods have been proposed for orthogonal frequency division multiple access [1] and multiple input multiple output systems [2]. Meanwhile, distributed antenna systems (DAS) have received considerable interests as a key technique to meet the increasing needs of spectral efficiency (SE) and the expanded coverage [3]. Unlike conventional antenna systems (CAS) with co-located antennas [3], the DAS have distributed antenna (DA) ports throughout a cell which are geographically separated and physically connected with each other by dedicated channels. Thus, the DAS exhibits benefits on power savings and the enhanced system capacity which results from the reduced transmit power and co-channel interference.

Lately, several efforts have been devoted to examine the DAS such as a design of antenna locations [4] and the SE analysis [5] [6]. Especially in [6], the ergodic capacity and the optimal beamforming were studied where each DA port has multiple antennas with per-DA port power constraint. From an EE point of view, however, most research has focused on the DAS where DA ports and users are equipped with a single antenna, and treated power allocation schemes in [7] and [8].

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This paper extends the work in [8] to a more general DAS where each DA port has multiple antennas by considering both the beamforming and the power control strategies. To the best of authors' knowledge, there is no reported work on a closed form expression of the beamforming and the power control design for the EE maximization in multiple-input single-output (MISO) DAS. Unlike conventional optimal design approach utilizing an iterative method [1], we propose a near optimal closed-form solution which provides an useful insight. To this end, we utilize the decomposition property of the joint optimization problem, and thereby the EE problem is solved in two steps. First we determine the beamforming design revealing a structure of the beamforming vectors. Then, we derive a closed-form solution for the EE maximizing power allocation problem by solving Karush-Kuhn-Tucker (KKT) conditions and present a simple power allocation scheme based on the obtained expression. Although our scheme does not ensure the optimality, numerical results demonstrate that we can achieve the performance very close to the jointly optimal solution with much reduced complexity.

Throughout this paper, we adopt lowercase boldface for vectors. The superscripts $(\cdot)^H$ and $\|\cdot\|$ stand for conjugate transpose and Euclidean 2-norm of a vector, respectively.

II. SYSTEM MODEL

We consider a downlink single cell DAS where N DA ports with M_i antennas $(i = 1, \ldots, N)$ support a single antenna user. We assume that all DA ports are physically connected with each other via fiber links. Moreover, it is assumed that all DA ports and the user know channel state information perfectly, and power allocation is centrally controlled. Then, the received signal for the user is written as

$$y = \sum_{i=1}^{N} \sqrt{p_i}\mathbf{g}_i^H \mathbf{w}_i s + z$$

where p_i is the transmit power consumed by the i-th DA port, $\mathbf{w}_i$ is defined as the beamforming column vector of length M_i for the i-th DA port with unit norm ($\|\mathbf{w}_i\|^2 = 1$), s stands for the transmitted signal with zero mean and unit variance, and z equals the additive white Gaussian noise with zero mean and variance σ_n^2. Here, we represent $\mathbf{g}_i = d_i^{-\frac{\alpha}{2}}\mathbf{h}_i$ as the $M_i \times 1$ channel vector between the i-th DA port and the user where $d_i^{-\frac{\alpha}{2}}$ denotes the propagation pathloss with the pathloss exponent α due to the distance d_i between the i-th DA port and the user, and $\mathbf{h}_i$ indicates the channel column vector for small scale fadings.

As a result, the achievable rate for the user is given as

$$R = \log_2\left(1 + \frac{\left|\sum_{i=1}^{N} \sqrt{p_i}\mathbf{g}_i^H \mathbf{w}_i\right|^2}{\sigma_n^2}\right).$$

It is assumed that per-DA port power constraint is applied as $p_i \leq P_{max}$, where P_{max} is the maximum transmit power available at each DA port. Then, the EE for DAS with N DA ports is defined as $\eta_{EE} = \dfrac{R}{\sum_{i=1}^{N} p_i + p_c}$ where p_c indicates the circuit power. The total power consumption for DAS includes the power consumption of power amplifiers at each DA port p_i for $i = 1, \ldots, N$, and that of all other circuit blocks p_c, which encompasses the power consumed by digital signal processors, frequency synthesizers, mixers, etc. Here, we assume that p_c is a fixed constant.

III. Joint Beamforming and Power Allocation for EE Maximization

In this section, we first formulate joint beamforming and power allocation for EE maximization in DAS with per-DA port power constraint. The EE maximizing beamforming and power allocation problem is expressed as

$$\max_{p_i, \mathbf{w}_i, \forall i} \quad \frac{\ln\left(1 + \frac{1}{\sigma_n^2}\left|\sum_{i=1}^{N} \sqrt{p_i}\mathbf{g}_i^H \mathbf{w}_i\right|^2\right)}{\sum_{i=1}^{N} p_i + p_c} \tag{1}$$
$$\text{subject to} \quad p_i \leq P_{max}, \;\; \|\mathbf{w}_i\|^2 = 1 \;\; \text{for } i = 1, \cdots, N.$$

In general, it is very difficult to obtain an explicit solution for this joint optimization problem since it is strictly non-convex. Instead, most previous works have adopted an alternating optimization method which iteratively identifies a local optimum solution. Although such methods may successfully maximize the EE, they result in high implementation complexity and hardly provide helpful insights. In order to derive an efficient optimization algorithm, we divide the joint optimization problem (1) into two individual optimization problems: beamforming and power allocation. The details will be given in the following subsections.

A. Beamforming Design

First, we introduce useful lemmas below to design the beamforming vectors $\mathbf{w}_i$'s.

Lemma 1: For given p_i's, the optimal beamforming solution for the EE maximization is identical to that of the SE maximization.

Proof: With fixed p_i's, the denominator of the EE metric in (1) can be treated as a constant, which means that the EE maximization problem is equivalent to the SE maximization with respect to $\mathbf{w}_i$'s. ■

Lemma 2: For a single user MISO DAS, the optimal beamforming strategy for the SE maximization is distributed maximum ratio transmission (D-MRT) with any given p_i's which is expressed as $\mathbf{w}_i^\star = \frac{\mathbf{g}_i}{\|\mathbf{g}_i\|}$ for $i = 1, \cdots, N$.

Proof: See [6]. ■

Different from conventional iterative algorithms which optimize p_i's and $\mathbf{w}_i$'s alternately after fixing each parameter [9], in our case, $\mathbf{w}_i$'s are set to $\frac{\mathbf{g}_i}{\|\mathbf{g}_i\|}$ regardless of the value of p_i's for $i = 1, \cdots, N$, and thus we do not need to update $\mathbf{w}_i$'s because of Lemmas 1 and 2. Motivated by these observations, the beamforming $\mathbf{w}_i$ is determined as the D-MRT for the EE maximization of MISO DAS. As will be shown later, although this beamforming strategy may be suboptimum, almost the same performance is achieved compared to the conventional optimal approach.

B. Power Allocation

To investigate the efficient optimal power allocation method with the beamforming vectors $\mathbf{w}_i = \frac{\mathbf{g}_i}{\|\mathbf{g}_i\|}$ for all i, we reformulate the problem (1) as

$$\max_{p_i, \forall i} \quad g(\{p_i\}) \tag{2}$$
$$\text{subject to} \quad p_i \leq P_{max} \qquad \text{for } i = 1, \cdots, N$$

where $g(\{p_i\})$ is defined as $\dfrac{\ln\left(1 + \frac{1}{\sigma_n^2}\left(\sum_{i=1}^{N} \sqrt{p_i}\gamma_i\right)^2\right)}{\sum_{i=1}^{N} p_i + p_c}$, and γ_i equals $d_i^{-\frac{\alpha}{2}}\|\mathbf{h}_i\|$.

Without loss of generality, we assume that all γ_i's are sorted in descending order as $\gamma_1 > \gamma_2 > \cdots > \gamma_N$. Then, the Lagrangian function for the EE maximizing power allocation problem is written by

$$L(\{p_i, \lambda_i, \nu_i\}) = \frac{\ln\left(1 + \frac{1}{\sigma_n^2}\left(\sum_{i=1}^{N} \sqrt{p_i}\gamma_i\right)^2\right)}{\sum_{i=1}^{N} p_i + p_c} + \sum_{j=1}^{N} \lambda_j p_j + \sum_{j=1}^{N} \nu_j (P_{max} - p_j),$$

where λ_i and ν_i are the Lagrange multipliers chosen to meet the conditions of $p_i \geq 0$ and $p_i \leq P_{max}$ for $i = 1, \ldots, N$.

According to the KKT conditions [10], the optimal values $\{p_i^\star, \lambda_i^\star, \nu_i^\star\}$ $(i = 1, \ldots, N)$ should satisfy the following equations:

$$\frac{\partial L}{\partial p_i} = f_i(p_1^\star, \ldots, p_N^\star) + \lambda_i^\star - \nu_i^\star = 0 \tag{3}$$
$$0 \leq p_i^\star \leq P_{max}, \;\; \lambda_i^\star \geq 0, \;\; \nu_i^\star \geq 0$$
$$\lambda_i^\star p_i^\star = \nu_i^\star (P_{max} - p_i^\star) = 0 \;\; \text{for } i = 1, \cdots, N \tag{4}$$

where

$$f_i(p_1^\star, \ldots, p_N^\star) = \frac{1}{(\sum_{j=1}^{N} p_j^\star + p_c)^2} \cdot \left(\frac{\gamma_i (\sum_{j=1}^{N} \sqrt{p_j^\star}\gamma_j)(\sum_{j=1}^{N} p_j^\star + p_c)}{\sqrt{p_i^\star}\left(\sigma_n^2 + (\sum_{j=1}^{N} \sqrt{p_j^\star}\gamma_j)^2\right)} - \ln\left(1 + \frac{(\sum_{j=1}^{N} \sqrt{p_j^\star}\gamma_j)^2}{\sigma_n^2}\right)\right). \tag{5}$$

For simplicity, we drop the arguments $p_i^\star$'s from $f_i(p_1^\star, \ldots, p_N^\star)$ from now on.

Next, we derive a power allocation solution which maximizes the EE for DAS with D-MRT. Based on the complementary slackness condition in (4), a possible set of solutions for the power p_i of the i-th DA port can be divided into three mutually exclusive cases as

$$(p_i^\star, \lambda_i^\star, \nu_1^\star) \in \left\{(0, \lambda_i^\star, 0), (x_i^\star, 0, 0)\,|_{0 < x_i^\star < P_{max}}, (P_{max}, 0, \nu_i^\star)\right\}. \tag{6}$$

Before further derivations, we first introduce the following Lemmas.

Lemma 3: The optimal power of the i-th DA port $p_i^\star$ is always non-zero.

Proof: In order to have $p_i^\star = 0$, we need $\nu_i = 0$ from (4), and it follows $f_i = -\lambda_i^\star + \nu_i^\star \leq 0$ for $i = 1, \ldots, N$. With $p_i^\star = 0$, however, f_i in (5) becomes infinity, which contradicts the KKT conditions. Thus, $p_i^\star > 0$ should be satisfied for $\forall i$. ■

Lemma 4: For any i and j, if the optimal power of the i-th DA port $p_i^\star$ is less than P_{max}, the power for the j-th DA ports having worse SNR than the i-th DA port is determined as $p_j^\star = \frac{\gamma_j^2}{\gamma_i^2} p_i^\star$ for $j > i$.

Proof: From (3) and (4), it is clear that $f_j = 0$ if $p_j^\star < P_{max}$, or $f_j \geq 0$ otherwise. So, to have $p_j^\star = P_{max}$ for $j > i$ when $p_i^\star$ is less than P_{max}, $\sqrt{\frac{p_j^\star}{p_i^\star}} \leq \frac{\gamma_j}{\gamma_i}$ should be fulfilled since f_k's are proportional to $\frac{\gamma_k}{\sqrt{p_k^\star}}$ based on (5). However, for $j > i$, this condition cannot be satisfied since we have $\gamma_i > \gamma_j$ and thus f_j should be zero, which is equivalent to $p_j^\star < P_{max}$. Then, combining (5) and the relation $f_i = f_{i+1} = \cdots = f_N = 0$, it follows $p_j^\star = \frac{\gamma_j^2}{\gamma_i^2} p_i^\star$ for $j > i$ when $p_i^\star < P_{max}$. ■

Let us consider the power p_1 of the first DA port which has the largest γ_i. For the first case $(p_1^\star, \lambda_1^\star, \nu_i^\star) = (0, \lambda_i^\star, 0)$, from Lemma 3, we can figure out that the optimal solution for the first case with $p_1^\star = 0$ never occurs. For the same reason, the case having $p_i^\star = 0$ cannot be a solution for the EE maximizing power allocation for MISO DAS.

Next, we examine the case $(p_1^\star, \lambda_1^\star, \nu_i^\star) = (x_1^\star, 0, 0)\,|_{0 < x_1^\star < P_{max}}$. To obtain a closed-form solution $\{x_i^\star\}$ based on the zero-gradient conditions, we insert $x_j^\star = \frac{\gamma_j^2}{\gamma_1^2} x_1^\star$ $(j = 2, 3, \cdots, N)$ into f_1 in (5) based on Lemma 4. After some mathematical manipulations, a closed-form expression for the optimal power of DA ports is computed as

$$x_i^\star = \frac{\gamma_i^2}{\gamma_1^4} \cdot \frac{\exp\left\{\omega\left(\frac{1}{e\sigma_n^2}\left(\gamma_1^2 p_c\left(1 + \frac{\gamma_2^2}{\gamma_1^2}\right) - \sigma_n^2\right)\right) + 1 + \ln \sigma_n^2\right\} - \sigma_n^2}{\left(1 + \frac{\gamma_2^2}{\gamma_1^2}\right)^2} \quad (7)$$

where $\omega(\cdot)$ denotes the principal branch of the Lambert ω function defined as the inverse function of $f(x) = xe^x$.

Here, it should be noted that if $x_1^\star$ computed in (7) exceeds P_{max}, $p_1^\star$ should be set to P_{max}, and we need to further investigate the optimal value of $p_2^\star$ according to λ_2 and ν_2 after fixing $p_1^\star = P_{max}$, since the power for the rest of DA ports has not been decided. Now, we know that the optimal power for the second DA port becomes either $x_2^\star$ or P_{max} for $0 < x_2^\star < P_{max}$, and we get $p_2^\star$ with $p_1^\star = P_{max}$. For the case $(p_2^\star, \lambda_2^\star, \nu_2^\star) = (x_2^\star, 0, 0)\,|_{0 < x_2^\star < P_{max}}$, we have $f_1 = -\lambda_1^\star + \nu_1^\star > 0$ and $f_2 = -\lambda_2^\star + \nu_2^\star = 0$, and the power of the rest DA ports are determined as $p_j^\star = \frac{\gamma_j^2}{\gamma_2^2} p_2^\star$ for $j = 3, 4, \cdots, N$ from Lemma 4. Also, combining the relation $f_1 > f_2$ and (5) yields $\frac{\gamma_1}{\sqrt{P_{max}}} > \frac{\gamma_2}{\sqrt{x_2^\star}}$, and thus $x_2^\star$ is lower bounded by $\frac{\gamma_2^2}{\gamma_1^2} P_{max}$. Since it is very hard to compute a closed-form solution $x_2^\star$ based on the zero-gradient condition by equating f_2 to zero with $p_1^\star = P_{max}$, we acquire $x_2^\star$ by examining the characteristics of the objective function $g(P_{max}, x_2, \cdots, x_N)$ where $x_j = \frac{\gamma_j^2}{\gamma_2^2} x_2$ for $j = 2, 3, \cdots, N$.

For simple explanations, with given i_0 and x, we define $\mathcal{P}(i_0, x)$ as the set for the power of N DA ports where the j-th element is set to $\frac{\gamma_j^2}{\gamma_{i_0}^2} x$ if $j \geq i_0$, or P_{max} otherwise for $j = 1, 2, \cdots, N$. Then, $g(P_{max}, x_2, \cdots, x_N)$ with $x_j = \frac{\gamma_j^2}{\gamma_2^2} x_2$ $(j = 2, 3, \cdots, N)$ can be simply expressed as $g(\mathcal{P}(2, x_2))$. For $i = 2, 3, \cdots, N$, since the objective function $g(\mathcal{P}(i, x_i))$ is strictly pseudo-concave as well as quasi-concave with respect to x_i, the following properties hold.

- Property 1: $x_i^\star$ which satisfies $f_i(\mathcal{P}(i, x_i^\star)) = 0$ is a unique globally optimal point, if a feasible $x_i^\star$ exists.

TABLE I
SIMULATION PARAMETERS

Noise power σ_n^2	-104 dBm
Cell radius R	1000 m
Pathloss exponent α	4
User distribution	Uniform
DA port deployment	Circular layout
Circuit power p_c	1 W
Number of channel realizations	3000

- Property 2: $g(\mathcal{P}(i, x_i^\star))$ monotonically increases with respect to x_i with $f_i(\mathcal{P}(i, x_i^\star)) > 0$ for $x_i < x_i^\star$, while it monotonically decreases with $f_i(\mathcal{P}(i, x_i^\star)) < 0$ for $x_i > x_i^\star$.

Based on Property 2, if a solution based on the zero-gradient condition $x_2^\star$ exists, $f_2(\mathcal{P}(2, \frac{\gamma_2^2}{\gamma_1^2} P_{max})) \cdot f_2(\mathcal{P}(2, P_{max}))$ is negative due to the pseudo-concavity of $g(\mathcal{P}(2, x_2^\star))$, and for this case, $x_2^\star$ on the feasible open set $\left(\frac{\gamma_2^2}{\gamma_1^2} P_{max},\, P_{max}\right)$ can be found efficiently using a bisection method. Otherwise, when $f_2(\mathcal{P}(2, \frac{\gamma_2^2}{\gamma_1^2} P_{max})) \cdot f_2(\mathcal{P}(2, P_{max})) > 0$, $g(\mathcal{P}(2, x_2^\star))$ monotonically increases in the feasible region of x_2, and thus the optimal power allocation is determined as $p_1^\star = p_2^\star = P_{max}$ while $p_i^\star$ has not been determined yet for $i = 3, 4, \cdots, N$. In a similar way, we can obtain solutions for the remaining DA ports and the overall algorithm is summarized in Algorithm 1.

Algorithm 1 Optimal power allocation method with D-MRT

Assume $\gamma_1 > \gamma_2 > \cdots > \gamma_N$.
Initialize $i = 1$ and compute $(x_1^\star, x_2^\star, \cdots, p_N^\star)$ using (7).
If $x_1^\star < P_{max}$
    $(p_1^\star, p_2^\star, \cdots, p_N^\star) = (x_1^\star, x_2^\star, \cdots, x_N^\star)$
else
    Set $p_1^\star = P_{max}$.
    While $(p_i^* = P_{max,i}$ and $i \leq N)$
        Set $i = i + 1$
        If $f_i(\mathcal{P}(i, \frac{\gamma_i^2}{\gamma_{i-1}^2} P_{max})) \cdot f_i(\mathcal{P}(i, P_{max})) > 0$
            Set $p_i^\star = P_{max}$.
        else
            Obtain $x_i^\star$ using a bisection method and set $(p_1^\star, p_2^\star, \cdots, p_N^\star) = (\mathcal{P}(i, x_i^\star))$.
        end
    end
end

Note that this simple algorithm is applicable to DAS regardless of the number of DA ports N and the number of antennas per DA port M_i. It should be emphasized that the derived solution becomes globally optimal from a power allocation perspective with a given D-MRT beamforming. Although the joint optimality is not guaranteed with respect to p_i's and $\mathbf{w}_i$'s, numerical results in Section IV demonstrate that the proposed method shows negligible performance loss compared to the optimum solution obtained by the conventional iterative scheme in [1].

IV. Simulation Results

In this section, we present the performance of the proposed beamforming and power control method through Monte Carlo simulations. The system parameters used in the simulations are listed in Table I. For conveniences, we assume $M_i = M$ for $\forall i$ throughout the simulations. For the DAS with N DA ports, the j-th DA port is located at $\left(r \cos \frac{2\pi(j-1)}{N}, r \sin \frac{2\pi(j-1)}{N}\right)$

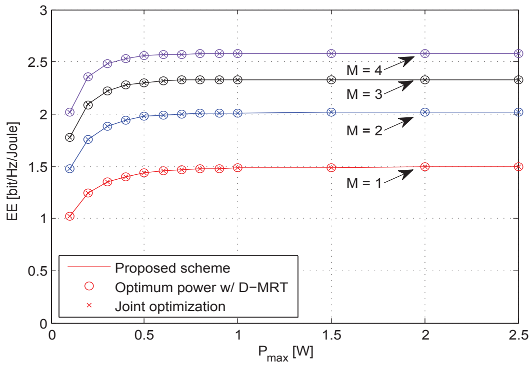

Fig. 1. Energy efficiency with $N = 2$.

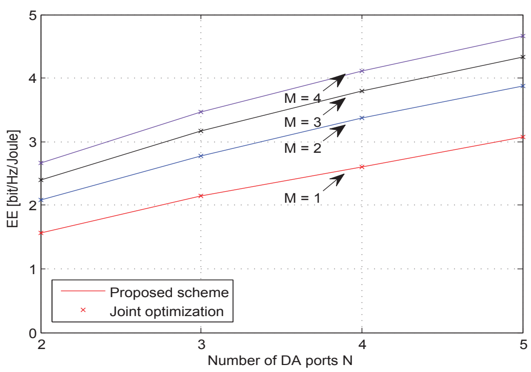

Fig. 2. Energy efficiency with different N.

for $j = 1, \cdots, N$ with $r = \sqrt{\frac{3}{7}}R$ as in [3]. To confirm the performance of our proposed scheme, we compare the EE of the following schemes:

- *Optimum power w/ D-MRT*: The solution for the optimal power allocation is obtained by examining all possible power allocation combinations with a resolution of 0.01 W after applying D-MRT to DAS.
- *Joint optimization*: The joint optimal solution of a beamforming and power allocation is found by the iterative Dinkelbach method described in [1].

In Figure 1, we plot the EE as a function of P_{max} for DAS with a different number of antennas per DA port. The EE performance of our scheme gradually improves and is saturated as P_{max} increases. This is due to the fact that $x_1^\star$ becomes smaller than P_{max} as P_{max} grows, and thus $p_i^\star$'s do not change any more. Moreover, the proposed scheme exhibits the performance identical to the optimum power with D-MRT and the joint optimization with significantly reduced complexity. This is quite interesting considering that our proposed scheme does not jointly optimize the beamforming and power allocation. Note that our simple scheme is based on a closed-form expression, while the joint optimization finds a solution iteratively requiring the convex optimization tools at each iteration.

Figure 2 depicts the EE performance of beamforming and power allocation methods for MISO DAS in terms of N with $P_{max} = 2$ W. It is shown that we can achieve higher EE as N increases since DAS reduce the transmit power along with the access distance using geographically separated DA ports. Through these numerical experiments, we conjecture the optimality of our proposed beamforming and power allocation scheme, and the rigorous proof of the joint optimality remains as an interesting future work.

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