# Beamforming Designs Based on an Asymptotic Approach in MISO Interference Channels

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***Abstract*—In this paper, we consider weighted sum-rate (WSR) maximization problems in multiple-input single-output (MISO) interference channels (IFC) and interfering broadcast channels (IFBC). Most of existing techniques have tried to improve the WSR performance by utilizing instantaneous channel state information. However, since these methods in general should be carried out for each channel realization, they require high computational complexity, which may not be suitable for practical systems. To overcome this issue, we propose a new low complexity beamforming scheme for IFC based on virtual signal-to-interference-plus-noise ratio with constant parameters which depend only on the long-term channel statistics. In our approach, to obtain the constant parameters, the asymptotic values of the leakage coefficients which control the interference signal power are derived by employing asymptotic results from random matrix theory. Moreover, based on the results in MISO IFC, we extend the algorithm to the MISO IFBC case by applying a power allocation algorithm. Numerical results confirm that the proposed schemes provide the near-optimal WSR performance with much reduced system complexity.**

***Index Terms*—Interference channels, beamforming, random matrix theory.**

## I. INTRODUCTION

INTERFERENCE channels (IFC) have attracted great interest in research since many important real world communication systems such as cellular and ad-hoc networks can be modeled as the IFC [1]–[4]. From the information theoretic aspect, several researches have been carried out to exactly identify the capacity region of this channel model for a long time. However, the capacity region of the IFC still remains unknown in general. The best achievable technique up to now is based on the scheme by Han and Kobayashi in [1] which considers rate splitting at transmitters and multiuser detection at receivers. However, the nonlinear multiuser detection incurs high complexity at the receiver side, and thus it may not be feasible in practice. In order to reduce the receiver complexity, it is reasonable to adopt single user detection (SUD) where receivers treat the interference signals as noise.

Under the assumption of the SUD, several papers have studied the Pareto boundary[1] of the achievable rate region for multiple input single output (MISO) IFC. An explicit parameterization of the Pareto boundary was proposed in [2]. Especially, for the two-user case, it was shown that any point on the Pareto boundary can be achieved using a linear combination of maximal-ratio transmission (MRT) and zero-forcing beamforming (ZFBF) vectors. Also, many researches have used a virtual signal-to-interference-plus-noise ratio (VSINR) [2] framework which guarantees a certain point in the Pareto boundary was first proved in [3] for the two-user case. For general $M$-user MISO IFC, the optimality of linear beamforming for achieving a certain Pareto-optimal point has been verified, and a method for the complete characterization of all Pareto optimal rates was proposed by adopting interference temperature constraints in [5]. In addition, distributed methods for sum rate maximization were presented in [6]–[9].

As an important metric which guarantees different quality-of-service and fairness among users, the weighted sum rate (WSR) performance has normally been considered [10]. The WSR measure is useful for prioritizing different users by associating a weight value to each user. For instance, the rate of a user with poor channel can be enhanced by increasing the weight value. Also, for a stricter problem regarding the signal-to-interference-plus-noise-ratio (SINR) constraint of users, the total power minimization subject to the target SINR constraint of each user can be taken into account as in [11] and [12]. In general, identifying a closed form solution for the WSR maximization problem is very difficult due to its non-convexity. The authors in [13] have developed an algorithm based on the monotonic optimization with outer polyblock approximations for two-user MISO IFC. Recently, for the $M$-user MISO IFC, a method based on the rate profile approach and the outer polyblock approximation was introduced in [14], which achieves the global optimality in terms of WSR. A distributed beamforming technique based on local channel state information (CSI) has been proposed by applying high SINR approximations in [15]. Also, a technique based on the VSINR was presented by establishing a relationship between the WSR and the VSINR in [10]. However, the aforementioned schemes employ iterative algorithms and should be carried out at each channel realization, which require high computational complexity.

In this work, our goal is to design a low complexity beamforming scheme which is applicable for practical systems. It was shown in [10] that an additional performance gain in terms of the WSR can be achieved by the beamforming

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[1]The Pareto boundary indicates the upper-right outmost boundary of that region [2].

[2]VSINR is defined as the ratio of the desired signal power at one's own user to the sum of noise plus leakage interference power generated at the other unintended users.

technique based on VSINR through the proper adaptation of the leakage coefficients depending on each channel realization. Motivated by this, we first propose a new low complexity beamforming scheme based on the VSINR with the fixed leakage coefficients which depend only on long-term channel statistics and signal-to-noise ratio (SNR) for MISO IFC.

To this end, we derive deterministic equivalents of the leakage coefficients by employing the asymptotic results of random matrix theory, which are widely used as an analytic tool in many literatures [12], [16]–[20]. The asymptotic expressions in large system limit often serve as a good approximation of the actual performance even in the finite dimensional case. The simulation results demonstrate that the performance of the proposed scheme is almost identical to the optimal WSR with much reduced computational complexity.

Furthermore, we extend our low complexity beamforming scheme in MISO IFC to the MISO interfering broadcast channels (IFBC) where each base station (BS) supports multiple users. In this setting, a power allocation among users in the same cell should be additionally considered, and thus the WSR problem becomes more complicated to solve. For the WSR problem in MISO IFBC, the authors in [21] have derived the structure of the beamforming vectors by adopting the Karush-Kuhn-Tucker conditions directly to the non-convex problem, which converges to a local optimal solution. The beamforming scheme which performs in a distributed manner has also been proposed in [22]. However, the computational complexity of the existing works is still high. To alleviate this, by extending the result of MISO IFC and applying power allocation, we propose an efficient low complexity algorithm for MISO IFBC. As in the case of the MISO IFC, our proposed method still provides the near-optimal WSR performance, which will be verified in the simulation section.

The rest of this paper is organized as follows: Section II describes a system model and the problem formulation. In Section III, we briefly review related works and propose a beamforming scheme using deterministic equivalents of the leakage coefficients in MISO IFC. Section IV presents a simple beamforming design for MISO IFBC, and simulation results are presented in Section V. Finally, in Section VI, this paper is terminated with conclusions.

Throughout the paper, we adopt uppercase boldface letters for matrices and lowercase boldface for vectors. The superscripts $(\cdot)^{\mathsf{T}}$ and $(\cdot)^{\mathsf{H}}$ stand for transpose and conjugate transpose, respectively. In addition, $\|\cdot\|$, $\mathsf{tr}(\cdot)$, $[\cdot]_k$ and $[\cdot]_{ij}$ represent 2-norm, trace, the $k$-th element of a vector and the $(i,j)$-th entry of a matrix, respectively. Also, $\mathbf{I}_d$ denotes an identity matrix of size $d$. A set of $N$ dimensional complex column vectors is defined by $\mathbb{C}^N$ and $|\mathcal{S}|$ indicates the cardinality of the set $\mathcal{S}$.

## II. System Model

We consider the $M$-cell MISO IFC where each BS equipped with $N$ transmit antennas communicates with its corresponding user with a single antenna as depicted in Fig. 1. Let us denote $\mathbf{v}_m$ as the beamforming vector for user $m$ $(m = 1,\ldots,M)$. Then, the received signal at user $m$ can be written

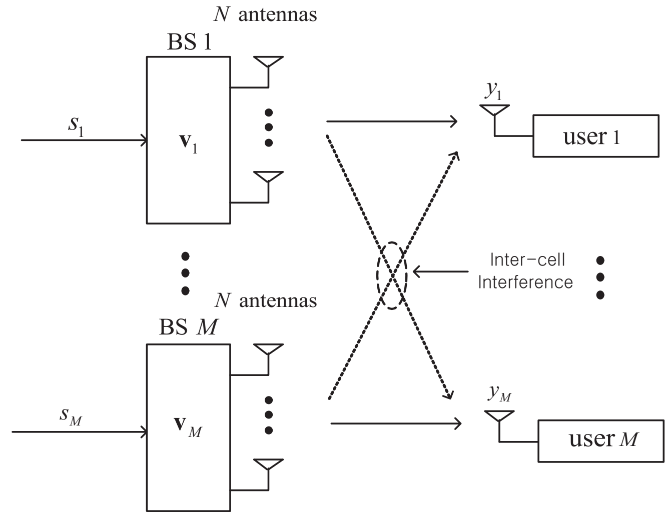


Fig. 1. The system model of $M$-cell MISO IFC.

as

$$y_m = \mathbf{h}_{m,m}^{\mathsf{H}}\mathbf{v}_m s_m + \sum_{j=1,j\neq m}^{M}\mathbf{h}_{m,j}^{\mathsf{H}}\mathbf{v}_j s_j + n_m$$

where $\mathbf{h}_{m,j}^{\mathsf{H}} \in \mathbb{C}^N$ stands for the channel vector from BS $j$ to user $m$, $s_m$ represents the data symbol intended for user $m$, and $n_m \sim \mathcal{CN}(0, N_0)$ indicates the additive white Gaussian noise at user $m$. It is assumed that entries of $\mathbf{h}_{m,j}$ have an independent and identically (i.i.d.) distributed complex Gaussian distribution with $\mathcal{CN}(0,1)$, but our result can easily be extended to a more general channel model. We also assume that the condition $\|\mathbf{v}_m\|^2 \leq 1$ for all BSs $(m = 1,\ldots,M)$ is imposed to satisfy per-BS power constraint.

Assuming a Gaussian codebook, i.e., $s_m \sim \mathcal{CN}(0,1)$, and SUD at receivers, an achievable rate of user $m$ is given as

$$R_m(\{\mathbf{v}_l\}) = \log_2\left(1 + \gamma_m(\{\mathbf{v}_l\})\right)$$

where $\gamma_m(\{\mathbf{v}_l\})$ denotes the SINR for user $m$ expressed as

$$\gamma_m(\{\mathbf{v}_l\}) = \frac{|\mathbf{h}_{m,m}^{\mathsf{H}}\mathbf{v}_m|^2}{N_0 + \sum_{j=1,j\neq m}^{M}|\mathbf{h}_{m,j}^{\mathsf{H}}\mathbf{v}_j|^2}.$$

Thus, we can write the WSR as

$$R_\Sigma(\{\mathbf{v}_l\}) = \sum_{m=1}^{M} w_m R_m(\{\mathbf{v}_l\})$$

where the positive weight term $w_m$ is determined by a scheduler according to the required quality of service for applications. Therefore, the WSR problem can be formulated as

$$\max_{\{\mathbf{v}_l\}} R_\Sigma(\{\mathbf{v}_l\}) \qquad \text{s.t. } \|\mathbf{v}_l\|^2 \leq 1 \quad \forall l. \tag{1}$$

In general, the problem (1) is non-convex and difficult to solve directly due to coupled interference among users [23]. However, since the optimal solution lies on the Pareto boundary of the achievable rate region, we can identify the optimal point by exhaustive search using the parameterizations in [2] and [3], or by jointly utilizing monotonic optimization and rate profile techniques in an iterative manner [14]. Here, the rate region is defined as the set of all possible rate-tuples $(R_1(\{\mathbf{v}_l\}), \cdots, R_M(\{\mathbf{v}_l\})$ which can be achieved simultaneously while satisfying $\|\mathbf{v}_l\|^2 \leq 1$ for all $l$.

In this work, we focus on the case of $N \geq M$ as in [10]. It is well known that for $N \geq M$, all Pareto-optimal points can be obtained by considering only the beamforming vector with full power transmission, i.e., $\|\mathbf{v}_m\|^2 = 1$ [2]. Therefore, the WSR maximization problem (1) reduces to

$$\max_{\mathbf{v}_1, \cdots, \mathbf{v}_M} R_\Sigma(\{\mathbf{v}_l\}) \qquad \text{s.t. } \|\mathbf{v}_l\|^2 = 1 \quad \forall l.$$

In this scenario, as an alternative technique for achieving Pareto-optimal points, the beamforming method based on the VSINR in [3] can be considered, where the VSINR for user $m$ is defined as

$$\text{VSINR}_m = \frac{|\mathbf{h}_{m,m}^{\mathsf{H}} \mathbf{v}_m|^2}{N_0 + \sum_{j \neq m} \alpha_{j,m} |\mathbf{h}_{j,m}^{\mathsf{H}} \mathbf{v}_m|^2}.$$

It was proved in [3] and [5] that if the non-negative weight coefficients $\alpha_{j,m}$ for $j \in \mathcal{M} \backslash \{m\}$ with $\mathcal{M} \triangleq \{1, \cdots, M\}$ are properly chosen, all Pareto-optimal points can be determined by adopting the VSINR maximizing beamformers as

$$\mathbf{v}_m = \frac{\bar{\mathbf{v}}_m}{\|\bar{\mathbf{v}}_m\|} \tag{2}$$

where $\bar{\mathbf{v}}_m = \left(N_0 \mathbf{I}_N + \sum_{j=1, j \neq m}^M \alpha_{j,m} \mathbf{h}_{j,m} \mathbf{h}_{j,m}^{\mathsf{H}}\right)^{-1} \mathbf{h}_{m,m}$. Therefore, it is obvious that the WSR performance can be improved by adjusting $\alpha_{j,m}$ according to channel realizations.

## III. Proposed Low Complexity Beamforming for MISO IFC

In this section, we propose a new low complexity beamforming scheme for MISO IFC while maintaining the near-optimal performance. Before presenting our new beamforming scheme, we briefly review the main results in [10] which motivate our work. Then, based on the results in [10], our proposed method are illustrated by utilizing random matrix theory [24].

### A. Related Works

In [10], instead of solving the WSR maximization directly, the VSINR maximization problem is defined by establishing the relationship between the WSR and the VSINR. The key idea is to identify $\alpha_{j,k}$ such that the zero gradient values for both WSR and VSINR maximization problems are achieved at the same point.

After inserting $\mathbf{v}_k = \bar{\mathbf{v}}_k / \|\bar{\mathbf{v}}_k\|$ in (2) into (1) and taking the derivative with respect to $\bar{\mathbf{v}}_k$, the gradient expressions for the WSR and VSINR maximization problems become

$$\begin{aligned} \nabla_{\bar{\mathbf{v}}_k} R_\Sigma \;\; = \;\; & \frac{w_k}{I_k + D_k} \left( \frac{\mathbf{h}_{k,k} \mathbf{h}_{k,k}^{\mathsf{H}}}{\|\bar{\mathbf{v}}_k\|^2} - \frac{|\mathbf{h}_{k,k}^{\mathsf{H}} \bar{\mathbf{v}}_k|^2}{\|\bar{\mathbf{v}}_k\|^4} \right) \bar{\mathbf{v}}_k \\ & - \sum_{j=1, j \neq k}^M \frac{1}{\beta_j} \left( \frac{\mathbf{h}_{j,k} \mathbf{h}_{j,k}^{\mathsf{H}}}{\|\bar{\mathbf{v}}_k\|^2} - \frac{|\mathbf{h}_{j,k} \bar{\mathbf{v}}_k|^2}{\|\bar{\mathbf{v}}_k\|^4} \right) \bar{\mathbf{v}}_k, \end{aligned}$$

$$\begin{aligned} \nabla_{\bar{\mathbf{v}}_k} \log(\text{VSINR}_k) \;\; = \;\; & \frac{1}{D_k} \left( \frac{\mathbf{h}_{k,k} \mathbf{h}_{k,k}^{\mathsf{H}}}{\|\bar{\mathbf{v}}_k\|^2} - \frac{|\mathbf{h}_{k,k}^{\mathsf{H}} \bar{\mathbf{v}}_k|^2}{\|\bar{\mathbf{v}}_k\|^4} \right) \bar{\mathbf{v}}_k \\ & - \sum_{j=1, j \neq k}^M \frac{\alpha_{j,k}}{N_0 + \sum_{i=1, i \neq k}^M \alpha_{i,k} \frac{|\mathbf{h}_{i,k}^{\mathsf{H}} \bar{\mathbf{v}}_k|^2}{\|\bar{\mathbf{v}}_k\|^2}} \\ & \times \left( \frac{\mathbf{h}_{j,k} \mathbf{h}_{j,k}^{\mathsf{H}}}{\|\bar{\mathbf{v}}_k\|^2} - \frac{|\mathbf{h}_{j,k} \bar{\mathbf{v}}_k|^2}{\|\bar{\mathbf{v}}_k\|^4} \right) \bar{\mathbf{v}}_k \end{aligned}$$

where $I_k = N_0 + \sum_{j=1, j \neq k}^M \frac{|\mathbf{h}_{k,j}^{\mathsf{H}} \bar{\mathbf{v}}_j|^2}{\|\bar{\mathbf{v}}_j\|^2}$, $D_k = \frac{|\mathbf{h}_{k,k}^{\mathsf{H}} \bar{\mathbf{v}}_k|^2}{\|\bar{\mathbf{v}}_k\|^2}$ and $\beta_k = \frac{I_k (I_k + D_k)}{w_k D_k}$. Then, the weight coefficient $\alpha_{j,k}$ which makes two gradient equal to zero simultaneously can be obtained as

$$\alpha_{j,k} = \frac{N_0}{\beta_j \left( \frac{I_k}{\beta_k} - \sum_{i=1, i \neq k}^M \frac{|\mathbf{h}_{i,k}^{\mathsf{H}} \bar{\mathbf{v}}_k|^2}{\beta_i \|\bar{\mathbf{v}}_k\|^2} \right)} \text{ for } j \in \mathcal{M} \backslash \{k\}. \tag{3}$$

Since $\alpha_{j,k}$'s are coupled with each other, we need to compute $\alpha_{j,k}$ iteratively until convergence. Besides, this approach requires global CSI. To alleviate this issue, a decentralized algorithm using only local CSI was also proposed by using a high SINR approximation in [10]. By optimizing $\alpha_{j,k}$'s, a performance gain was achieved compared to the conventional methods in [3]. However, both the centralized and decentralized algorithms in [10] should be carried out in every channel realizations, which leads to high complexity. Thus, we propose a new beamforming scheme with low complexity which calculates $\alpha_{j,k}$'s based only on long-term channel statistics and SNR.

### B. Proposed Beamforming

In the following, we propose a new low complexity beamforming technique with constant $\alpha_{j,k}$'s which are updated only when the channel statistics or SNR change. To this end, we employ useful results of random matrix theory developed in [25]. More specifically, we derive a deterministic equivalent of $\alpha_{j,k}$ which is used for constructing new beamforming vectors. First, we describe the deterministic equivalent as follows:

*Definition 1 (Deterministic Equivalent [25]):* Let $\{\mathbf{X}_N\}$ be a series of complex random matrices $\mathbf{X}_N$ of size $N \times N$ $(N = 1, 2, \ldots)$. For some functional $f$, we define a deterministic equivalent $m^\circ_{\mathbf{X}_N}$ of $m_{\mathbf{X}_N} \triangleq f(\mathbf{X}_N)$ as any series $m^\circ_{\mathbf{X}_1}, m^\circ_{\mathbf{X}_2}, \ldots$ such that

$$m_{\mathbf{X}_N} - m^\circ_{\mathbf{X}_N} \xrightarrow{N \to \infty} 0$$

almost surely.

To derive the deterministic equivalent of $\alpha_{j,k}$, we adopt equation (3). By adopting an asymptotic approach based on the random matrix theory, we arrive at the following theorem which is the main result of this paper.

*Theorem 1:* As $N \to \infty$ with the fixed ratio $\frac{M}{N}$, we have

$$\alpha_{j,k} - \alpha^\circ_{j,k} \xrightarrow{N \to \infty} 0$$

almost surely, where $\alpha^\circ_{j,k}$ is given by

$$\alpha^\circ_{j,k} = \frac{N_0}{\beta^\circ_j \left( \frac{I^\circ_k}{\beta^\circ_k} - \sum_{i=1, i \neq k}^M \frac{\Upsilon^\circ_{ik}}{\beta^\circ_i \Psi^\circ_k} \right)} \qquad \text{for } j \in \mathcal{M} \backslash \{k\}. \tag{4}$$

Here, $\beta_k^\circ, I_k^\circ$ and $D_k^\circ$ are $\beta_k^\circ = \frac{I_k^\circ(I_k^\circ + D_k^\circ)}{w_k D_k^\circ}$, $I_k^\circ = N_0 + \sum_{j=1,j\neq k}^{M} \frac{\Upsilon_{kj}^\circ}{\Psi_j^\circ}$, $D_k^\circ = \frac{(m_k^\circ)^2}{\Psi_k^\circ}$. The definitions of $\Upsilon_{kj}^\circ, \Psi_j^\circ$ and $m_k^\circ$ are provided in the proof.

*Proof:* The leakage coefficient $\alpha_{j,k}$ in (3) consists of three terms: the desired signal power $|\mathbf{h}_{k,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2$, the power normalization term $\|\bar{\mathbf{v}}_k\|^2$ and the leakage interference power $|\mathbf{h}_{j,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2$. For each of these three terms, we will subsequently derive the deterministic equivalent which together constitutes the final expression for $\alpha_{j,k}^\circ$.

*1) Deterministic Equivalent for $|\mathbf{h}_{k,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2$:* For given $\alpha_{j,k}$'s, $\mathbf{h}_{k,k}^{\mathsf{H}}\bar{\mathbf{v}}_k$ can be expressed as

$$\begin{aligned}\mathbf{h}_{k,k}^{\mathsf{H}}\bar{\mathbf{v}}_k &= \mathbf{h}_{k,k}^{\mathsf{H}}\left(N_0\mathbf{I}_N + \sum_{j=1,j\neq k}^{M}\alpha_{j,k}\mathbf{h}_{j,k}\mathbf{h}_{j,k}^{\mathsf{H}}\right)^{-1}\mathbf{h}_{k,k}\\ &= \frac{1}{N}\mathbf{h}_{k,k}^{\mathsf{H}}\mathbf{\Sigma}_k\mathbf{h}_{k,k}\end{aligned}$$

where $\mathbf{\Sigma}_k$ is defined as $\mathbf{\Sigma}_k = \left(\frac{N_0}{N}\mathbf{I}_N + \sum_{j=1,j\neq k}^{M}\frac{\alpha_{j,k}}{N}\mathbf{h}_{j,k}\mathbf{h}_{j,k}^{\mathsf{H}}\right)^{-1}$. Then, employing Lemma 1 in Appendix, we have

$$\mathbf{h}_{k,k}^{\mathsf{H}}\bar{\mathbf{v}}_k - \frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_k \xrightarrow{N\to\infty} 0.$$

By setting $\mathbf{R}_k$ to $\alpha_{j,k}\mathbf{I}_N$ in Corollary 2 in Appendix, it follows

$$\mathbf{h}_{k,k}^{\mathsf{H}}\bar{\mathbf{v}}_k - m_k^\circ \xrightarrow{N\to\infty} 0$$

where $m_k^\circ = \frac{1}{N}\mathrm{tr}\mathbf{T}(\mathcal{L}_k, \frac{N_0}{N})$ and $\mathcal{L}_k$ is denoted as $\mathcal{L}_k = \{\alpha_{1,k},\ldots,\alpha_{k-1,k},\alpha_{k+1,k},\ldots,\alpha_{M,k}\}$. Here, $\mathbf{T}(\mathcal{S},\rho)$ is given as

$$\mathbf{T}(\mathcal{S},\rho) = \left(\frac{1}{N}\sum_{s_i\in\mathcal{S}}\frac{s_i}{1+e_i} + \rho\right)^{-1}\mathbf{I}_N \tag{5}$$

where $\mathcal{S}$ is a set with non-negative elements $s_i$ for $i = 1,\ldots,|\mathcal{S}|$, $\rho$ represents a positive scalar value and $e_i$'s are unique positive solutions to the fixed-point equations

$$e_i = s_i\left(\frac{1}{N}\sum_{s_j\in\mathcal{S}}\frac{s_j}{1+e_j} + \rho\right)^{-1}. \tag{6}$$

*2) Deterministic Equivalent for $\|\bar{\mathbf{v}}_k\|^2$:* The power normalization term $\|\bar{\mathbf{v}}_k\|^2$ is obtained as

$$\|\bar{\mathbf{v}}_k\|^2 = \mathbf{h}_{k,k}^{\mathsf{H}}\left(N_0\mathbf{I}_N + \sum_{j=1,j\neq k}^{M}\alpha_{j,k}\mathbf{h}_{j,k}\mathbf{h}_{j,k}^{\mathsf{H}}\right)^{-2}\mathbf{h}_{k,k}.$$

Using Lemma 1, we can show that

$$\|\bar{\mathbf{v}}_k\|^2 - \frac{1}{N^2}\mathrm{tr}\mathbf{\Sigma}_k^2 \xrightarrow{N\to\infty} 0.$$

Then, applying Corollary 3 in Appendix into $\frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_k^2$, we can write

$$\|\bar{\mathbf{v}}_k\|^2 - \Psi_k^\circ \xrightarrow{N\to\infty} 0$$

where $\Psi_k^\circ = \frac{1}{N}\mathrm{tr}\mathbf{T}'(\mathcal{L}_k,\rho)$. Here, $\mathbf{T}'(\mathcal{S},\rho)$ is defined as

$$\mathbf{T}'(\mathcal{S},\rho) = \left(1 + \frac{1}{N}\sum_{s_i\in\mathcal{S}}\frac{s_i e_i'}{(1+e_i)^2}\right)\mathbf{T}(\mathcal{S},\rho)^2 \tag{7}$$

and $\mathbf{e}' = [e_1',\ldots,e_{|\mathcal{S}|}']^{\mathsf{T}}$ is expressed by

$$\mathbf{e}' = \left(\mathbf{I}_{|\mathcal{S}|} - \mathbf{J}\right)^{-1}\mathbf{v},$$

where $\mathbf{J}$ and $\mathbf{v}$ are computed as

$$\begin{aligned}[\mathbf{J}]_{ij} &= \frac{s_i s_j}{N^2(1+e_j)^2}\mathrm{tr}\mathbf{T}(\mathcal{S},\rho)^2 \qquad \text{for } i,j = 1,\ldots,|\mathcal{S}|\\ \mathbf{v} &= \left[\frac{s_1}{N}\mathrm{tr}\mathbf{T}(\mathcal{S},\rho)^2,\ldots,\frac{s_{|\mathcal{S}|}}{N}\mathrm{tr}\mathbf{T}(\mathcal{S},\rho)^2\right]^{\mathsf{T}}.\end{aligned}$$

*3) Deterministic Equivalent for $|\mathbf{h}_{j,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2$:* The unnormalized leakage terms $|\mathbf{h}_{j,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2$ is written as

$$|\mathbf{h}_{j,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2 = \frac{1}{N^2}\mathbf{h}_{k,k}^{\mathsf{H}}\mathbf{\Sigma}_k\mathbf{h}_{j,k}\mathbf{h}_{j,k}^{\mathsf{H}}\mathbf{\Sigma}_k\mathbf{h}_{k,k}.$$

Due to the independence between $\mathbf{h}_{k,k}$ and $\mathbf{h}_{j,k}$ for $j \in \mathcal{M}\backslash\{k\}$, we can adopt Lemma 1 and obtain

$$|\mathbf{h}_{j,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2 - \frac{1}{N^2}\mathbf{h}_{j,k}^{\mathsf{H}}\mathbf{\Sigma}_k^2\mathbf{h}_{j,k} \xrightarrow{N\to\infty} 0.$$

Since $\mathbf{\Sigma}_k$ includes $\mathbf{h}_{j,k}$, they are not independent. Thus, employing the Sherman-Morrison matrix inversion lemma yields

$$\frac{1}{N^2}\mathrm{tr}\mathbf{h}_{j,k}^{\mathsf{H}}\mathbf{\Sigma}_k^2\mathbf{h}_{j,k} = \frac{1}{N^2}\mathrm{tr}\frac{\mathbf{h}_{j,k}^{\mathsf{H}}\mathbf{\Sigma}_{jk}\mathbf{\Sigma}_{jk}\mathbf{h}_{j,k}}{(1+\frac{\alpha_{j,k}}{N}\mathbf{h}_{j,k}^{\mathsf{H}}\mathbf{\Sigma}_{jk}\mathbf{h}_{j,k})^2}$$

where $\mathbf{\Sigma}_{jk}$ is denoted as $\mathbf{\Sigma}_{jk} = \left(\frac{N_0}{N}\mathbf{I}_N + \sum_{i=1,i\neq j,k}^{M}\frac{\alpha_{i,k}}{N}\mathbf{h}_{i,k}\mathbf{h}_{i,k}^{\mathsf{H}}\right)^{-1}$.

Then, applying Lemma 1 again to both the denominator and the numerator, it follows

$$\frac{1}{N^2}\mathrm{tr}\mathbf{h}_{j,k}^{\mathsf{H}}\mathbf{\Sigma}_k^2\mathbf{h}_{j,k} - \frac{1}{N}\frac{\frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_{jk}^2}{\left(1+\frac{\alpha_{j,k}}{N}\mathrm{tr}\mathbf{\Sigma}_{jk}\right)^2} \xrightarrow{N\to\infty} 0. \tag{8}$$

Using Corollary 2 and (5), $\frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_{jk}$ in the denominator of the second term in (8) is given by

$$\frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_{jk} - \bar{m}_{jk}^\circ \xrightarrow{N\to\infty} 0,$$

where $\bar{m}_{jk}^\circ = \frac{1}{N}\mathrm{tr}\mathbf{T}(\mathcal{U}_{jk},\frac{N_0}{N})$ and $\mathcal{U}_{jk}$ is defined as $\mathcal{U}_{jk} = \mathcal{L}_k\backslash\{\alpha_{j,k}\}$. Also, according to Corollary 3 and (7), $\frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_{jk}^2$ in the numerator of the second term in (8) can be expressed as

$$\frac{1}{N}\mathrm{tr}\mathbf{\Sigma}_{jk}^2 - \bar{\Psi}_{jk}^\circ \xrightarrow{N\to\infty} 0$$

where $\bar{\Psi}_{jk}^\circ = \frac{1}{N}\mathrm{tr}\mathbf{T}'(\mathcal{U}_{jk},\frac{N_0}{N})$.

Then, we can finally obtain

$$|\mathbf{h}_{j,k}^{\mathsf{H}}\bar{\mathbf{v}}_k|^2 - \Upsilon_{jk}^\circ \xrightarrow{N\to\infty} 0$$

where $\Upsilon_{jk}^\circ$ is written as

$$\Upsilon_{jk}^\circ = \frac{1}{N}\frac{\bar{\Psi}_{jk}^\circ}{(1+\alpha_{j,k}\bar{m}_{jk}^\circ)^2}.$$

Using these three deterministic equivalent quantities $m_k^\circ, \Psi_k^\circ$ and $\Upsilon_{jk}^\circ$, we arrive at the deterministic equivalent quantities $\alpha_{j,k}^\circ$'s in (4), and this concludes the proof. ■

According to our main result in Theorem 1, $\alpha_{j,k}^\circ$ can be calculated in an iterative manner. For each $k$, $e_i$'s from (6) for $\mathbf{T}(\mathcal{L}_k,\frac{N_0}{N})$ and $\mathbf{T}(\mathcal{U}_{jk},\frac{N_0}{N})$ can be solved by fixed-point iterations, which converge very quickly with high accuracy.

TABLE I
THE COMPLEXITY COMPARISON FOR THE PROPOSED SCHEMES AND EXISTING APPROACHES

| | scheme [14] | scheme [10] | Conventional VSINR [3] | Proposed scheme |
|---|---|---|---|---|
| CSI | global | global or local | local | local |
| Coefficients update | instantaneous | instantaneous | none | long-term |
| Algorithm type | iterative | iterative | non-iterative | iterative |

Once $e_i$'s are obtained, we can compute $\mathbf{T}(\mathcal{L}_k, \frac{N_0}{N})$ and $\mathbf{T}(\mathcal{U}_{jk}, \frac{N_0}{N})$, and successively obtain $m_k^\circ$, $\Psi_k^\circ$ and $\Upsilon_{jk}^\circ$ which lead to $\alpha_{j,k}^\circ$. Then, the above process is repeated until $\alpha_{j,k}^\circ$'s converge. Finally, the beamforming vector $\mathbf{v}_k$ is determined according to (2) for all $k$. The whole algorithm of the proposed scheme is summarized as below.

1. Initialize $\alpha_{j,k}$ for all $k \in \mathcal{M}$.
2. Compute $e_i$'s from (6) for $\mathbf{T}(\mathcal{L}_k, \frac{N_0}{N})$ and $\mathbf{T}(\mathcal{U}_{jk}, \frac{N_0}{N})$ by using a fixed-point algorithm for all $k$.
3. Compute $m_k^\circ, \Psi_k^\circ$ and $\Upsilon_{jk}^\circ$ based on $e_i$'s for all $k$.
4. Update the weight coefficients $\alpha_{j,k}^\circ$ according to (4) for all $k$.
5. Go back to step 2 and repeat until convergence.

Note that in our algorithm, the leakage coefficient $\alpha_{j,k}^\circ$ is a function of the weight term $w_k$, the number of transmit antennas $N$, the number of users $M$ and SNR. Therefore, the proposed scheme relies only on local CSI, which allows distributed implementations. More importantly, our algorithm is calculated only once for a fixed SNR, since $\alpha_{j,k}^\circ$ does not depend on instantaneous channel realizations. Thus, the computational complexity is significantly reduced compared to the scheme in [10] which requires the recalculation of $\alpha_{j,k}$ for each channel realization. In other words, the leakage coefficients in our algorithm are updated with long-term statistics compared to the conventional methods. The complexity comparison for the proposed schemes and the existing approaches for MISO IFC is summarized in Table I. As will be shown in the simulation section, the proposed approach exhibits near-optimal WSR performance and provides a significant performance gain over the conventional VSINR scheme in [3].

### C. Rate Approximations

In this subsection, we provide the WSR approximation based on a deterministic equivalent of SINR. Using a similar approach of the proof in Theorem 1, the deterministic equivalent of the SINR at the user in cell $m$ can be easily derived as below.

*Theorem 2:* As $N \to \infty$ with the fixed ratio $\frac{M}{N}$, for given $\alpha_{j,m}^\circ$'s, $\gamma_m$ converges to

$$\gamma_m^\circ = \frac{D_m^\circ}{I_m^\circ}$$

almost surely, where $D_m^\circ$ are $I_m^\circ$ are defined in Theorem 1.

*Proof:* This theorem can be easily obtained from the derivation of Theorem 1, and thus the proof is omitted. ■

By applying the continuous mapping theorem [26] as in [25], it follows from the almost sure convergence that the individual rate $R_m$ converges almost surely to $R_m^\circ = \log_2(1+\gamma_m^\circ)$ as $N$ goes to infinity. Then, the approximated WSR $\hat{R}_\Sigma$ can be written as

$$\hat{R}_\Sigma = \sum_{m=1}^{M} w_m \log_2(1+\gamma_m^\circ). \tag{9}$$

## IV. BEAMFORMING DESIGNS FOR IFBC

In this section, we propose a low complexity beamforming technique for MISO IFBC where each BS supports $K$ users. We show that the proposed algorithm for IFC in Section III can be extended to IFBC case by employing power allocation among users in each cell. For the MISO IFBC, let us denote $\mathbf{v}_{m,k}$ and $\mathbf{h}_{m,k,j}^{\mathsf{H}}$ as the beamforming vector for user $k$ in the $m$-th cell and the channel vector from BS $j$ to user $k$ in the $m$-th cell ($k = 1, \ldots, K$ and $m = 1, \ldots, M$), respectively. Then, we can express the WSR maximization problem as

$$\max_{\{\mathbf{v}_{l,u}\}} R_\Sigma(\{\mathbf{v}_{l,u}\}) \qquad \text{s.t.} \sum_{u=1}^{K} \|\mathbf{v}_{l,u}\|^2 \le 1 \quad \forall l, u, \tag{10}$$

where $R_\Sigma(\{\mathbf{v}_{l,u}\}) = \sum_{m=1}^{M}\sum_{k=1}^{K} w_{m,k} R_{m,k}(\{\mathbf{v}_{l,u}\})$. Here, $R_{m,k}(\{\mathbf{v}_{l,u}\})$ is defined as $R_{m,k}(\{\mathbf{v}_{l,u}\}) = \log_2(1+\gamma_{m,k}(\{\mathbf{v}_{l,u}\}))$ and the SINR of user $k$ in the $m$-th cell, $\gamma_{m,k}$ becomes

$$\gamma_{m,k}(\{\mathbf{v}_{l,u}\}) = \frac{|\mathbf{h}_{m,k,m}^{\mathsf{H}} \mathbf{v}_{m,k}|^2}{N_0 + \sum_{(j,u)\neq(m,k)} |\mathbf{h}_{m,k,j}^{\mathsf{H}} \mathbf{v}_{j,u}|^2}.$$

It is assumed that $N \ge MK$ as in the MISO IFC case, and then full power transmission $\sum_{k=1}^{K} \|\mathbf{v}_{m,k}\|^2 = 1$ is optimal in terms of WSR [22]. Therefore, the WSR maximization problem (10) can be rephrased as

$$\max_{\{\mathbf{v}_{l,u}\}} R_\Sigma(\{\mathbf{v}_{l,u}\}) \qquad \text{s.t.} \sum_{u=1}^{K} \|\mathbf{v}_{l,u}\|^2 = 1 \quad \forall l, u. \tag{11}$$

Unlike the case of MISO IFC, power allocation among users in the same cell should be additionally considered. Since this requires a joint optimization with respect to beamforming and power allocation, it is more complicated to solve the WSR problem for the IFBC.

From now on, we tackle the WSR problem by using a suboptimal but simple approach. Under the assumption of equal power allocation for all users, the beamforming vector for user $k$ in the $m$-th cell for MISO IFBC can be represented as

$$\mathbf{v}_{m,k} = \frac{1}{\sqrt{K}} \frac{\bar{\mathbf{v}}_{m,k}}{\|\bar{\mathbf{v}}_{m,k}\|} \tag{12}$$

where

$$\bar{\mathbf{v}}_{m,k} = \left( K N_0 \mathbf{I}_N + \sum_{(j,u)\neq(m,k)} \alpha_{j,u,m,k} \mathbf{h}_{j,u,m} \mathbf{h}_{j,u,m}^{\mathsf{H}} \right)^{-1} \mathbf{h}_{m,k,m}.$$

Here, similar to (3) in the MISO IFC case, $\alpha_{j,u,m,k}$'s for user $k$ in the $m$-th cell for MISO IFBC are computed as

$$\alpha_{j,u,m,k} = \frac{KN_0}{\beta_{j,u}\left(\frac{I_{m,k}}{\beta_{m,k}} - \sum_{(n,i)\neq(m,k)} \frac{|\mathbf{h}_{n,i,m}^{\mathsf{H}}\bar{\mathbf{v}}_{m,k}|^2}{\beta_{n,i}\|\bar{\mathbf{v}}_{m,k}\|^2}\right)} \quad (13)$$

where $\beta_{m,k} = \frac{I_{m,k}(I_{m,k}+D_{m,k})}{w_{m,k}D_{m,k}}$, $I_{m,k} = KN_0 + \sum_{(j,u)\neq(m,k)} \frac{|\mathbf{h}_{m,k,j}^{\mathsf{H}}\bar{\mathbf{v}}_{j,u}|^2}{\|\bar{\mathbf{v}}_{j,u}\|^2}$, $D_{m,k} = \frac{|\mathbf{h}_{m,k,k}^{\mathsf{H}}\bar{\mathbf{v}}_{m,k}|^2}{\|\bar{\mathbf{v}}_{m,k}\|^2}$. Then, the deterministic equivalent of $\alpha_{j,u,m,k}^{\circ}$ can be obtained in the following corollary.

*Corollary 1:* As $N \to \infty$ with the fixed ratio $\frac{MK}{N}$, it follows

$$\alpha_{j,u,m,k} - \alpha_{j,u,m,k}^{\circ} \xrightarrow{N\to\infty} 0$$

almost surely, where $\alpha_{j,u,m,k}^{\circ}$ is given by

$$\alpha_{j,u,m,k}^{\circ} = \frac{KN_0}{\beta_{j,u}\left(\frac{I_{m,k}^{\circ}}{\beta_{m,k}^{\circ}} - \sum_{(n,i)\neq(m,k)} \frac{\Upsilon_{n,i,m,k}^{\circ}}{\beta_{n,i}^{\circ}\Psi_{m,k}^{\circ}}\right)}. \quad (14)$$

Here, $\beta_{m,k}^{\circ}, I_{m,k}^{\circ}$ and $D_{m,k}^{\circ}$ are $\beta_{m,k}^{\circ} = \frac{I_{m,k}^{\circ}(I_{m,k}^{\circ}+D_{m,k}^{\circ})}{w_{m,k}D_{m,k}^{\circ}}$, $I_{m,k}^{\circ} = KN_0 + \sum_{(j,u)\neq(m,k)} \frac{\Upsilon_{m,k,j,u}^{\circ}}{\Psi_{j,u}^{\circ}}$, $D_{m,k}^{\circ} = \frac{(m_{m,k}^{\circ})^2}{\Psi_{m,k}^{\circ}}$. Also, $m_{m,k}^{\circ}, \Psi_{m,k}^{\circ}$ and $\Upsilon_{n,i,m,k}^{\circ}$ are denoted by

$$m_{m,k}^{\circ} = \frac{1}{N}\mathrm{tr}\mathbf{T}(\mathcal{G}_{m,k}, \frac{KN_0}{N}),\ \Psi_{m,k}^{\circ} = \frac{1}{N}\mathrm{tr}\mathbf{T}'(\mathcal{G}_{m,k}, \frac{KN_0}{N}),$$
$$\Upsilon_{n,i,m,k}^{\circ} = \frac{1}{N}\frac{\bar{\Psi}_{n,i,m,k}^{\circ}}{(1+\alpha_{n,i,m,k}\bar{m}_{n,i,m,k}^{\circ})^2}$$

where $\mathcal{G}_{m,k}$ is a set with elements $\alpha_{j,u,m,k}$ for $j = 1,\ldots,M$ and $u = 1,\ldots,K$ and $(j,u) \neq (m,k)$, and we have $\bar{m}_{n,i,m,k}^{\circ} = \frac{1}{N}\mathrm{tr}\mathbf{T}(\mathcal{G}_{m,k}^{n,i}, \frac{KN_0}{N})$, $\bar{\Psi}_{n,i,m,k}^{\circ} = \frac{1}{N}\mathrm{tr}\mathbf{T}'(\mathcal{G}_{m,k}^{n,i}, \frac{KN_0}{N})$, and $\mathcal{G}_{m,k}^{n,i} = \mathcal{G}_{m,k}\backslash\{\alpha_{n,i,m,k}\}$.

*Proof:* This corollary can be proved similarly using the derivation of Theorem 1, and thus the proof is omitted. ■

From (14), $\alpha_{j,u,m,k}^{\circ}$'s can be calculated in an iterative fashion. Then, the beamforming vectors $\mathbf{v}_{m,k}$'s are determined based on $\alpha_{j,u,m,k}^{\circ}$'s. However, one might expect a performance loss in terms of WSR due to the assumption of equal power allocation. To solve this issue, we additionally perform power control after obtaining the beamformer. Now, we present a power control method based on the beamforming in (12). Similar to Theorem 2 and (9), the approximated WSR $\hat{R}_{\Sigma}$ can be written as

$$\hat{R}_{\Sigma} = \sum_{m=1}^{M}\sum_{k=1}^{K} w_{m,k}\log_2(1+\gamma_{m,k}^{\circ}) \quad (15)$$

where $\gamma_{m,k}^{\circ}$ is expressed as $\gamma_{m,k}^{\circ} = \frac{D_{m,k}^{\circ}}{I_{m,k}^{\circ}}$.

By redefining $\mathbf{v}_{m,k}$ in (12) as $\mathbf{v}_{m,k} = \sqrt{p_{m,k}}\frac{\bar{\mathbf{v}}_{m,k}}{\|\bar{\mathbf{v}}_{m,k}\|}$, the WSR $\hat{R}_{\Sigma}$ can be reformulated as

$$\hat{R}_{\Sigma} = \sum_{m=1}^{M}\sum_{k=1}^{K} w_{m,k}\log_2(1+p_{m,k}\nu_{m,k}^{\circ}) \quad (16)$$

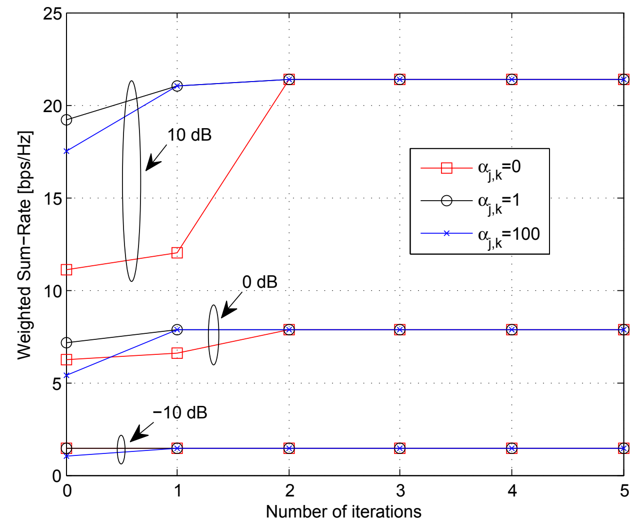


Fig. 2. Average WSR performance of different initial $\alpha_{j,k}$ with $M = N = 2$ and $[w_1, w_2] = [5, 1]$.

where $\nu_{m,k}^{\circ} = \gamma_{m,k}^{\circ}/p_{m,k}$ and $\sum_{k=1}^{K} p_{m,k} = 1$. Then, by applying the water-filling to maximize $\hat{R}_{\Sigma}$, the optimal solution $p_{m,k}^{\star}$ for each $m$ BS is given as

$$p_{m,k}^{\star} = \left[w_{m,k}\,\mu - \frac{1}{\nu_{m,k}^{\circ}}\right]^{+} \quad \text{for } k = 1,\ldots,K \quad (17)$$

where $[x]^{+} \triangleq \max(0,x)$ and $\mu$ is the water level such that $\sum_{k=1}^{K} p_{m,k} = 1$.

It is emphasized again that power loading as well as the leakage coefficients for MISO IFBC are determined by long-term channel statistics and can be carried out based on only local CSI. Therefore, a reduction in the computational complexity is significant compared to the existing methods in [21] and [22] which need to be computed for each channel realization. Furthermore, the proposed beamforming provides the WSR performance quite close to that of the scheme [21] which will be shown in the following section.

## V. Numerical Results

In this section, we evaluate the WSR performance of the proposed beamforming schemes. In all simulations, spatially uncorrelated Rayleigh fading channels with unit variance are assumed. First, we start with the MISO IFC case. In Fig. 2, we verify the robustness of the proposed method to the choice of initial points. The average WSR performance with $N = M = 2$ and $[w_1, w_2] = [5, 1]$ is presented as a function of the number of iterations at various SNR levels for different initial $\alpha_{j,k}$. In this plot, zero iteration indicates the case where the fixed $\alpha_{j,k}$ is applied without any update. As shown in [3], when $\alpha_{j,k}$ is zero, the corresponding beamforming becomes MRT, while as $\alpha_{j,k}$ approaches infinity, the solution behaves like ZFBF. Thus, as initial points, we set $\alpha_{j,k}$ to $0, 1, 100$ to represent MRT, conventional VSINR and the ZFBF, respectively. Interestingly, we can check that the WSR performance of the proposed scheme converges within only 2 iterations regardless of any initial $\alpha_{j,k}$ for all SNR region.

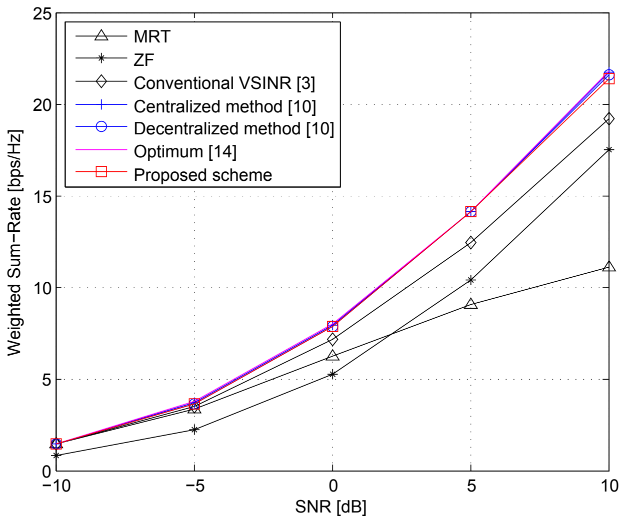


Fig. 3. Average WSR performance with $[w_1, w_2] = [5, 1]$ and $M = N = 2$.

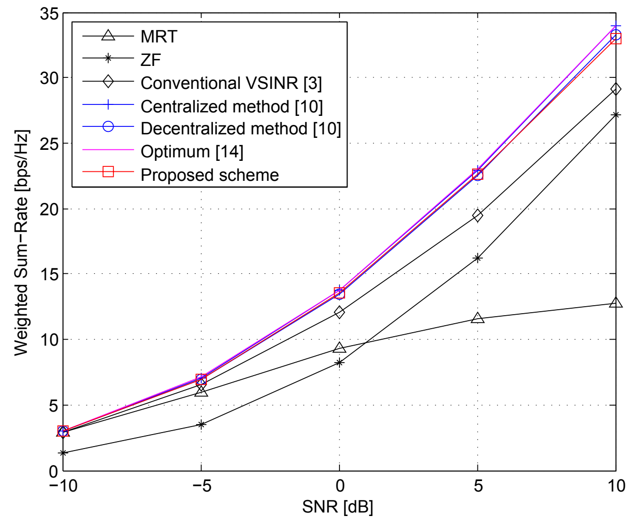


Fig. 4. Average WSR performance with $[w_1, w_2, w_3] = [5, 3, 1]$ and $M = N = 3$.

Figs. 3 and 4 exhibit the average WSR performance of various schemes as a function of SNR for $N = M = 2$ and $N = M = 3$, respectively. Here, the weight terms $\{w_k\}$ are set to $[w_1, w_2] = [5, 1]$ and $[w_1, w_2, w_3] = [5, 3, 1]$, respectively. For both our proposed scheme and the method in [10], the ZFBF is adopted as an initial point. Also, by using the algorithm proposed in [14], the global optimal WSR performance which serves as an upper bound on the performance of the beamforming schemes is plotted. Surprisingly, we can see that the proposed scheme achieves near-optimal performance with much reduced complexity for all simulated SNR regions. It is emphasized again that our proposed algorithm is performed only when SNR or channel statistics changes and the constant $\alpha_{j,k}$ is employed for the same SNR. This results in a significant computational complexity reduction compared to the scheme in [10]. We can also observe that the WSR performance of the proposed scheme outperforms the conventional VSINR method, while the complexity of the proposed scheme is the same as that of the VSINR method after obtaining $\alpha_{j,k}$'s from our one-shot algorithm.

In what follows, we evaluate the proposed scheme for MISO IFBC compared to the existing techniques. In this scenario, as an upper bound of the WSR performance, we employ a gradient decent (GD) algorithm which requires full CSI. In order to increase the probability of finding the global optimal solution, two solutions of MRT and ZFBF as well as 10 random points are applied to the GD as initial points. For comparison, we also present the WSR performance of the ZFBF with water-filling, the conventional VSINR and the scheme proposed in [21]. For simple notations, we denote $M \times (N \times K)$ MISO IFBC as $M$-cell multiuser systems where each BS with $N$ transmit antennas supports $K$ users.

Figs. 5 and 6 compare the average WSR of the proposed scheme with that of the existing schemes for $2 \times (6 \times 3)$ MISO IFBC systems with the symmetric weight terms and the asymmetric weight terms, respectively. Here, the symmetric weight means that all BSs have the same user weight vector, i.e., $[w_{m,1}, \ldots, w_{m,K}] = [w_{j,1}, \ldots, w_{j,K}]$ for all $m \neq j$, while the asymmetric weight indicates that the weights of users in the same cell are equal but different from those of the other cells. In Fig. 5, the weight terms are set to $[w_{m,1}, w_{m,2}, w_{m,3}] = [10, 5, 1]$ and $[4, 2, 1]$ for all $m$. As expected, it is observed that our scheme with equal power allocation has a large performance loss compared to the near-optimal GD due to the absence of power optimization. In contrast, the proposed method with a water-filling algorithm provides the performance almost identical to the GD and outperforms the conventional VSINR and ZFBF with water-filling. In Fig. 6, the weight terms are set to $[w_{1,1}, w_{1,2}, w_{1,3}, w_{2,1}, w_{2,2}, w_{2,3}] = [10, 10, 10, 1, 1, 1]$ and $[4, 4, 4, 1, 1, 1]$. In contrast to the symmetric weight case in Fig. 5, the proposed schemes with and without water-filling allocation have the similar performance. This is due to the fact that the same weights are employed for all users in each cell. It can also be seen that the performance gaps from the GD are negligible.

Finally, Fig. 7 presents the WSR performance for $3 \times (6 \times 2)$ MISO IFBC systems. Here, the weight terms are given as $[w_{m,1}, w_{m,2}] = [10, 1]$ and $[4, 1]$. In the plot, we can observe that the performance curves exhibit the similar trend as Fig. 5, and the performance gap between the proposed scheme and GD becomes smaller compared to curves in Fig. 5. We can check that as in the case of MISO IFC, the proposed method for MISO IFBC provides the near-optimal performance with much reduced complexity in various configurations.

## VI. Conclusions

In this paper, we have proposed low complexity beamforming methods for both MISO IFC and IFBC. We have first derived the deterministic equivalent of the leakage coefficients to compute the parameters in our proposed beamforming structure for MISO IFC. Then, combining the results in MISO IFC and power allocation, the beamforming scheme for the MISO IFBC scenario has also been proposed. Note that our proposed schemes are carried out in a fully distributed manner, and the parameters are computed only when channel statistics or SNR changes. Thus, the computational complexity

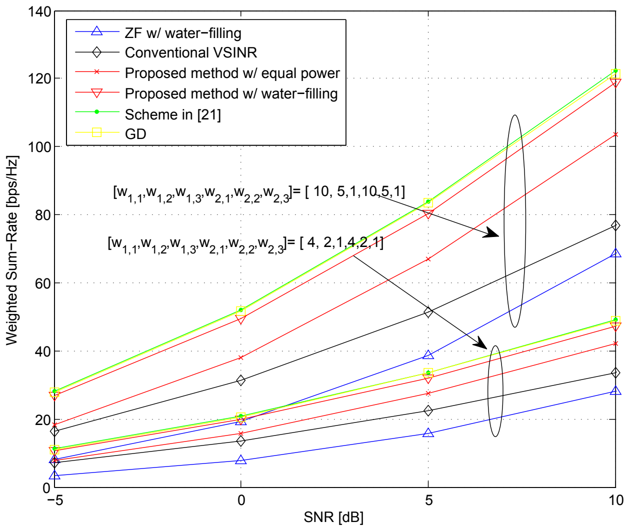


Fig. 5. Average WSR performance with symmetric weight terms for $2 \times (6 \times 3)$ MISO IFBC systems.

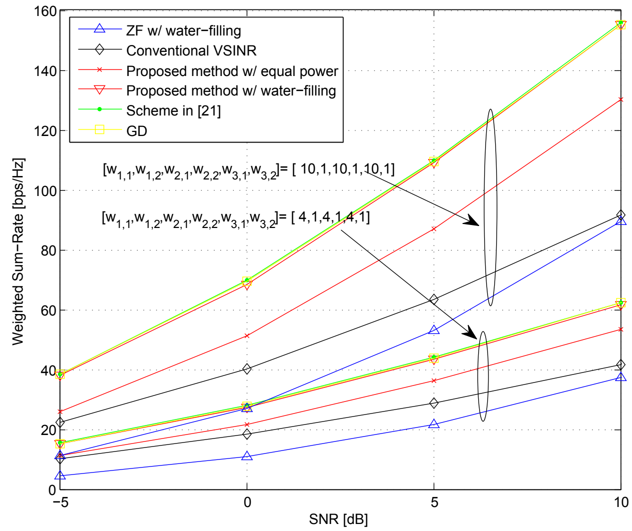


Fig. 7. Average WSR performance with symmetric weight terms for $3 \times (6 \times 2)$ MISO IFBC systems.

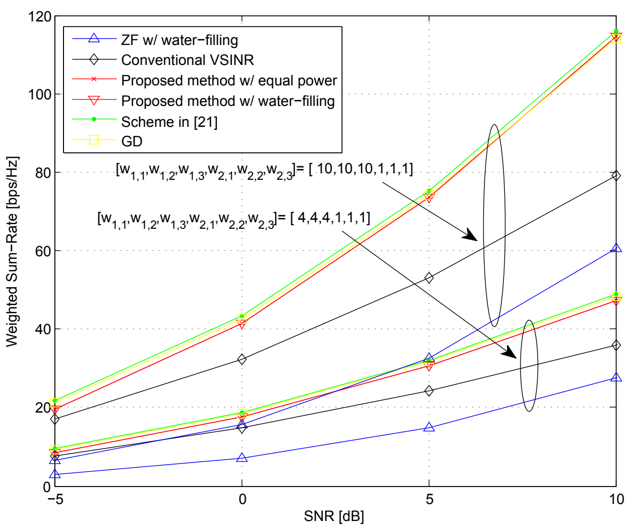


Fig. 6. Average WSR performance with asymmetric weight terms for $2 \times (6 \times 3)$ MISO IFBC systems.

is significantly reduced compared to the conventional method. Through simulations, we have confirmed that the proposed schemes with the asymptotic results are efficient beamforming strategies even for the finite system case and provide the near-optimal WSR performance.

## Appendix

*Lemma 1 ([24, Theorem 3.4]):* Let $\mathbf{A}_1, \mathbf{A}_2, \ldots,$ with $\mathbf{A}_N \in \mathbb{C}^{N \times N}$ be a series of matrices with uniformly bounded spectral norm. Also let us denote $\mathbf{x}_1, \mathbf{x}_2, \ldots,$ with $\mathbf{x}_N \in \mathbb{C}^N$ as a series of random vectors of i.i.d. entries with zero mean, variance $\frac{1}{N}$, and the eighth order moment of order $O(1/N^4)$, which are independent of $\mathbf{A}_N$. Then, we have

$$\mathbf{x}_N^{\mathsf{H}} \mathbf{A}_N \mathbf{x}_N - \frac{1}{N} \mathrm{tr} \mathbf{A}_N \xrightarrow{N \to \infty} 0$$

almost surely.

*Corollary 2:* Let $\mathbf{H} \in \mathbb{C}^{N \times K}$ be a random matrix with columns $\mathbf{h}_k = \frac{1}{\sqrt{N}} \mathbf{R}_k^{1/2} \mathbf{u}_k$, where $\mathbf{R}_k \in \mathbb{C}^{N \times N}$ is a deterministic covariance matrix with uniformly bounded spectral norm and $\mathbf{u}_k \in \mathbb{C}^N$ represents a random vector with i.i.d. elements with zero mean, unit variance and the finite eighth order moment. Let $N, K \to \infty$ such that $0 \leq \liminf \frac{K}{N} \leq \limsup \frac{K}{N} < \infty$. Then, for any $\rho > 0$, it follows

$$\frac{1}{N} \mathrm{tr} \left( \mathbf{H}\mathbf{H}^{\mathsf{H}} + \rho \mathbf{I}_N \right)^{-1} - \frac{1}{N} \mathrm{tr} \mathbf{T}(\rho) \xrightarrow{a.s.} 0$$

where $\mathbf{T}(\rho) \in \mathbb{C}^{N \times N}$ is defined as

$$\mathbf{T}(\rho) = \left( \frac{1}{N} \sum_{k=1}^{K} \frac{\mathbf{R}_k}{1 + \delta_k(\rho)} + \rho \mathbf{I}_N \right)^{-1}$$

and $\delta_k(\rho)$ is a non-negative unique solution of

$$\delta_k(\rho) = \frac{1}{N} \mathrm{tr} \left( \mathbf{R}_k \mathbf{T}(\rho) \right) \qquad \text{for } k = 1, \ldots, K.$$

*Proof:* This corollary is a direct result of [25, Theorem 1] for uncorrelated Rayleigh channels. ■

*Corollary 3:* Under the same conditions as in Corollary 2, the following relation holds:

$$\frac{1}{N} \mathrm{tr} \left( \mathbf{H}\mathbf{H}^{\mathsf{H}} + \rho \mathbf{I}_N \right)^{-2} - \frac{1}{N} \mathrm{tr} \mathbf{T}^{'}(\rho) \xrightarrow{a.s.} 0$$

where $\mathbf{T}^{'}(\rho) \in \mathbb{C}^{N \times N}$ is denoted as

$$\mathbf{T}^{'}(\rho) = \mathbf{T}(\rho)^2 + \mathbf{T}(\rho) \frac{1}{N} \sum_{k=1}^{K} \frac{\mathbf{R}_k \delta_k^{'}(\rho)}{(1 + \delta_k(\rho))^2} \mathbf{T}(\rho)$$

with $\boldsymbol{\delta}^{'}(\rho) = [\delta_1^{'}(\rho) \cdots \delta_K^{'}(\rho)]^{\mathsf{T}}$ given by

$$\boldsymbol{\delta}^{'}(\rho) = (\mathbf{I}_K - \mathbf{J}(\rho))^{-1} \mathbf{v}(\rho).$$

Here, $\mathbf{J}(\rho) \in \mathbb{C}^{K \times K}$ and $\mathbf{v}(\rho) \in \mathbb{C}^K$ are expressed by

$$[\mathbf{J}(\rho)]_{kl} = \frac{\mathrm{tr} \left( \mathbf{R}_k \mathbf{T}(\rho) \mathbf{R}_l \mathbf{T}(\rho) \right)}{N^2 (1 + \delta_l(\rho))^2}, \; [\mathbf{v}(\rho)]_k = \frac{1}{N} \mathrm{tr} \left( \mathbf{R}_k \mathbf{T}(\rho)^2 \right).$$

*Proof:* This corollary can be easily obtained from [25, Theorem 2]. ■

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