Downlink Vertical Beamforming Designs for Active Antenna Systems

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***Abstract*—In this paper, we study a vertical beamforming technique for multiple-input multiple-output downlink multi-user systems. In general, the transmit antenna gain is controlled by adjusting the boresight of antennas in directional antennas, and thus the cell average rate varies according to the angle of the boresight. First, we compute the tilting angles for directional antenna systems which maximize the cell average rate. To this end, the probability density function of a three-dimensional user distribution is derived. Based on the result, we analyze the average rate gain of active antenna systems over passive antenna systems for a single user case. Furthermore, for a multi-user active antenna system, beamforming designs to maximize the weighted sum rate are proposed by optimizing the transmit antenna gain and power allocation. Since finding joint optimal parameters requires prohibitively high computational complexity, we separate the optimization problem into two sub-problems of the vertical beamforming and the power allocation. Then a simple vertical beamforming algorithm based on a high signal-to-noise ratio assumption is presented. Also, for a multi-user passive antenna system, we provide a beamforming scheme based on a multi-sector concept. Simulation results show that the proposed beamforming schemes outperform the conventional beamforming schemes.**

***Index Terms*—Broadcast channels (BC), MIMO, multiuser, 3D beamforming.**

I. Introduction

ONE of the most important design considerations in next generation cellular networks is to support the explosive growth of demand for the data rate. Several approaches have been introduced to tackle this challenge. Among promising solutions, multiple-input multiple-output (MIMO) methods [1]–[12] and directional antenna techniques [13]–[19] have been highlighted in the past. Traditionally, MIMO precoding[1] schemes which achieve high data rate transmission have been intensively investigated for isotropic or omni-directional antennas.

Multi-user MIMO precoding methods allow us to support several users simultaneously [20]. Downlink multi-user transmission is often modeled as a MIMO broadcast channel (BC) where a base station (BS) with multiple antennas transmits to geographically separated users. In the MIMO BC scenario, an important issue is efficient multi-user interference (MUI) management, which has been addressed with various precoding techniques [1], [6]–[12]. One of these techniques is the simple zero-forcing beamforming (ZF-BF) scheme [6], which perfectly eliminates the MUI at each receiver.

In general, there are two types of directional antennas: a passive antenna and an active antenna where the former cannot change the directional antenna pattern dynamically, while the latter can do. For the case of the passive antenna system, which has a fixed downtilting angle, a design of directional antenna pattern settings or the cell architecture planning has often been considered as a deployment issue. For example, the directional antenna pattern has been normally determined based on field tests or the cell modeling in practice.

On the other hand, the active antenna system can change its directional antenna pattern for each transmission. For example, in the active antenna array, each antenna element can be connected to a separate transceiver component, and thus the active antenna system can support an electronic beam-tilt feature by controlling the phases, amplitudes, and delays of individual antenna elements [21], [22]. The electronic beam-tilt feature of active antenna systems enables the antenna pattern and the MIMO precoding to be optimized simultaneously.

In this paper, we study a system where a BS with multiple directional antennas supports users with a single omni-directional antenna. Instead of determining the transmit antenna pattern itself as in [16], we investigate how to jointly optimize the BS tilting angle and the precoding design for active antenna systems, and provide efficient solutions for both single user and multi-user systems. To this end, we first derive the probability density function (PDF) of the vertical angle of a user. Then, for the single user case, we offer the vertical beamforming solutions to maximize the cell average rate for the active and passive antenna systems by employing maximum ratio transmission (MRT) as the precoding technique.

Moreover, we analyze an average rate gain of the active antenna system over the passive antenna system. The accuracy of the analysis will be verified by numerical results. Note that, in our previous work [19], we provided the PDF of a vertical

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[1]To avoid confusion, we employ the terminology “precoding” for closed-loop MIMO systems, while “beamforming” is adopted for directional antenna pattern designs.

angle distribution when users are located uniformly in the three-dimensional (3-D) space, and then proposed solutions assuming all users are under the line of sight (LoS) condition. In contrast, in this paper, we make a more realistic assumption that users are uniformly distributed in horizontal domain while linearly distributed in vertical domain, which includes the previous work [19] as a special case. Furthermore, we adopt the 3D channel model defined in [23] and [24] with narrow band transmission.

Next, for the multi-user case, we formulate the weighted sum rate maximization problem assuming the ZF-BF method. Since a proper power allocation scheme introduces a considerable performance improvement in the MIMO BC scenario [25], joint optimization for the directional antenna pattern and the closed-loop precoding design including power allocation should be considered. However, in case of the active antenna system, the joint optimization requires prohibitively high computational complexity.

Thus, we propose a suboptimal but simple algorithm by separating the overall optimization problem into two sub-problems, each of which corresponds to the vertical beamforming and the power allocation. It is shown that the complexity can be further reduced by employing a simple vertical beamforming algorithm. Simulation results show that the proposed scheme provides the performance quite close to that of the joint optimization scheme with much reduced complexity, and outperforms the conventional beamforming method in [18]. Also for multi-user passive antenna systems, we provide a simple beamforming method utilizing the multiple sector concept. Simulation results demonstrate the proposed tilting angles provide better performance without any additional complexity and overhead.

The rest of the paper is organized as follows: In Section II, we introduce a wireless downlink system with directional antennas. For the single user case, we present the solutions for active and passive antenna systems and the analysis of the average rate gain of active antenna systems over passive antenna systems in Section III. Then, we propose algorithms to maximize the sum rate of multi-user systems in Section IV. Section V illustrates the simulation results of the proposed schemes. Section VI concludes the paper.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices and lowercase boldface for vectors. For any general matrix $\mathbf{A}$, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and complex-transpose of the matrix, respectively. $\mathbb{E}[\cdot]$, $|a|$ and $\|\mathbf{a}\|$ indicate the expectation operation, the absolute value of a and the Euclidean 2-norm of a vector $\mathbf{a}$, respectively. Also, $\mathrm{diag}\{a_1, \cdots, a_K\}$ represents a $K \times K$ diagonal matrix with the k-th diagonal element a_k.

II. System Model and Vertical Angle Distribution

Consider a wireless downlink system consisting of a BS with M directional antennas and a user equipped with a single receive antenna as shown in Fig. 1. In the figure, we denote $\Delta z = h_{BS} - h_U$ as the height difference between the user and the BS, and we define x and y as the relative distance between the user and the BS in the x and y coordinate, respectively. Then, the distance between the user and the BS can be expressed as $r = \sqrt{x^2 + y^2}$. In this case, the LoS vertical angle with respect to the horizontal plane at the user can be written as $\theta_U = \tan^{-1}(\Delta z / r)$. In addition, θ_{BS} represents the vertical boresight angle of the BS (also known as the tilting angle). In what follows, we will refer to θ_U and θ_{BS} as the vertical angle and the tilting angle, respectively.

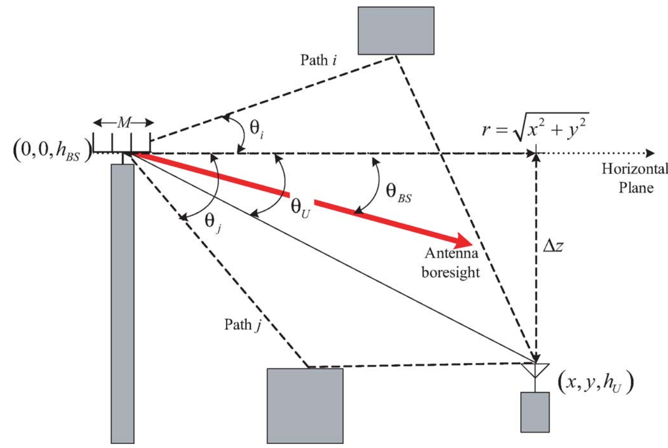

Fig. 1. Vertical view of a 3-D channel model.

Typically, a directional antenna is often implemented using a panel antenna consisting of multiple antenna elements within a single radome enclosure [17]. Unlike isotropic antenna systems whose antenna gain is the same for all directions, directional antenna systems have antenna gain which changes according to the transmit antenna pattern. In general, the transmit antenna pattern can be divided into the horizontal and vertical parts [26]. However, to simplify the analysis, we assume the omni-directional antenna pattern in the horizontal region, which is valid when the horizontal 3 dB beamwidth is large [19] or when all antenna elements in a panel antenna are stacked up serially in the vertical domain.

Defining the vertical angle with respect to the horizontal plane at the l-th obstacle location by θ_l, the vertical antenna attenuation can be expressed in dB scale as [26]

$$A_V(\theta_l, \theta_{BS}) = \min\left(12\left(\frac{\theta_l - \theta_{BS}}{\theta_{3\ \mathrm{dB}}}\right)^2, A_m\right) \tag{1}$$

where $\theta_{3\ \mathrm{dB}}$ and A_m stand for the vertical 3 dB beamwidth of the BS antenna and the maximum vertical antenna attenuation, respectively. The transmit antenna pattern in (1) can be obtained by controlling the phases, amplitudes, and delays of individual antenna elements in a panel antenna.

Let us denote $g_{\max}$ as the maximum antenna gain at the antenna boresight. Then, after combining the antenna attenuation and the maximum antenna gain, the resultant antenna gain in linear scale for the l-th signal path with the vertical angle θ_l can be formulated as

$$g(\theta_l, \theta_{BS}) = g_{\max}\, a_V(\theta_l, \theta_{BS}) \approx g_{\max}\, 10^{-1.2(\theta_l - \theta_{BS})^2/\theta_{3\ \mathrm{dB}}^2} \tag{2}$$

where $a_V(\theta_l, \theta_{BS})$ indicates the vertical antenna attenuation in linear scale at (θ_l, θ_{BS}), and the approximation holds when $A_V(\theta_l, \theta_{BS}) \leq A_m$ or A_m is large enough.[2]

It is assumed that the system bandwidth is so narrow that all signal paths from the BS to the user arrive within the sampling time of the system. In this case, the composite channel between the antenna of the BS and the user is given by

$$h(\theta_{BS}) = \sum_{l=1}^{L} \sqrt{g(\theta_l, \theta_{BS})} h_l \tag{3}$$

where L and h_l stand for the number of signal paths and the small scale channel between the antenna of the BS and the user for the l-th signal path, respectively. Here we assume $\mathbb{E}[\sum_{l=1}^{L} |h_l|^2] = 1$ for the scaling purpose. Note that since the magnitude of the l-th signal path $|\sqrt{g(\theta_l, \theta_{BS})} h_l|$ varies according to θ_{BS}, the composite channel also changes in terms of θ_{BS}.

Now we assume that users are linearly distributed in the vertical domain while uniformly distributed over the horizontal domain. In this case, the PDF of the random variable Δz can be expressed as $f_{\Delta z}(z) = az + b$ for $z_0 \leq z \leq z_m$ where z_0 and z_m denote the minimum and the maximum height difference between the BS and the user, respectively. Note that a depends on the environment of interest, and b is determined based on the value a. For example, in an apartment environment, users are likely to be located uniformly over the vertical domain, which leads to $a = 0$. Another example is a hotspot scenario where more users are located closer to the ground, which results in $a > 0$. Moreover, since the number of users with the distance r from the BS is proportional to the circumference of a circle with the radius r, the PDF of the random variable r is given by $f_r(\gamma) = 2\gamma/(r_m^2 - r_0^2) = d \cdot \gamma$ for $r_0 \leq \gamma \leq r_m$ where r_0 and r_m represent the minimum distance and the cell radius, respectively, and $d \triangleq 2/(r_m^2 - r_0^2)$.[3]

With these assumptions, we can derive the PDF of the vertical angle according on z_0, z_m, r_0, and r_m. In case of $z_0 > 0$, i.e. all users are located below the BS antennas, the PDF of the vertical angle $\theta = \tan^{-1}(\Delta z/r)$ is written as (4), shown at the bottom of the page, for $z_0/r_0 > z_m/r_m$ or (4) for $z_0/r_0 \leq z_m/r_m$. On the other hand, in the case of $z_0 < 0$ and $z_m > 0$, i.e. some users are located above the BS antenna, the PDF of the vertical angel is derived as (6) shown at the bottom of the page.

Detail derivations of (4), (5), and (6) are provided in the Appendix.

In the following sections, based on the derived distributions, we examine two antenna technologies: the active antenna, where θ_{BS} is determined instantaneously based on each channel realization, and the passive antenna, where θ_{BS} is obtained statistically. From now on, we refer to the former case as the active beamforming scheme, and the latter case as the passive beamforming scheme.

III. Single-User Vertical Beamforming

In this section, we consider a single user system with the same θ_{BS} for all transmit antennas at a given time. For the frequency-flat fading channels, the received signal $y \in \mathbb{C}$ is expressed by

$$y = \sqrt{r^{-\alpha}}\, \mathbf{h}^H \mathbf{s} + n \tag{7}$$

where r is the distance between the user and the BS, α equals the pathloss exponent, $\mathbf{h} \in \mathbb{C}^{M \times 1}$ stands for the channel vector

[2]For example, for the antenna model in [26], A_m is given as $A_m = 20$ dB. In this case, the difference between $a_V(\theta_l, \theta_{BS})$ and $10^{-1.2(\theta_l - \theta_{BS})^2/\theta_3^2}$ dB is less than 1%.

[3]The density of r can be determined by $f_r(\gamma)d\gamma = Pr\{\gamma < r \leq \gamma + d\gamma\} = \int\int_{\Delta D_\gamma} f_{x,y}(x,y)dxdy$ where ΔD_γ is the region of the x-y plane such that $\gamma < \sqrt{x^2 + y^2} \leq \gamma + d\gamma$ and $f_{x,y}(x,y)$ equals the joint distribution of x and y which is a constant within the region.

$$f_\theta(\theta) = \begin{cases} \frac{z_0^2 d\left(3az_0^2+8bz_0+12c\right)}{12\tan\theta\sin^2\theta} + \frac{d\left(4br_m^3+3ar_m^4\tan\theta\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_0}{r_m} \leq \theta \leq \tan^{-1}\frac{z_m}{r_m} \\ \frac{z_0^2 d\left(3az_0^2+8bz_0+12c\right) - z_m^2 d\left(3az_m^2+8bz_m+12c-12\right)}{12\tan\theta\sin^2\theta} & \text{for } \tan^{-1}\frac{z_m}{r_m} \leq \theta \leq \tan^{-1}\frac{z_0}{r_0} \\ -\frac{z_m^2 d\left(3az_m^2+8bz_m+12c-12\right)}{12\tan\theta\sin^2\theta} - \frac{d\left(4br_0^3+3ar_0^4\tan\theta\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_0}{r_0} \leq \theta \leq \tan^{-1}\frac{z_m}{r_0} \\ 0 & \text{else} \end{cases} \tag{4}$$

$$f_\theta(\theta) = \begin{cases} \frac{z_0^2 d\left(3az_0^2+8bz_0+12c\right)}{12\tan\theta\sin^2\theta} + \frac{d\left(4br_m^3+3ar_m^4\tan\theta\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_0}{r_m} \leq \theta \leq \tan^{-1}\frac{z_0}{r_0} \\ \frac{3ad\left(r_m^4-r_0^4\right)\tan\theta+4bd\left(r_m^3-r_0^3\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_0}{r_0} \leq \theta \leq \tan^{-1}\frac{z_m}{r_m} \\ -\frac{z_m^2 d\left(3az_m^2+8bz_m+12c-12\right)}{12\tan\theta\sin^2\theta} - \frac{d\left(4br_0^3+3ar_0^4\tan\theta\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_m}{r_m} \leq \theta \leq \tan^{-1}\frac{z_m}{r_0} \\ 0 & \text{else} \end{cases} \tag{5}$$

$$f_\theta(\theta) = \begin{cases} -\frac{z_0^2 d\left(3az_0^2+8bz_0+12c\right)}{12\tan\theta\sin^2\theta} - \frac{d\left(4br_0^3+3ar_0^4\tan\theta\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_0}{r_0} \leq \theta \leq \tan^{-1}\frac{z_0}{r_m} \\ \frac{3ad\left(r_m^4-r_0^4\right)\tan\theta+4bd\left(r_m^3-r_0^3\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_0}{r_m} \leq \theta \leq \tan^{-1}\frac{z_m}{r_m} \\ -\frac{z_m^2 d\left(3az_m^2+8bz_m+12c-12\right)}{12\tan\theta\sin^2\theta} - \frac{d\left(4br_0^3+3ar_0^4\tan\theta\right)}{12\cos^2\theta} & \text{for } \tan^{-1}\frac{z_m}{r_m} \leq \theta \leq \tan^{-1}\frac{z_m}{r_0} \\ 0 & \text{else} \end{cases} \tag{6}$$

from the BS to the user whose entry is given as in (3) as a function of θ_{BS}, $\mathbf{s} \in \mathbb{C}^{M\times 1}$ represents the transmit signal vector, and n indicates the complex additive white Gaussian noise (AWGN) with variance σ_n^2 at the user.

The transmit signal vector $\mathbf{s}$ is defined as $\mathbf{s} = \sqrt{P}\mathbf{w}d$ where $\mathbf{w} \in \mathbb{C}^{M\times 1}$ and d denote the transmit precoding vector with $\|\mathbf{w}\|^2 = 1$ and the transmit data symbol with unit variance, respectively, and P equals the transmit power of the BS. With perfect channel state information (CSI) at the transmitter, it is well known that MRT, i.e. $\mathbf{w} = \mathbf{h}/\|\mathbf{h}\|$, is the optimal scheme for the single user single stream precoding [25]. Using the MRT as the transmit precoding vector, the resultant received signal-to-noise ratio (SNR) at the user can be written as

$$\text{SNR} = \frac{r^{-\alpha}P\|\mathbf{h}\|^2}{\sigma_n^2} = \frac{r^{-\alpha}Pf(\theta_{BS})}{\sigma_n^2} \tag{8}$$

where $f(\theta_{BS}) = \|\mathbf{h}\|^2$ represents the channel gain for a given θ_{BS}.

Now we want to find the optimal tilting angle θ_{BS}^* for the active and the passive beamforming cases which maximize the average rate. In case of the active beamforming, since the BS can adjust the tilting angle for each transmission time, the optimal tilting angle can be obtained by

$$\theta_{BS}^* = \arg\max \log_2(1 + \text{SNR}) = \arg\max f(\theta_{BS}) \tag{9}$$

which can be determined by the user using the training signals with different θ_{BS} and is fed back to the BS.

For the case of the passive beamforming, identifying the optimal tilting angle is not trivial. Since the direct optimization of θ_{BS} over all SNR regions is intractable, we focus on the high SNR region. For high SNR, the average rate is expressed as $\mathbb{E}_{r,\theta,\mathbf{h}}[\log_2(1 + \text{SNR})] \approx \mathbb{E}_{r,\theta,\mathbf{h}}[\log_2 \text{SNR}]$. Then θ_{BS} can be determined as

$$\arg\max_{\theta_{BS}} \mathbb{E}_{r,\theta,\mathbf{h}}\left[\log_2 \frac{r^{-\alpha}P\|\mathbf{h}\|^2}{\sigma_n^2}\right]. \tag{10}$$

Since finding the tilting angle based on the composite channel in (3) is quite challenging, we rewrite the composite channel as

$$h(\theta_{BS}) = \sum_{l=1}^{L}\sqrt{g(\theta_l, \theta_{BS})}h_l = \sqrt{g(\theta, \theta_{BS})}\tilde{h}(\theta_{BS}) \tag{11}$$

where $g(\theta, \theta_{BS})$ represents the transmit antenna gain of the user at the vertical angle θ (i.e., the line-of-sight direction of the user) and the tilting angle θ_{BS}, and $\tilde{h}(\theta_{BS}) = \sum_{l=1}^{L}(\sqrt{g(\theta_l, \theta_{BS})}/\sqrt{g(\theta, \theta_{BS})}h_l)$ is the small scale fading channel for θ_{BS}.

Then, we can reformulate (10) as

$$\arg\max_{\theta_{BS}} \mathbb{E}_{r,\theta,\tilde{\mathbf{h}}}\left[\log_2 \frac{r^{-\alpha}g_{\max}\, 10^{-1.2(\theta-\theta_{BS})^2/\theta_{3\text{ dB}}^2}P\|\tilde{\mathbf{h}}\|^2}{\sigma_n^2}\right].$$

Since the objective function of the above equation is a monotonically increasing function, after removing the non θ_{BS} related parts from the equation, we can rewrite the above equation as

$$\arg\max_{\theta_{BS}} \mathbb{E}\left[-1.2\frac{(\theta-\theta_{BS})^2}{\theta_{3\text{ dB}}^2}\log_2 10\right] + \mathbb{E}\left[\log_2\|\tilde{\mathbf{h}}\|^2\right]. \tag{12}$$

After applying Jensen's inequality, we obtain θ_{BS}^* which maximizes an upper bound of the objective function of (12) as

$$\theta_{BS}^* = \arg\max_{\theta_{BS}} \mathbb{E}\left[-1.2\frac{(\theta-\theta_{BS})^2}{\theta_{3\text{ dB}}^2}\log_2 10\right] + \log_2 \mathbb{E}\left[\|\tilde{\mathbf{h}}\|^2\right]. \tag{13}$$

Now we assume that the vertical angle of each signal path is a random variable where the mean value equals θ as modeled in [23] and [24].

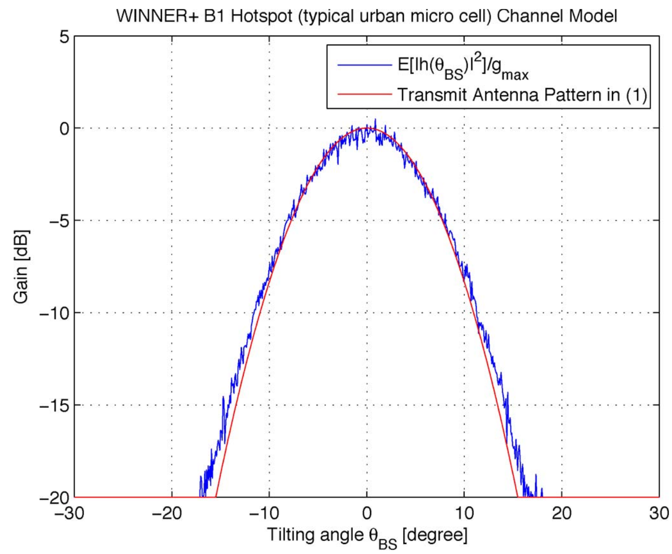

Fig. 2. Average small scale fading channel gain with different θ_{BS} for $\theta = 0$.

Fig. 2 plots $\mathbb{E}[|h(\theta_{BS})|^2]/g_{\max}$ in dB scale with respect to the tilting angle θ_{BS} when $\theta = 0$. Note that the maximum number of signal paths L in the WINNER+B1 hotspot model is set to 16, and only the signal paths with at least -25 dB above that of the strongest signal path are selected for simulation. From the figure and (11), we can assume that $\mathbb{E}[|h(\theta_{BS})|^2] \approx g(\theta, \theta_{BS})$ or $\mathbb{E}[|\tilde{h}(\theta_{BS})|^2] \approx 1$ for any tilting angle θ_{BS}. In other words, $\mathbb{E}[\|\tilde{\mathbf{h}}\|^2]$ is not a function of θ_{BS}.

Thus, the above equation is equivalent to

$$\theta_{BS}^* = \arg\min_{\theta_{BS}} \mathbb{E}\left[(\theta - \theta_{BS})^2\right]. \tag{14}$$

Since the second derivative of the expectation in (14) with respect to θ_{BS} is always positive, a solution of (14) can be determined by setting the first derivative of the expectation to zero, which is

$$\frac{d}{d\theta_{BS}}\int(\theta-\theta_{BS})^2 f_\theta(\theta)d\theta = 2\theta_{BS}\int f_\theta(\theta)d\theta - 2\int \theta f_\theta(\theta)d\theta = 0. \tag{15}$$

Then the solution of (15) is obtained as $\theta_{BS}^* = \int \theta f_\theta(\theta) d\theta = \mathbb{E}[\theta]$.

Note that $\mathbb{E}[\theta]$ can be calculated using the PDF of the vertical angle, e.g. (4), (5) or (6) depending on the sign of z_0, z_0/r_0 and z_m/r_m. For example, when $a = 0$, after applying the trigonometry function equality [27], for all cases, θ_{BS}^* is given as

$$\begin{aligned}\theta_{BS}^* = \frac{1}{3(z_m - z_0)(r_m^2 - r_0^2)} \Bigg(&\left(z_0^2 - z_m^2\right)(r_0 - r_m) \\ &+ z_0^3 \left(\tan^{-1}\frac{z_0}{r_0} - \tan^{-1}\frac{z_0}{r_m}\right) - z_m^3 \left(\tan^{-1}\frac{z_m}{r_0} - \tan^{-1}\frac{z_m}{r_m}\right) \\ &+ r_m^3 \ln\frac{z_0^2 + r_m^2}{z_m^2 + r_m^2} - r_0^3 \ln\frac{z_0^2 + r_0^2}{z_m^2 + r_0^2} + 3r_m^2 z_m \tan^{-1}\frac{z_m}{r_m} \\ &+ 3r_0^2 z_0 \tan^{-1}\frac{z_0}{r_0} - 3r_m^2 z_0 \tan^{-1}\frac{z_0}{r_m} - 3r_0^2 z_m \tan^{-1}\frac{z_m}{r_0} \Bigg).\end{aligned}$$

Now we want to provide an average rate gain of the active beamforming over the passive beamforming using the above results. For the single user system, the average rate in the high SNR region can be derived as

$$\begin{aligned}R &\approx \mathbb{E}_{r,\theta,\tilde{\mathbf{h}}}[\log_2 \mathrm{SNR}] \\ &= \mathbb{E}_{R,\theta,\tilde{\mathbf{h}}}\left[\log_2 \frac{r^{-\alpha} g_{\max}\, a_V(\theta,\theta_{BS}) P \|\tilde{\mathbf{h}}\|^2}{\sigma_n^2}\right] \\ &= \log_2 P + \mathbb{E}\left[\log_2 \|\tilde{\mathbf{h}}\|^2\right] + \mathbb{E}\left[\log_2 a_V(\theta,\theta_{BS})\right] \\ &\quad + \log_2 g_{\max} - \alpha\mathbb{E}[\log_2 r] - \mathbb{E}\left[\log_2 \sigma_n^2\right]. \qquad (16)\end{aligned}$$

Note that only $\mathbb{E}[\log_2 a_V(\theta,\theta_{BS})]$ is related to θ_{BS}. Therefore, we can obtain the average rate gain of the active beamforming by measuring $\mathbb{E}[\log_2 a_V(\theta,\theta_{BS})]$.

In case of the active beamforming, we can assume $\mathbb{E}[\log_2 a_V(\theta,\theta_{BS})] = 0$ since the maximum value of $a_V(\cdot)$ equals 1 which can be obtained by setting the tilting angle as $\theta_{BS} = \theta$. For the passive beamforming, after plugging $\theta_{BS}^* = \mathbb{E}[\theta]$ into $a_V(\cdot)$, $\mathbb{E}[\log_2 a_V(\theta,\theta_{BS})]$ can be computed as

$$\begin{aligned}\mathbb{E}\left[\log_2 a_V(\theta,\mathbb{E}[\theta])\right] &= \mathbb{E}\left[\log_2 10^{-1.2(\theta-\mathbb{E}[\theta])^2/\theta_{3\text{ dB}}^2}\right] \\ &= -\frac{1.2\log_2 10}{\theta_{3\text{ dB}}^2}\left(\int \theta^2 f_\theta(\theta)d\theta - \mathbb{E}^2\theta\right) \\ &= -\frac{1.2\sigma_\theta^2 \log_2 10}{\theta_{3\text{ dB}}^2} = -3.986\frac{\sigma_\theta^2}{\theta_{3\text{ dB}}^2} \qquad (17)\end{aligned}$$

where σ_θ^2 is the variance of the vertical angle which can be calculated from the PDF of the vertical angle. In this case, the average rate gain of the active beamforming over the passive beamforming in the high SNR region is equal to $3.986(\sigma_\theta^2/\theta_{3\text{ dB}}^2)$. Thus, we expect the average rate gain to be proportional to $\sigma_\theta^2/\theta_{3\text{ dB}}^2$, which will be verified in the simulation section.

IV. Multi-User Vertical Beamforming

In this section, we consider a multi-user downlink system which consists of a BS and N users. All other assumptions remain the same as in Sections II and III, if not stated otherwise. First of all, we assume that the BS selects K out of N users ($K \leq N$ and M). Then, the received signal of the k-th scheduled user ($k = 1, \cdots, K$) is given by

$$y_k = \sqrt{r_k^{-\alpha}}\, \mathbf{h}_k^H \mathbf{s} + n_k \qquad (18)$$

where r_k is the distance between the k-th user and the BS, $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ stands for the complex channel vector from the BS to the k-th user which is a function of θ_{BS}, $\mathbf{s} \in \mathbb{C}^{M\times 1}$ represents the transmit signal vector, and n_k indicates the complex AWGN at the k-th user with variance σ_n^2.

The transmit signal vector is expressed as $\mathbf{s} = \mathbf{WPd}$ where $\mathbf{W}$ denotes the transmit precoding matrix $\mathbf{W} = [\mathbf{w}_1\mathbf{w}_2\cdots\mathbf{w}_K] \in \mathbb{C}^{M\times K}$ with $\|\mathbf{w}_k\|^2 = 1$, the power allocation matrix $\mathbf{P}$ is given by $\mathbf{P} = \mathrm{diag}\{\sqrt{P_1}\sqrt{P_2}\cdots\sqrt{P_K}\} \in \mathbb{C}^{K\times K}$ which satisfies $\sum_{k=1}^K P_k \leq P$, and the transmit data vector $\mathbf{d}$ is defined by $\mathbf{d} = [d_1 d_2 \cdots d_K]^T \in \mathbb{C}^{K\times 1}$. Here, we adopt the ZF-BF scheme [25] which perfectly eliminates the MUI at the receiver, i.e. $\mathbf{h}_i^H \mathbf{w}_j = 0$ for all $i \neq j$. Then, the received SNR at the k-th user becomes

$$\mathrm{SNR}_k = \frac{r_k^{-\alpha} P_k \left|\mathbf{h}_k^H \mathbf{w}_k\right|^2}{\sigma_n^2}. \qquad (19)$$

A. Optimization for Active Beamforming

Now we want to maximize the weighted sum rate of the system for the active beamforming. Unlike the single user case, finding the optimal tilting angle for the active beamforming is no longer trivial. Defining all possible sets of scheduled users by $\mathcal{S}$ and the n-th set in $\mathcal{S}$ by $\mathcal{S}_n$ where the cardinality of $\mathcal{S}_n$ is K, the optimization problem for the active beamforming can be formulated as

$$\begin{aligned}&\max_{\mathcal{S}_n\in\mathcal{S}} \max_{\theta_{BS},\mathbf{P}} \sum_{k=1}^K w_k \log_2(1 + \mathrm{SNR}_k) \\ &\text{s.t.} \sum_{k=1}^K P_k \leq P \qquad (20)\end{aligned}$$

where w_k is the weight for the k-th user.

For a given scheduled user set $\mathcal{S}_n$, the sub-problem of (20) is rephrased as

$$\begin{aligned}&\max_{\theta_{BS},\mathbf{P}} \sum_{k=1}^K w_k \log_2\left(1 + \frac{r_k^{-\alpha} P_k f_k(\theta_{BS})}{\sigma_n^2}\right) \\ &\text{s.t.} \sum_{k=1}^K P_k \leq P \qquad (21)\end{aligned}$$

where $f_k(\theta_{BS}) = |\mathbf{h}_k^H \mathbf{w}_k|^2$ represents the channel gain for the k-th user for a given θ_{BS}. Note that unlike the case of single user active antenna transmission, the value of $f_k(\theta_{BS})$ depends on the scheduled user set as well, and thus the scheduler needs to obtain CSI for each case, increasing the feedback overhead.

To identify the optimal value, we should examine all possible sets of scheduled users.[4] In other words, a solution of the original problem in (20) can be obtained by solving (21) for every $\mathcal{S}_n$. To this end, we first illustrate the joint optimization of the tilting angle and power allocation. The joint optimization problem (21) can be reformulated in a standard optimization form as

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) = -\sum_{k=1}^{K} w_k \log_2\left(1 + \frac{r_k^{-\alpha} P_k f_k(\theta_{BS})}{\sigma_n^2}\right) \\ \text{s.t.} \quad & \mathbf{a}^T\mathbf{x} - P \leq 0 \\ & x_k \geq 0 \text{ for } k = 1, \cdots, K \end{aligned} \tag{22}$$

where we have $\mathbf{x} = [P_1 P_2 \cdots P_K \theta_{BS}]^T$ and $\mathbf{a}$ is the column vector $\mathbf{a} = [11 \cdots 10]^T$ of length $K+1$. Let us denote μ and $\nabla f_0(\mathbf{x})$ as the step size of the gradient descent method and the gradient of $f_0(\mathbf{x})$ at point $\mathbf{x}$, respectively. Then a solution of the joint optimization algorithm can be obtained by applying the gradient descent and the projection methods which correspond to step 2 and step 3 in the following algorithm, respectively.

Joint Optimization

1. Initialize $\mathbf{x}$ to any point in a feasible set.
2. Compute $\mathbf{g} = \mathbf{x} - \mu\nabla f_0(\mathbf{x})$.
3. Determine $\mathbf{x} = \arg\min_{\mathbf{x}} \|\mathbf{g} - \mathbf{x}\|$ subject to $\mathbf{a}^T\mathbf{x} - P \leq 0$ and $x_k \geq 0$ for $k = 1, \cdots, K$.
4. Go back to step 2 and repeat until convergence.

Although the projection method in step 3 is a convex problem which can be efficiently solved by a convex optimization tool [28], we need to evaluate the joint optimization algorithm with multiple initial points since the overall problem is not convex.

Because of the iterative nature, the above joint optimization solution may become prohibitively complex. To decrease the computational complexity, we propose a simple multi-user active beamforming (MUAB) by separating the joint problem into two sub-problems which determine the power allocation $\mathbf{P}$ and the tilting angle θ_{BS} separately. Since the power allocation problem can be solved by the conventional water-filling algorithm for a given θ_{BS}, we focus on finding the optimal tilting angle. Suppose that the power allocation matrix is given as $\mathbf{P} = \text{diag}\{\sqrt{P_1}, \sqrt{P_2}, \cdots, \sqrt{P_K}\}$. Then, we can rewrite (21) using a high SNR approximation as

$$\max_{\theta_{BS}} \sum_{k=1}^{K} w_k \log_2\left(f_k(\theta_{BS})\right). \tag{23}$$

[4]We can also adopt suboptimal user selection solutions such as the semiorthogonal user selection (SUS) algorithm [3] or the greedy user selection algorithm [2].

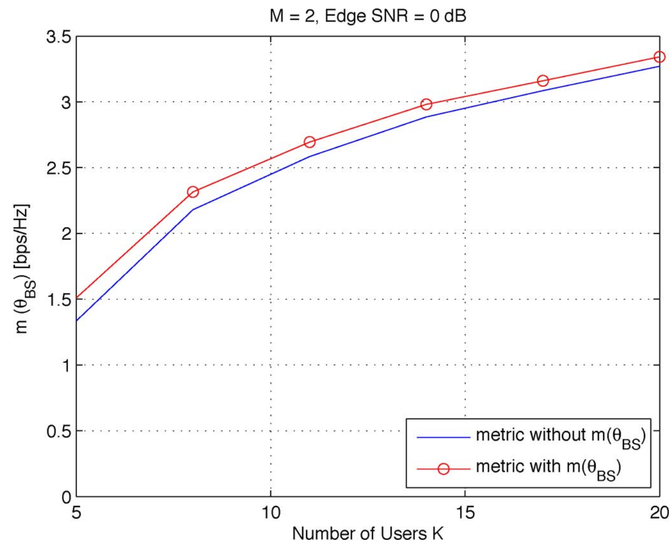

Fig. 3. $m(\theta_{BS})$ for scheduled users with $M = 2$ and $\theta_{3\text{ dB}} = 10°$.

From (11), we can express the above problem as

$$\max_{\theta_{BS}} \sum_{k=1}^{K} w_k \log_2\left(10^{-1.2(\theta_k - \theta_{BS})^2/\theta_{3\text{ dB}}^2}\right) + m(\theta_{BS}) \tag{24}$$

where θ_k is the vertical angle of the k-th user and the second term of the cost function in (24) is defined as

$$\begin{aligned} m(\theta_{BS}) &= \sum_{k=1}^{K} w_k \log_2\left(\frac{f_k(\theta_{BS})}{g(\theta_k, \theta_{BS}})\right) \\ &= \sum_{k=1}^{K} w_k \log_2\left(\left|\frac{\mathbf{h}_k^H \mathbf{w}_k}{\sqrt{g(\theta_k, \theta_{BS})}}\right|^2\right) \\ &= \sum_{k=1}^{K} w_k \log_2\left(\left|\tilde{\mathbf{h}}_k^H \mathbf{w}_k\right|^2\right). \end{aligned} \tag{25}$$

From the above equation, we can see that $m(\theta_{BS})$ is related only to the ZF-BF precoding gain, and not to the antenna gain. However, since the channel $\tilde{\mathbf{h}}_k$ varies according to the tilting angle, $m(\theta_{BS})$ also changes. Unfortunately, this variation is not computationally tractable.

Fig. 3 plots $m(\theta_{BS})$ for scheduled users with and without considering $m(\theta_{BS})$ in the optimization metric. From the figure, we can observe that whether $m(\theta_{BS})$ is considered in the metric or not, these two optimization results show only a small difference in performance. Since including $m(\theta_{BS})$ significantly complicates the optimization problem with only a negligible improvement, we ignore the second term of (24) from now on. It will be shown in Section V that in spite of these assumptions and simplifications, the proposed low-complexity scheme achieves almost the same performance as the joint optimization method for all SNR regions.

Then, the optimal tilting angle can be obtained by solving

$$\min_{\theta_{BS}} \sum_{k=1}^{K} w_k (\theta_k - \theta_{BS})^2. \tag{26}$$

Then, a solution of (26) is derived as

$$\theta_{BS}^* = \frac{\sum_{k=1}^{K} w_k \theta_k}{\sum_{k=1}^{K} w_k} \tag{27}$$

which is equal to the weighted arithmetic mean value of the vertical angles of the scheduled users.

Summarizing the proposed low-complexity MUAB, for all possible sets in $\mathcal{S}$, we can find the user set which has the maximum $\sum_{k=1}^{K} w_k \log_2(1+\mathrm{SNR}_k)$. When calculating $\sum_{k=1}^{K} w_k \log_2(1+\mathrm{SNR}_k)$ for each user set $\mathcal{S}_n$, we set θ_{BS} to the weighted arithmetic mean value of the vertical angles for all users within the set, and we can compute $\mathbf{P}$ based on the water-filling algorithm for the given θ_{BS}. In this case, the overall complexity of (20) becomes almost the same as that of the conventional ZF-BF technique. Thus, the proposed scheme is much less complex than the joint optimization scheme.

B. Optimization for Passive Beamforming

For the case of the passive beamforming, once θ_{BS}^* is computed, it remains as a static value, while the optimal user selection set $\mathcal{S}_n^*$ and the optimal power allocation matrix $\mathbf{P}^*$ can be dynamically determined. Thus, we need to rewrite the optimization problem in (20) as

$$\begin{aligned} \max_{\theta_{BS}} \quad & \mathbb{E}_{r,\theta,\tilde{\mathbf{h}}}\left[\max_{\mathcal{S}_n\in\mathcal{S}} \max_{\mathbf{P}} \sum_{k=1}^{K} w_k \log_2(1+\mathrm{SNR}_k)\right] \\ \text{s.t.} \quad & \sum_{k=1}^{K} P_k \leq P. \end{aligned} \tag{28}$$

Note that the above problem is not convex either. Moreover, in some cases, w_k may vary over time. For example, in the case of proportional fair scheduling [3], we have $w_k(t+1) = (1-(1/t_c))(1/w_k(t)) + (1/t_c)\log_2(1+\mathrm{SNR}_k)$ where t_c is the averaging window size. Thus, solving the above problem may not be feasible. Instead, we provide a solution for this problem based on the result in Section III and the multi-sector concept, which is called a multi-sector multi-user passive beamforming (MS-MUPB) scheme.

The procedure of the MS-MUPB is as follows: We first divide a cell into multiple sectors. For example, the cell can be split into two sectors so that each sector covers half of users. In other words, the sectors are divided according to users' elevation in the LoS direction (vertical angle) in the proposed scheme, while a geometrical division is used in [18]. Then, we calculate the mean value of the vertical angle for each sector. In this case, the tilting angle in the first sector can be determined using the PDF of the vertical angle θ', i.e. $f_{\theta'}(\theta) = 2f_\theta(\theta)$ for $\theta_{\min} \leq \theta \leq \theta_{half}$ where $f_\theta(\theta)$ is the PDF of the vertical angle in the entire cell, $\theta_{\min}$ denotes the minimum vertical angle of the cell, and θ_{half} represents the vertical angle which satisfies $F_\theta(\theta_{half}) = 0.5$. Here $F_\theta(\cdot)$ equals the cumulative distribution function (CDF) of the vertical angle in the cell. Similarly, the second tilting angle can be calculated using the PDF of the vertical angle in the second sector θ'', i.e. $f_{\theta''}(\theta) = 2f_\theta(\theta)$ for $\theta_{half} \leq \theta \leq \theta_{\max}$ where $\theta_{\max}$ indicates the maximum vertical angle of the cell.

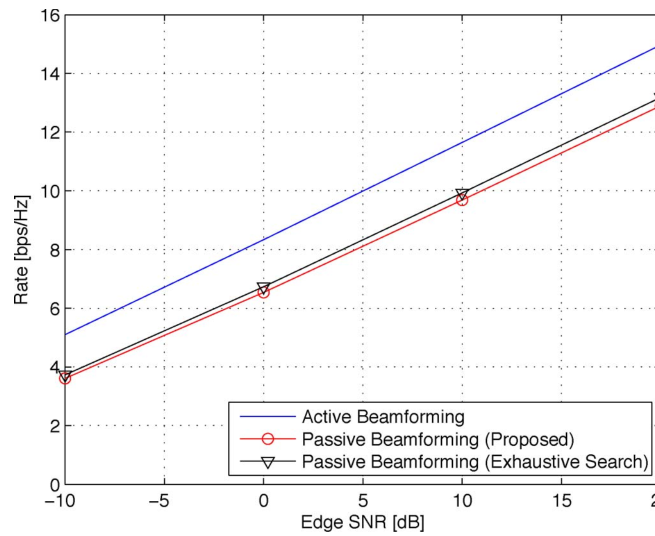

Fig. 4. Performance for single user systems ($M = 2$, $\theta_{3\text{ dB}} = 10°$).

After computing the tilting angles, we divide the transmit antennas to several groups and set the tilting angle for each group of antennas to the mean value of the vertical angle for the given sector. Then, we can find the user set which has the best $\sum_{k=1}^{K} w_k \log_2(1+\mathrm{SNR}_k)$ among all possible set $\mathcal{S}_n \in \mathcal{S}$. For each user set $\mathcal{S}_n$, we calculate $\mathbf{P}$ based on the water-filling algorithm for the given tilting angles. Note that compared with vertical sectorization in [16] which reduces inter-sector correlation by adjusting the vertical sector beam pattern, in MS-MUPB, we try to cover the equal amount of users per each sector while maintaining the vertical sector beam pattern as in (1).

V. Simulation Results

In this section, we demonstrate the efficiency of our proposed algorithms compared to those of the joint optimization scheme and the exhaustive search method where the best performance is chosen among all possible tilting angles with 0.01 radian resolution through Monte-Carlo simulations. It is also assumed that the maximum antenna gain at the antenna boresight $G_{\max}$ is 17 dB. We adopt the typical urban micro cell environment (B1) in [23] and [24] with slight modifications as listed in Table I. Here we denote h_{BS}, $h_{\mathrm{U,min}}$, and $h_{\mathrm{U,max}}$ by the BS antenna height, the minimum user antenna height, and the maximum user antenna height, respectively. In this case, the minimum and the maximum height difference are given by $z_0 = h_{\mathrm{BS}} - h_{\mathrm{U,max}}$ and $z_m = h_{\mathrm{BS}} - h_{\mathrm{U,min}}$, respectively. Moreover, we assume that the BS antenna structure is the uniform linear array and the antenna spacing between two adjacent directional antenna elements is 10 λ where λ represents the wavelength of the system. Also, we define the edge SNR as $r_m^{-\alpha} P/\sigma_n^2$.

Fig. 4 shows simulation results for systems with two transmit antennas and a single user ($M = 2$). The variance of the vertical angle becomes $\sigma_\theta^2 = 0.0155$ according to the simulation setting in Table I. As expected, we confirm from the plot that the active beamforming outperforms the passive beamforming.

TABLE I
SIMULATION SETTINGS

Maximum antenna gain	$G_{\max} = 17$ dB
Vertical 3 dB beamwidth	$\theta_{3\text{dB}} = 10^\circ$
BS antenna height	$h_{\text{BS}} = 10$ m
Minimum user antenna height	$h_{\text{U,min}} = 1.5$ m
Maximum user antenna height	$h_{\text{U,max}} = 15$ m
Minimum distance between a user and a BS	$r_0 = 10$ m
Cell radius	$r_m = 50$ m
Number of transmit antennas at a BS	$M = 2$ or 4
Number of receive antennas at a user	1
Pathloss exponent	3.4
a	$\frac{2}{(z_m - z_0)^2}$

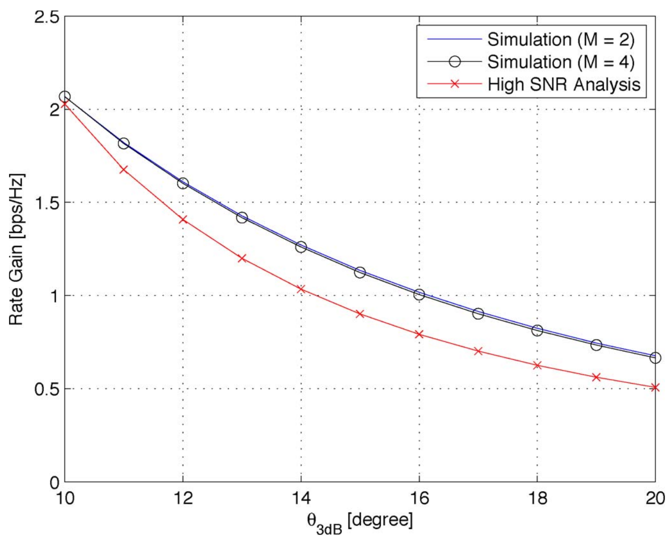

Fig. 5. Rate gains for different $\theta_{3\text{ dB}}$ at the edge SNR = 20 dB.

Particularly, a 2 bps/Hz gain is observed at medium to high SNR which accurately matches with the analysis in (17), i.e. $3.986(\sigma_\theta^2/\theta_{3\text{ dB}}^2) \approx 2$ bps/Hz. As we can see from the figure, the performance of the proposed passive beamforming scheme is quite close to that of the exhaustive search with much reduced complexity.

To show the effect of the vertical 3 dB beamwidth of the BS antennas $\theta_{3\text{ dB}}$, we evaluate the active beamforming and the passive beamforming for different $\theta_{3\text{ dB}}$, and plot the rate gain of the acive beamforming over the passive beamforming at the edge SNR of 20 dB as shown in Fig. 5. From (17), the gain is shown to be proportional to $1/\theta_{3\text{ dB}}^2$, and the curves of the actual rate gains in the plot have similar slopes to those of the high SNR analysis. Thus we can conclude that the rate gain is proportional to $1/\theta_{3\text{ dB}^2}$. Similarly, Fig. 6 depicts the effect of the variance of the vertical angle σ_θ^2. The simulation results support our analysis that the rate gain is proportional to σ_θ^2. From both plots, we can confirm that the trend of the rate gain matches well with the result derived in (17), i.e. $\sigma_\theta^2/\theta_{3\text{ dB}}^2$. Moreover, we verify that the rate gain is independent of the number of transmit antennas in the single user case.

Figs. 7 and 8 show the performance of multi-user systems for $M = 2$ and 4 using the proportional fair scheduling algorithm with $N = 5$. In particular, Fig. 7 presents the performance of multi-user active antenna systems. In the figure, we compare the MUAB and the joint optimization scheme as well as the switched beam tilting scheme (SBT) in [18] with two sectors

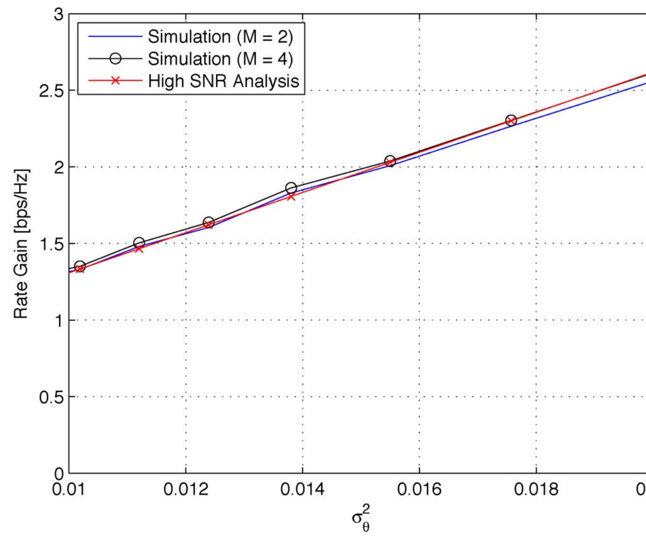

Fig. 6. Rate gain with different σ_θ^2 at the edge SNR = 20 dB.

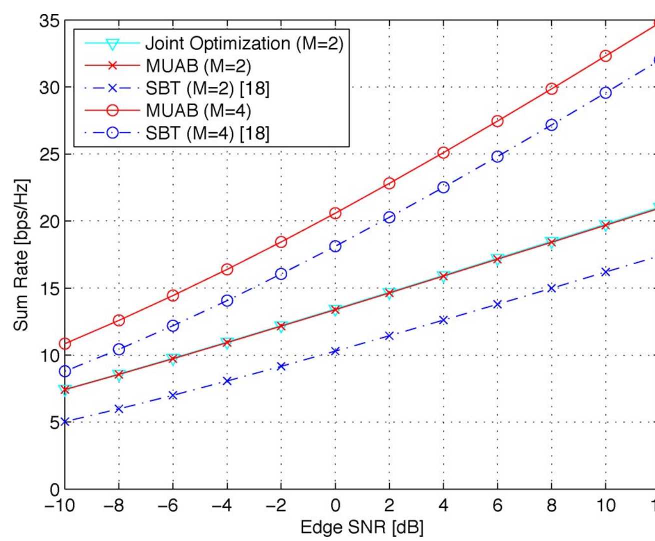

Fig. 7. Performance of multi-user active antenna systems.

as a conventional method. Note that, for a comparison purpose, we assume that $\theta_{3\text{ dB}}$ is the same for all systems and all sectors. Here, the joint optimization scheme for $M = 4$ is not included due to its extremely high complexity. In the figure, we observe gains of 5.7 dB and 2.1 dB for the MUAB over the SBT for $M = 2$ and 4, respectively. It is important to note that the proposed active antenna scheme achieves performance almost identical to that of the joint optimization, but with substantially reduced complexity.

Similarly, in Fig. 8, we present the performance of the two sector MS-MUPB systems using the tilting angles derived in Section IV-B and those in [18]. Note that the SBT is an active beamforming scheme where the tilting angles change within a set of predefined angles. In this simulation, to provide a conventional multi-user passive beamforming method, we apply the tilting angles in [18] to MS-MUPB. From the figure, we observe that the proposed scheme achieves about 1.1 dB gain at the sum rate of 15 bps/Hz over that of the conventional multi-user passive beamforming scheme for both $M = 2$ and 4.

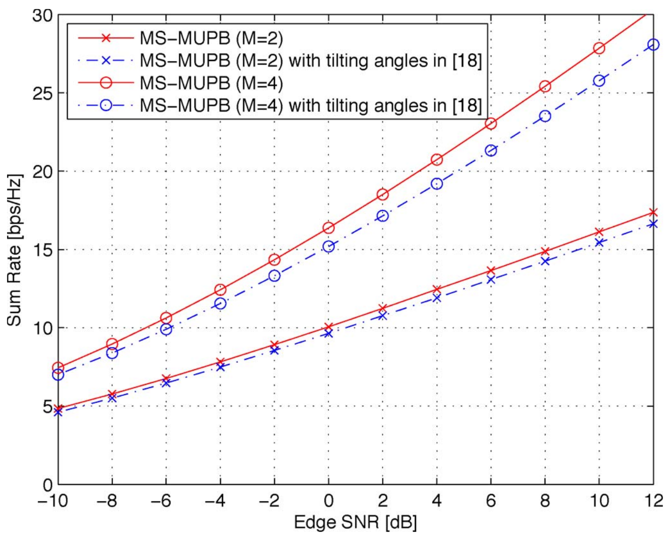

Fig. 8. Performance of multi-user passive antenna systems.

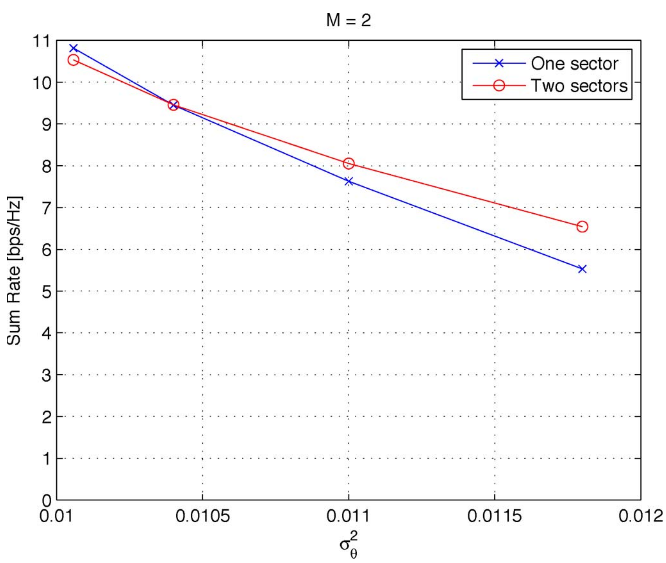

Fig. 9. Performance of MS-MUPB with different σ_θ^2 at the edge SNR $= 0$ dB.

It is worth mentioning that the overhead and complexity are identical in both schemes.

Finally we evaluate MS-MUPB with different σ_θ^2 in Fig. 9. We can confirm from the results in the figure that a large number of sectors perform better with larger vertical angle variations. The reason for this is that many users suffer from a low antenna gain with large vertical angle variations. On the other hand, if the number of sectors increases, the number of antennas per sector will be decreased. Thus, with small vertical angle variations, a small number of sectors will perform better.

VI. Conclusion

In this paper, we have studied transmit beamforming techniques for MIMO downlink single and multi-user systems with directional antennas where a transmit antenna gain is determined in 3-D coordinates. We have provided the PDF of the vertical angle of a user and the tilting angles for active and passive antenna systems for a flat fading channel. Moreover, we have derived the average rate gain of the active system over the passive antenna system for the single user case. For multi-user systems, we have proposed a new beamforming method by using a parameter separation method for the active beamforming system, greatly reducing the computational complexity. Also, for the passive beamforming system, we have introduced the multi-sector multi-user passive beamforming scheme. Simulation results show that the proposed multi-user active beamforming schemes achieve performance similar to that of the joint optimization scheme, but at much reduced complexity. Moreover, we have observed that the proposed beamforming schemes outperform the conventional beamforming scheme. An extension to multicell scenario would be meaningful, but we leave this as a future work.

Appendix A
Derivation of the PDF of θ

We derive the PDF of the random variable $\theta = \tan^{-1}(\Delta z/r)$ with $f_{\Delta z}(z) = az + b$ for $z_0 \leq z \leq z_m$ where $b = (2 - a(z_m^2 - z_0^2)/2(z_m - z_0))$, and $f_r(\gamma) = (2\gamma/r_m^2 - r_0^2)$ for $r_0 \leq \gamma \leq r_m$. First, the CDF of θ is computed as

$$\begin{aligned} F_\theta(\theta) &= Pr\left\{\tan^{-1}\frac{\Delta z}{r} \leq \theta\right\} \\ &= Pr\{\Delta z \leq r\tan\theta\} \\ &= \mathbb{E}\left[Pr\{\Delta z \leq \gamma\tan\theta\} | r = \gamma\right]. \end{aligned}$$

To obtain the above CDF, in case of $z_0 > 0$, we need to examine the following six cases.

i) For $r_m \tan\theta < z_0$:

$$F_\theta(\theta) = 0.$$

ii) For $r_0 \tan\theta < z_0$ and $z_0 < r_m \tan\theta < z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{\frac{z_0}{\tan\theta}}^{r_m} \gamma\left(\frac{a\gamma^2\tan^2\theta}{2} + b\gamma\tan\theta + c\right) d\gamma \\ &= -\frac{z_0^2 d\left(3az_0^2 + 8bz_0 + 12c\right)}{24\tan^2\theta} + \frac{cd}{2}r_m^2 \\ &\quad + \frac{bd}{3}r_m^3\tan\theta + \frac{ad}{8}r_m^4\tan^2\theta \end{aligned}$$

where $c = (a/2)z_0 z_m - (z_0/z_m - z_0)$ and $d = (2/r_m^2 - r_0^2)$.

iii) For $r_0 \tan\theta < z_0$ and $r_m \tan\theta > z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{\frac{z_0}{\tan\theta}}^{\frac{z_m}{\tan\theta}} \gamma\left(\frac{a\gamma^2\tan^2\theta}{2} + b\gamma\tan\theta + c\right) d\gamma \\ &\quad + d \cdot \int_{\frac{z_m}{\tan\theta}}^{r_m} \gamma d\gamma \\ &= \frac{z_m^2 d\left(3az_m^2 + 8bz_m + 12c - 12\right)}{24\tan^2\theta} \\ &\quad - \frac{z_0^2 d\left(3az_0^2 + 8bz_0 + 12c\right)}{24\tan^2\theta} + \frac{r_m^2 d}{2}. \end{aligned}$$

iv) For $z_0 < r_0 \tan\theta < z_m$ and $z_0 < r_m \tan\theta < z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{r_0}^{r_m} \gamma \left(\frac{a\gamma^2 \tan^2\theta}{2} + d\gamma \tan\theta + c \right) d\gamma \\ &= \frac{ad}{8}\left(r_m^4 - r_0^4\right)\tan^2\theta + \frac{bd}{3}\left(r_m^3 - r_0^3\right)\tan\theta \\ &\quad + \frac{cd}{2}\left(r_m^2 - r_0^2\right). \end{aligned}$$

v) For $z_0 < r_0 \tan\theta < z_m$ and $r_m \tan\theta > z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{r_0}^{\frac{z_m}{\tan\theta}} \gamma \left(\frac{a\gamma^2 \tan^2\theta}{2} + b\gamma \tan\theta + c \right) d\gamma \\ &\quad + d \cdot \int_{\frac{z_m}{\tan\theta}}^{r_m} \gamma \, d\gamma \\ &= \frac{z_m^2 d\left(3az_m^2 + 8bz_m + 12c - 12\right)}{24\tan^2\theta} - \frac{cd}{2}r_0^2 \\ &\quad + \frac{d}{2}r_m^2 - \frac{bd}{3}r_0^3\tan\theta - \frac{ad}{8}r_0^4\tan^2\theta. \end{aligned}$$

vi) For $r_0 \tan\theta > z_m$:

$$F_\theta(\theta) = d \cdot \int_{r_0}^{r_m} \gamma \, d\gamma = 1.$$

In case of $z_0 < 0$ and $z_m > 0$, we need to check the following five cases.

i) For $r_0 \tan\theta < z_0$:

$$F_\theta(\theta) = 0.$$

ii) For $r_m \tan\theta < z_0$ and $z_0 < r_0 \tan\theta < z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{r_0}^{\frac{z_0}{\tan\theta}} \gamma \left(\frac{a\gamma^2 \tan^2\theta}{2} + b\gamma \tan\theta + c \right) d\gamma \\ &= \frac{z_0^2 d\left(3az_0^2 + 8bz_0 + 12c\right)}{24\tan^2\theta} - \frac{cd}{2}r_0^2 \\ &\quad - \frac{bd}{3}r_0^3\tan\theta - \frac{ad}{8}r_0^4\tan^2\theta. \end{aligned}$$

iii) For $z_0 < r_m \tan\theta < z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{r_0}^{r_m} \gamma \left(\frac{a\gamma^2 \tan^2\theta}{2} + d\gamma \tan\theta + c \right) d\gamma \\ &= \frac{ad}{8}\left(r_m^4 - r_0^4\right)\tan^2\theta + \frac{bd}{3}\left(r_m^3 - r_0^3\right)\tan\theta \\ &\quad + \frac{cd}{2}\left(r_m^2 - r_0^2\right). \end{aligned}$$

iv) For $z_0 < r_0 \tan\theta < z_m$ and $r_m \tan\theta > z_m$:

$$\begin{aligned} F_\theta(\theta) &= d \cdot \int_{r_0}^{\frac{z_m}{\tan\theta}} \gamma \left(\frac{a\gamma^2 \tan^2\theta}{2} + b\gamma \tan\theta + c \right) d\gamma \\ &\quad + d \cdot \int_{\frac{z_m}{\tan\theta}}^{r_m} \gamma \, d\gamma \\ &= \frac{z_m^2 d\left(3az_m^2 + 8bz_m + 12c - 12\right)}{24\tan^2\theta} \\ &\quad - \frac{cd}{2}r_0^2 + \frac{d}{2}r_m^2 - \frac{bd}{3}r_0^3\tan\theta - \frac{ad}{8}r_0^4\tan^2\theta. \end{aligned}$$

v) For $r_0 \tan\theta > z_m$:

$$F_\theta(\theta) = d \cdot \int_{r_0}^{r_m} \gamma \, d\gamma = 1.$$

Then, the PDF of the random variable θ can be calculated by differentiating the CDF which results in (4), (5) and (6) depending on the value of z_0, z_m, r_0, and r_m.

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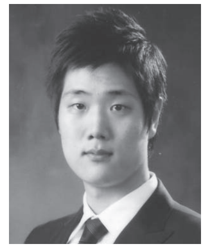

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