Capacity and Error Probability Analysis of Diversity Reception Schemes Over Generalized-K Fading Channels Using a Mixture Gamma Distribution

Jaehoon Jung, *Student Member, IEEE*, Sang-Rim Lee, *Student Member, IEEE*,
Haewook Park, *Student Member, IEEE*, Sunho Lee, and Inkyu Lee, *Senior Member, IEEE*

***Abstract*—In this paper, we analyze the error probability and ergodic capacity performance for diversity reception schemes over generalized-K fading channels using a mixture gamma (MG) distribution. With high accuracy, the MG distribution can approximate a variety of composite fading channel models and provide mathematically tractable properties. In contrast to previous analysis approaches that require complicated signal-to-noise ratio (SNR) statistics, it is shown that a distribution of the received SNR for diversity reception schemes is composed of a weighted sum of gamma distributions by exploiting the properties of the MG distribution. Then, based on this result, we can derive the exact average symbol error probability and simple closed-form expressions of diversity and array gains for maximal ratio combining and selection combining. In addition, an expression of the ergodic capacity for these schemes is obtained in independent and identically distributed fading channels. Our results lead to meaningful insights for determining the system performance with parameters of the MG distribution. We show that our analysis can be expressed with any number of receiver branches over various fading conditions. Numerical results confirm that the derived error probability and ergodic capacity expressions match well with the empirical results.**

***Index Terms*—Generalized-K fading, mixture gamma distribution, diversity technique, error probability, capacity, statistical metrics.**

I. Introduction

THE performance of wireless communication systems is generally affected by characteristics of the radio-wave propagation effect which includes shadowing and multipath fadings [1]. In a realistic wireless situation, the shadowing and the multipath fading occur simultaneously and they are often modeled by composite fading channels such as Rayleigh-lognormal (RL) and Nakagami-lognormal (NL) distribution [2]. However, in general, such lognormal based fading models are not given in a closed-form, which makes the performance analysis complicated [3].

In order to represent wireless propagation properties more tractably compared to the RL and NL distributions, several channel models have been proposed such as the K distribution [4], the generalized-K (K_G) distribution [5]–[7], and the generalized gamma distribution [8]–[10]. In particular, many researches focused on the K_G distribution which includes the K distribution as a special case, since a variety of fading cases can be covered simply by adjusting two shaping parameters with a closed-form expression [5]. For this reason, in this paper, we study the K_G distribution as a system channel model.

Many efforts have been devoted to analyze the average symbol error probability (ASEP) of diversity reception schemes over the K_G fading channel [11]–[18]. The authors in [14] analyzed the diversity gain by applying the moment generating function (MGF) based on Padé approximations. Due to the mathematically complex form, it is hard to recognize what parameters determine the system performance. Moreover, in the derived results, the generalized analysis of selection combining (SC) with any number of branches is difficult. In [19], it is noted that the evaluation of wireless performance becomes complicated because the K_G distribution includes the modified Bessel function of order $k - m$. Recently, the authors in [20] analyzed diversity and array gains by applying the MG distribution for maximal ratio combining (MRC) [21] and SC.

From a perspective of the information theoretic analysis, the capacity is an important measure of the system performance in wireless communication systems, as it provides significant intuition on the theoretical transmission limit [22], [23]. For L-branch diversity reception schemes over generalized fading channels, researchers in [24] and [25] analyzed the capacity with the MGF-based approaches by using complex functions such as the Meijer-G function [26] and the Fox-H function [27]. Also, the capacity of multiple-input multiple-output channels over K_G fadings was examined in [28]. In this case, in general, the probability density function (PDF) of the overall instantaneous signal-to-noise ratio (SNR) is in general not available in simple form, and the ergodic capacity analysis becomes intractable, since it apparently involves the L-fold integral defined by the joint multivariate PDF of the instantaneous SNRs for the L branch case. For these reasons, the PDF-based approaches are difficult for the performance analysis generally [25].

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J. Jung, S.-R. Lee, and I. Lee are with the School of Electrical Engineering, Korea University, Seoul 136-713, Korea (e-mail: jhnjung@korea.ac.kr; sangrim78@korea.ac.kr; inkyu@korea.ac.kr).

H. Park is with Incujector Inc., Seoul 135-812, Korea (e-mail: jetaime01@korea.ac.kr).

S. Lee is with the Department of Applied Mathematics, Sejong University, Seoul 143-747, Korea (e-mail: leesh@sejong.ac.kr).

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To solve these problems, a mixture gamma (MG) distribution can be adopted for the analysis. According to the results in [19], the MG distribution which is composed of a weighted sum of gamma distributions can approximate the K_G distribution as well as a variety of fadings with high accuracy by adjusting its parameters. Therefore, the MGF and the cumulative distribution function (CDF) can be obtained with closed form expressions that allow simple performance analysis.

In this paper, we perform the error probability and ergodic capacity analysis for diversity reception schemes over K_G fading channels using the MG distribution. First, we derive the PDF of the MRC and the SC by utilizing the properties of the MG distribution. We note that each PDF is also expressed as a form of the weighted sum of gamma distributions. Then, we derive the exact ASEP and meaningful closed-form expressions of the diversity gain as well as the array gain for the MRC and the SC. It is remarkable that unlike the work in [14], our derived results are applicable to systems with any number of receiver branches and various signal modulations over both independent and identically distributed (i.i.d.) and independent and non-identically distributed (i.n.d.) channel environments. Furthermore, we present expressions of the capacity for the MRC and the SC by applying the PDF-based approach in the i.i.d. case. As a result, the ergodic capacity of these diversity reception schemes can be obtained in an efficient way using the multinomial theorem in [29]. Experiments confirm that our derived analysis is well matched with the numerical results.

The organization of the paper is as follows: The system configuration and the MG distribution are described in Section II. As for the analytical framework, Section III derives the PDF of the MRC and the SC by adopting the MG distribution. Section IV presents the error probability and ergodic capacity analysis of diversity reception schemes over K_G fading channels. Through the numerical results in Section V, we confirm the validity of our proposed analysis. Finally, this paper is terminated with conclusions in Section VI.

Throughout the paper, we employ uppercase boldface letters for matrices and lowercase boldface letters for vectors. $\mathbb{E}[\cdot]$ denotes the expectation operator, $|\cdot|$ stands for the absolute value, and $(\cdot)^T$ means the transpose. Also, $\mathbf{I}_t$ indicates a $t \times t$ identity matrix and $n(\cdot)$ represents the number of elements in a set.

II. System Model

We consider single-input multiple-output systems where a diversity reception technique at a receiver is adopted over K_G fading channels. Assuming that the receiver is equipped with L antennas, the received signal $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{h}s + \mathbf{n} \tag{1}$$

where $\mathbf{h} = [h_1, \ldots, h_L]^T \in \mathbb{C}^L$ indicates the K_G fading channel vector, s is the transmitted complex signal symbol with energy $E_s = \mathbb{E}[|s|^2]$, and $\mathbf{n} = [n_1, \ldots, n_L]^T \sim \mathcal{CN}(0, N_0\mathbf{I}_L)$.

In order to analyze the performance, we first investigate the PDF of SNR over K_G fading channels with $L = 1$. Then, the PDF of the SNR ρ can be obtained by [13]

$$f_\rho(x) = \frac{2\nu^{(k+m)/2} x^{(k+m-2)/2}}{\Gamma(m)\Gamma(k)} K_{k-m}(2\sqrt{\nu x}), \quad \text{for } x \geq 0 \tag{2}$$

where k and m stand for the shaping parameters of K_G fading channels which represent the multipath fading and the shadowing effect, respectively, $\Gamma(\cdot)$ denotes the gamma function, $K_{k-m}(\cdot)$ is the modified Bessel function of order $k-m$ [26], ν is defined as $\nu = km/\bar{\rho}$, and $\bar{\rho}$ equals the average SNR. From (2), important channel model statistics such as the CDF and the MGF can be derived [13]. However, in [19], it noted that mathematical complications arise in the evaluation of wireless performance since the K_G distribution includes the modified Bessel function of order $k-m$. It causes difficulty in proceeding to further steps for the performance analysis.

In order to tackle these problems, we adopt the MG distribution of the SNR in this model. As mentioned before, the MG distribution has attractive properties such that an accurate approximation is possible for a variety of composite fadings by utilizing mathematically tractable expressions. The PDF of ρ in the form of the MG distribution is expressed by [19]

$$f_\rho(x) = \sum_{i=1}^{N} w_i f_i(x) = \sum_{i=1}^{N} \alpha_i x^{\beta_i - 1} e^{-\zeta_i x} \tag{3}$$

where $w_i = \alpha_i \Gamma(\beta_i)/\zeta_i^{\beta_i}$ means the normalization factor with $\sum_{i=1}^{N} w_i = 1$, $f_i(x) = \zeta_i^{\beta_i} x^{\beta_i - 1} e^{-\zeta_i x}/\Gamma(\beta_i)$ denotes the standard gamma distribution, and α_i, β_i, and ζ_i are the parameters of the ith mixture gamma component. Here, the number of terms N determines the accuracy measured by the mean square error $\mathbb{E}[|f_{ext}(x) - f_{MG}(x)|^2]$ between the exact PDF $f_{ext}(x)$ and the MG distribution $f_{MG}(x)$, or the Kullback-Leibler divergence $\mathcal{D}_{KL} = \int_{-\infty}^{\infty} f_{ext}(x) \log(f_{ext}(x)/f_{MG}(x))dx$ [30].

For the K_G fading channel model, the PDF of ρ in (2) can be approximated by the form of (3) with parameters α_i, β_i, and ζ_i as [19]

$$\alpha_i = \frac{\theta_i}{\sum_{j=1}^{N} \theta_j \Gamma(\beta_j) \zeta_j^{-\beta_j}}, \ \beta_i = m, \ \zeta_i = \frac{\nu}{t_i}, \ \theta_i = \frac{\nu^m y_i t_i^{\lambda - 1}}{\Gamma(m)\Gamma(k)} \tag{4}$$

where $\lambda = k - m$, and y_i and t_i are the weight factor and the abscissas for the Gaussian-Laguerre integration [31], respectively. Also, the CDF of ρ can be computed as

$$F_\rho(x) = \int_0^x f_\rho(t)dt = \sum_{i=1}^{N} \alpha_i \zeta_i^{-\beta_i} \gamma(\beta_i, \zeta_i x) \tag{5}$$

where $\gamma(\cdot,\cdot)$ indicates the lower incomplete gamma function defined as $\gamma(c, \sigma) \triangleq \int_0^\sigma t^{c-1} e^{-t} dt$ [26].

Based on (3), the MGF is evaluated as [13]

$$\mathcal{M}_\rho(s) = \int_0^\infty e^{-sx} f_\rho(x)dx = \sum_{i=1}^{N} \frac{\alpha_i \Gamma(\beta_i)}{(s + \zeta_i)^{\beta_i}}. \tag{6}$$

Compared to the exact MGF in [13], it is worth noting that (6) has a simpler form. The MGF in [13] includes a complicated Whittaker function. Using the PDF, the CDF, and the MGF obtained from the MG distribution, we will provide simple and clear solutions for diversity and array gains with parameters α_i, β_i, and ζ_i in the following section.

III. Probability Density Function of Diversity Schemes

In this section, we derive expressions of the PDFs of diversity reception schemes to provide the mathematical framework for analyzing the performance. As mentioned before, the PDFs of the MRC and the SC can be expressed by the weighted sum of gamma distributions. Thus, the analysis of the exact error probability and ergodic capacity becomes manageable with the derived PDFs, which will be addressed in Section IV. Now, we investigate the PDFs of two diversity schemes.

A. Maximal Ratio Combining

The received signals from all branches are weighted and combined to maximize the output SNR in the MRC scheme [32]. Let ρ_i denote the SNR for the i th branch for $i = 1, \ldots, L$. Then, the combined SNR for the MRC is written by $\rho_{MRC} = \sum_{i=1}^{L} \rho_i$. Assuming that $\{\rho_i\}$ are independent, the MGF of ρ_{MRC} can be computed as

$$\mathcal{M}_{\rho_{MRC}}(s) = \mathbb{E}[e^{-s\rho_{MRC}}] = \prod_{i=1}^{L} \mathcal{M}_{\rho_i}(s). \tag{7}$$

From (7), the MGF of the MRC can be alternatively represented by using the multinomial theorem [29], which is given as

$$\left(\sum_{i=1}^{N} x_i\right)^L = \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} x_i^{k_i} \tag{8}$$

where $\binom{L}{k_1,\ldots,k_N} = L!/k_1!\ldots k_N!$. By applying this theorem, the MGF of the MRC is rewritten as

$$\begin{aligned}\mathcal{M}_{\rho_{MRC}}(s) &= \left(\sum_{i=1}^{N} \frac{\alpha_i \Gamma(m)}{(s+\zeta_i)^m}\right)^L \\ &= \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \left(\alpha_i \Gamma(m)(s+\zeta_i)^{-m}\right)^{k_i}.\end{aligned} \tag{9}$$

From the relationship between the PDF and the MGF $\mathcal{L}^{-1}\{\mathcal{M}_\rho(s)\} = f_\rho(x)$, the PDF of the MRC (10), shown at the bottom of the page, can be obtained by the inverse Laplace transform of (9), $f_{\rho_{MRC}}(x) = \mathcal{L}^{-1}\{\mathcal{M}_{\rho_{MRC}}(s)\}$. By using the identity,

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)^m}\right\} = \frac{x^{m-1}}{(m-1)!} e^{-ax}, \tag{12}$$

the PDF of the MRC follows as (11), shown at the bottom of the page, where R_{jl} denotes the coefficient of the l th term for $a = \zeta_j$.

B. Selection Combining

In the SC scheme, we choose the branch with the largest SNR, e.g., $\rho_{SC} = \max(\rho_1, \rho_2, \ldots, \rho_L)$. The CDF of the SC is formulated as [13]

$$F_{\rho_{SC}}(x) = \prod_{i=1}^{L} F_{\rho_i}(x). \tag{13}$$

For generalized L branches, the PDF of the SC is composed of the product of the PDF in (3) and the CDF in (5) as

$$f_{\rho_{SC}}(x) = L F_\rho(x)^{L-1} f_\rho(x). \tag{14}$$

Similar to the MRC case, the PDF of the SC is represented by the method of the multinomial theorem as (15), shown at the top of the next page. In order to make (15) more tractable, we transform the product form of the lower incomplete gamma function $\prod_{1\le l\le N} \gamma(m, \zeta_l x)^{k_l}$ into a summation form. Applying the recurrence relation $\gamma(m, x) = (m-1)\gamma(m-1, x) - x^{m-1}e^{-x}$ [26], $\gamma(m, \zeta_l x)$ is converted into

$$\gamma(m, \zeta_l x) = (m-1)\left[1 - \sum_{k=0}^{m-1} \frac{\zeta_t^k}{k!} x^k e^{-\zeta_l x}\right]. \tag{18}$$

In what follows, we will show that after inserting (18) into (15), the PDF of the SC can be represented by the summation form which simplifies the PDF-based capacity analysis. For illustrative purposes, we begin by presenting the product form of $\gamma(m, \zeta_l x)$ for the case of $L = 3$ and $l = 1,\ 2$ as an example. In this case, the product form of $\gamma(m, \zeta_l x)$ is given as

$$\begin{aligned}&\gamma(m, \zeta_1 x)\gamma(m, \zeta_2 x) \\ &\quad = \{(m-1)!\}^2 \left[1 - K(\zeta_1, \zeta_2; m) + T(\zeta_1, \zeta_2; m)\right]\end{aligned} \tag{19}$$

where

$$\begin{aligned}K(\zeta_1, \zeta_2; m) &= \sum_{j=1}^{2} \sum_{k=0}^{m-1} \frac{\zeta_j^k}{k!} x^k e^{-\zeta_j x}, \\ T(\zeta_1, \zeta_2; m) &= \sum_{k=0}^{m-1} \frac{\zeta_1^k}{k!} x^k \sum_{p=0}^{m-1} \frac{\zeta_2^p}{p!} x^p e^{-(\zeta_1+\zeta_2)x}.\end{aligned} \tag{20}$$

$$f_{\rho_{MRC}}(x) = (\Gamma(m))^L \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \alpha_i^{k_i} \mathcal{L}^{-1}\left\{\prod_{1\le i\le N} (s+\zeta_i)^{-k_i m}\right\} \tag{10}$$

$$f_{\rho_{MRC}}(x) = (\Gamma(m))^L \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \alpha_i^{k_i} \sum_{j=1}^{N} \sum_{l=0}^{k_j m} R_{jl} \frac{x^l}{l!} e^{-\zeta_j x} \tag{11}$$

$$f_{\rho_{SC}}(x)=L\sum_{i=1}^{N}\alpha_i\sum_{k_1+\ldots+k_N=L-1}\binom{L-1}{k_1,\ldots,k_N}\prod_{1\le j\le N}(\alpha_j\zeta_j^{-m})^{k_j}\prod_{1\le l\le N}\gamma(m,\zeta_l x)^{k_l}x^{m-1}e^{-\zeta_i x}. \quad (15)$$

$$\begin{aligned}H(x,\mathcal{S},i)&\triangleq\prod_{1\le j\le N}\gamma(m,\zeta_j x)^{k_j}x^{m-1}e^{-\zeta_i x}\\&=\{(m-1)!\}^{L-1}\left[x^{m-1}e^{-\zeta_i x}+\sum_{r=1}^{L-1}\left\{(-1)^r\sum_{\substack{\mathcal{S}\\ p_k\in\mathcal{P}}}\sum_{l=0}^{r(m-1)}\sum_{h=1}^{B}\prod_{k=1}^{r}\frac{(\zeta_{\tilde{\mathcal{S}}_{hk}})^{p_k}}{p_k!}x^{l+m-1}e^{-(\sum_{k=1}^{r}\zeta_{\tilde{\mathcal{S}}_{hk}}+\zeta_i)x}\right\}\right]\end{aligned} \quad (16)$$

$$f_{\rho_{SC}}(x)=L\sum_{i=1}^{N}\alpha_i\sum_{k_1+\ldots+k_N=L-1}\binom{L-1}{k_1,\ldots,k_N}\prod_{1\le j\le N}(\alpha_j\zeta_j^{-m})^{k_j}H(x,\mathcal{S},i). \quad (17)$$

After some manipulations, $T(\zeta_1,\zeta_2;m)$ can also be written with the weighted sum of gamma distributions as

$$T(\zeta_1,\zeta_2;m)=\sum_{s=0}^{2(m-1)}\sum_{\substack{i+j=s\\0\le i,j\le m-1}}\frac{\zeta_1^i\zeta_2^j}{i!j!}x^s e^{-(\zeta_1+\zeta_2)x}. \quad (21)$$

Now, we consider the L branches case for simplifying the PDF of the SC. By observing the characteristics of $K(\cdot)$ and $T(\cdot)$ for given L, we define $H(x,\mathcal{S},i)$ as (16), shown at the top of the page, where $\mathcal{S}$ is a set with k_j repeated elements for each j $(j=1,\ldots,N)$, $\mathcal{P}$ is given by $\mathcal{P}=\{p_1,\ldots,p_r\}$ such that $\sum_{k=1}^{r}p_k=l, n(\mathcal{P})=r$, $0\le p_k\le m-1$, B denotes ${}_{n(\mathcal{S})}C_r$, $\tilde{\mathcal{S}}$ represents a set of subsets that are organized by choosing r elements from the given set $\mathcal{S}$, and $\tilde{\mathcal{S}}_{hk}$ stands for the k th element of the h th subset of $\tilde{\mathcal{S}}$. For example, in case of $k_1=2$, $k_3=1$, and $r=2$ for $L=4$, $\mathcal{S}$ and $\tilde{\mathcal{S}}$ are given by $\mathcal{S}=\{1,1,3\}$ and $\tilde{\mathcal{S}}=\{(1,1),(1,3),(1,3)\}$, respectively.

Note that the PDF of the SC is represented by the weighted sum of $x^{m-1}e^{-\zeta_i x}$ and $x^{l+m-1}e^{-(\sum_{k=1}^{r}\zeta_{\tilde{\mathcal{S}}_{hk}}+\zeta_i)x}$ as (17), shown at the top of the page. It can be seen that the PDF of the SC in (14) is directly derived by the PDF and the CDF of a single branch utilizing the MG distribution. This is due to the fact that the product form of the lower incomplete gamma function can be modified by the weighted sum of gamma distributions. Then, the performance analysis of the SC follows a similar procedure of the MRC in the following section.

IV. Performance Analysis of Diversity Schemes

In this section, we analyze the ASEP and ergodic capacity for diversity reception schemes. First, the exact ASEP is presented with a closed form. Then, to give meaningful insight on the performance, we also introduce the asymptotic ASEP analysis which provides diversity and array gains. Lastly, the analysis of the ergodic capacity is derived in an efficient manner. The key point making the PDF-based analysis tractable is that unlike the MGF containing other complex functions, the inverse Laplace transform can be calculated by exploiting the MGF formulated with the MG distribution. Thus, with the derived expression of the PDF which consists of the form of gamma distributions, an efficient computation of the performance analysis is enabled. In the following, we investigate the analysis of the ASEP and the ergodic capacity.

A. Average Symbol Error Probability Analysis

1) Exact ASEP: To begin with, we introduce the ASEP P_E as

$$P_E=\int_0^\infty P_{E|\rho}(x)f_\rho(x)dx \quad (22)$$

where $P_{E|\rho}(x)$ denotes the symbol error probability (SEP) for a given SNR ρ, which is usually expressed with a Q-function, defined as $Q(x)=(1/\sqrt{2\pi})\int_x^\infty e^{-(u^2/2)}du$ [34]. By employing the Craig's formula for the Gauss Q-function $Q(x)=(1/\pi)\int_0^{\pi/2}\exp(-(x^2/2\sin^2\theta))d\theta$ [35], we can rewrite $P_{E|\rho}(x)$ as [35]

$$P_{E|\rho}(x)=Q(\sqrt{2\omega x})=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{\omega x}{\sin^2\theta}\right)d\theta \quad (23)$$

where ω is a positive constant which depends on modulation schemes.

By utilizing the Craig's form and the MG distribution that consist of the weighted sum of gamma distributions, the exact ASEP (22) is calculated as

$$P_E=\frac{1}{\pi}\sum_{i=1}^{N}\alpha_i\int_0^{\frac{\pi}{2}}\int_0^\infty x^{\beta_i-1}\exp\left(-\left(\zeta_i+\frac{\omega}{\sin^2\theta}\right)x\right)dxd\theta. \quad (24)$$

Adopting $\int_0^\infty x^p e^{-qx}dx=\Gamma(p+1)q^{-(p+1)}$ [26] and the definition from [35]

$$J_\lambda(c)\triangleq\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta+c}\right)^\lambda d\theta, \quad (25)$$

an expression of P_E for the K_G fading channel is rewritten as

$$P_E=\frac{1}{\pi}\sum_{i=1}^{N}\alpha_i\zeta_i^{-m}\Gamma(m)J_m\left(\frac{\omega}{\zeta_i}\right). \quad (26)$$

From (26), it is remarkable that the exact ASEP of diversity schemes can be calculated in a similar manner with a closed form expression, since the derived PDFs in Section III also have the properties of the gamma distribution. Therefore, by exploiting the expression $J_\lambda(c)$, the ASEP of the MRC and the SC are calculated by (27), shown at the bottom of the page. In (28), shown at the bottom of the page, the integral term is simply presented as (29), shown at the bottom of the page.

2) Asymptotic ASEP: Next, we investigate the diversity and array gains of the MRC and the SC. It is well known that the ASEP for fading channels is approximated in the high SNR region by [36]

$$P_E \approx (G_a \cdot \bar{\rho})^{-G_d} \tag{30}$$

where G_a is an array gain which accounts for the shift of ASEP curves and G_d represents a diversity order which indicates the slope of ASEP curves with respect to the average SNR in a log-log scale. Consequently, the ASEP can be quantitatively parameterized with G_a and G_d, which are key factors of the diversity analysis. In this sense, we derive the diversity gain and the array gain for K_G fading channel models by applying the MG distribution.

It was shown in [1] that the MGF is a convenient tool to analyze the system performance in comparison to the PDF approach. Thus, we examine the characteristics of the MGF of the output SNR. At high SNR, P_E is dominated by the worst event, which means that the behavior of $f_\rho(x)$ at $x \to 0$ determines the system characteristics in the high SNR region. This property is related to the decaying behavior of the MGF [37]. The lemma in [37] characterizes the relation between the ASEP and the MGF, which describes a diversity gain and an array gain as

$$\begin{aligned} G_d &= \delta, \\ G_a &= \omega \left(\frac{2^{\delta-1}\mu\Gamma(\delta+\frac{1}{2})}{\sqrt{\pi}\Gamma(\delta+1)} \right)^{-\frac{1}{\delta}}, \end{aligned} \tag{31}$$

where ω is a positive constant for modulation schemes. Since the ASEP is calculated by the integration of the MGF, the diversity and array gains are expressed in terms of the parameters of $|\mathcal{M}_\rho(s)|$, i.e., μ and δ. Based on this result, we will derive the MGF of the MRC and the SC over composite fading channels with the MG distribution in the following.

In the MRC scheme, as mentioned before, the MGF of the MRC can be expressed as (7). Then, $|\mathcal{M}_\rho(s)|$ over K_G fading channels is given as [19]

$$|\mathcal{M}_\rho(s)| = \sum_{i=1}^{N} \alpha_i \Gamma(\beta_i) \left(s^{-\beta_i} + \left(\sum_{k=1}^{\infty} \binom{\beta_i}{k} \zeta_i^k s^{\beta_i+k} \right)^{-1} \right) \tag{32}$$

where $\binom{\beta_i}{k} = {}_{\beta_i}C_k$. Therefore, inserting (32) into (7), we can obtain $|\mathcal{M}_{\rho_{MRC}}(s)|$ as

$$\begin{aligned} |\mathcal{M}_{\rho_{MRC}}(s)| &= \prod_{i=1}^{L} \left| \sum_{j=1}^{N} \alpha_{ij}\Gamma(\beta_{ij}) s^{-\beta_{ij}} + o\left(s^{-(\beta_{ij}+1)}\right) \right| \\ &= \prod_{i=1}^{L} \left(\Gamma(m_i) \sum_{j=1}^{N} \alpha_{ij} \right) |s|^{-\sum_{i=1}^{L} m_i} \\ &\quad + o(s^{-(\sum_{i=1}^{L} m_i + 1)}) \end{aligned} \tag{33}$$

$$\begin{aligned} P_{E\rho_{MRC}} &= (\Gamma(m))^L \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \alpha_i^{k_i} \sum_{j=1}^{N} \sum_{l=0}^{k_j m} \frac{R_{jl}}{l!} \int_0^\infty P_{E|\rho}(x) x^{(l+1)-1} e^{-\zeta_j x} dx \\ &= \frac{(\Gamma(m))^L}{\pi} \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \alpha_i^{k_i} \sum_{j=1}^{N} \sum_{l=0}^{k_j m} \frac{R_{jl}}{l!} \frac{\Gamma(l+1)}{\zeta_j^{l+1}} J_{l+1}\left(\frac{\omega}{\zeta_j}\right) \end{aligned} \tag{27}$$

$$P_{E\rho_{SC}} = L \sum_{i=1}^{N} \alpha_i \sum_{k_1+\ldots+k_N=L-1} \binom{L-1}{k_1,\ldots,k_N} \prod_{1\le j\le N} \left(\alpha_j \zeta_j^{-m}\right)^{k_j} \int_0^\infty P_{E|\rho}(x) H(x,\mathcal{S},i) dx \tag{28}$$

$$\begin{aligned} &\int_0^\infty P_{E|\rho}(x) H(x,\mathcal{S},i) dx \\ &= \frac{\{(m-1)!\}^{L-1}}{\pi} \left(\frac{\Gamma(m)}{\zeta_i^m} J_m\left(\frac{\omega}{\zeta_i}\right) + \sum_{r=1}^{L-1} (-1)^r \sum_{\substack{\mathcal{S} \\ p_k \in \mathcal{P}}} \sum_{l=0}^{r(m-1)} \sum_{h=1}^{B} \prod_{k=1}^{r} \frac{\zeta_{\tilde{\mathcal{S}}_{hk}}^{p_k}}{p_k!} \frac{\Gamma(l+m)}{\left(\sum_{k=1}^{r} \zeta_{\tilde{\mathcal{S}}_{hk}} + \zeta_i\right)^{-(l+m)}} J_{l+m}\left(\frac{\omega}{\sum_{k=1}^{r} \zeta_{\tilde{\mathcal{S}}_{hk}} + \zeta_i} \right) \right) \end{aligned} \tag{29}$$

where α_{ij} and β_{ij} denote the parameters of the j th term in the MG distribution at the i th branch channel model ($i = 1, \ldots, L$ and $j = 1, \ldots, N$). It is noted that the high order term $o(s^{-(\sum_{i=1}^{L} m_i+1)})$ in (33) does not affect the results of the diversity analysis. In case of K_G fading channels, the parameter β_i is equal to the multipath fading effect parameter m_i for all terms of the MG distribution, i.e., $\beta_{ij} = m_i$.

Consequently, $|\mathcal{M}_{\rho_{MRC}}(s)|$ is expressed in the form of $\mu|s|^{-\delta}$ as

$$|\mathcal{M}_{\rho_{MRC}}(s)| \approx \mu_{MRC}|s|^{-\delta_{MRC}} \tag{34}$$

where

$$\mu_{MRC} = \prod_{i=1}^{L}\left(\Gamma(m_i)\sum_{j=1}^{N}\alpha_{ij}\right),$$
$$\delta_{MRC} = \sum_{i=1}^{L} m_i. \tag{35}$$

By substituting (35) into (31), we can determine G_a and G_d for the MRC as a function of α_{ij} and m_i in quantitative manner.

In the SC scheme, we compute the MGF of the SC as $\mathcal{M}_{\rho_{SC}}(s) = \int_0^\infty e^{-sx} f_{\rho_{SC}}(x)dx$. For simple presentations, we set $L = 2$ and assume an i.i.d. case. An extension to the general case is straightforward. Using (3) and (5), the PDF of the SC is given as

$$\begin{aligned} f_{\rho_{SC}}(x)|_{L=2} &= 2F_\rho(x)f_\rho(x) \\ &= 2\left[\sum_{i=1}^{N}\alpha_i\zeta_i^{-\beta_i}\gamma(\beta_i,\zeta_i x)\right]\left[\sum_{i=1}^{N}\alpha_i x^{\beta_i-1}e^{-\zeta_i x}\right]. \end{aligned} \tag{36}$$

Here, note that the lower incomplete gamma function $\gamma(a, \sigma)$ can be given by an alternative form $\gamma(c, \sigma) = e^{-\sigma}\sum_{k=0}^{\infty}(\sigma^{c+k}/c(c+1)\ldots(c+k))$ [26].

From this expression, the first summation term in (36) can be represented as

$$\begin{aligned} &\alpha_i\zeta_i^{-\beta_i}\gamma(\beta_i,\zeta_i x) \\ &= \alpha_i\zeta_i^{-\beta_i}e^{-\zeta_i x}\left[\frac{1}{\beta_i}(\zeta_i x)^{\beta_i} + \frac{1}{\beta_i(\beta_i+1)}(\zeta_i x)^{\beta_i+1} + \ldots\right] \\ &= \alpha_i\beta_i^{-1}x^{\beta_i}e^{-\zeta_i x} + o(x^{\beta_i+1}). \end{aligned} \tag{37}$$

Since the high order term $o(x^{\beta_i+1})$ in (37) does not affect the diversity, we can ignore this term. Substituting (37) into (36), we obtain the PDF of the SC as

$$\begin{aligned} f_{\rho_{SC}}(x)|_{L=2} &= \frac{2}{m}x^{2m-1} \\ &\times \sum_{i=1}^{N}\left(\alpha_i^2 e^{-2\zeta_i x} + 2\sum_{\substack{j=1 \\ j\neq i}}^{N}\alpha_i\alpha_j e^{-(\zeta_i+\zeta_j)x}\right). \end{aligned} \tag{38}$$

In order to provide a simpler result, using (38), the first term of the MGF can be written as

$$\begin{aligned} &\int_0^\infty e^{-sx}\frac{2}{m}x^{2m-1}\alpha_1^2 e^{-2\zeta_1 x}dx \\ &= \frac{2\alpha_1^2}{m}(2m-1)!(s+2\zeta_1)^{-(2m-1)-1} \\ &= \frac{2\alpha_1^2}{m}\Gamma(2m)\frac{1}{\sum_{k=0}^{2m}\binom{2m}{k}s^{2m-k}\zeta_1^k} \end{aligned} \tag{39}$$

where $\int_0^\infty t^n e^{-\psi t}dt = n!\psi^{-n-1}$ [26]. Applying the calculation in (39) for all terms leads to

$$\begin{aligned} &|\mathcal{M}_{\rho_{SC}}(s)|\Big|_{L=2} \\ &= \frac{2}{m}\left(\sum_{i=1}^{N}\alpha_i\right)^2\Gamma(2m)|s|^{-2m} + o\left(|s|^{-(2m+1)}\right). \end{aligned} \tag{40}$$

In the process of deriving (40), the terms of ζ are not considered for computing the parameters μ and δ because ζ is contained only in the high order term $o(|s|^{-(2m+1)})$.

For the general case of L, $|\mathcal{M}_{\rho_{SC}}(s)|$ in the i.n.d. case is obtained in a similar manner by

$$|\mathcal{M}_{\rho_{SC}}(s)| \approx \mu_{SC}|s|^{-\delta_{SC}} \tag{41}$$

where

$$\mu_{SC} = \frac{\sum_{i=1}^{L} m_i}{\prod_{i=1}^{L} m_i}\Gamma\left(\sum_{l=1}^{L} m_l\right)\prod_{j=1}^{L}\left(\sum_{k=1}^{N}\alpha_{jk}\right),$$
$$\delta_{SC} = \sum_{i=1}^{L} m_i. \tag{42}$$

It is clear that G_a and G_d for SC are determined with the parameterized expressions by plugging (42) into (31).

B. Ergodic Capacity Analysis

In this subsection, we derive expressions of the ergodic capacity using the derived PDFs of the MRC and the SC. First, the ergodic capacity C_{erg} is defined as

$$C_{erg} = \int_0^\infty \log_2(1+x)f_\rho(x)dx. \tag{46}$$

For simple calculation of the capacity, we can apply a useful identity in [1] as

$$\begin{aligned} \int_0^\infty \ln(1+x)x^{n-1}e^{-\mu x}dx &= (n-1)!e^\mu\sum_{k=1}^{n}\frac{\Gamma(k-n,\mu)}{\mu^k} \\ &\triangleq I_n(\mu) \end{aligned} \tag{47}$$

where $\Gamma(\cdot,\cdot)$ indicates the upper incomplete gamma function defined as $\Gamma(a, b) = \int_b^\infty t^{a-1}e^{-t}dt$ [26]. By utilizing the function $I_n(\mu)$, the ergodic capacity analysis for the MRC and the SC is achieved in a simple manner. For succint explanations, we consider the i.i.d. case. However, our result can be easily extended to the i.n.d. case.

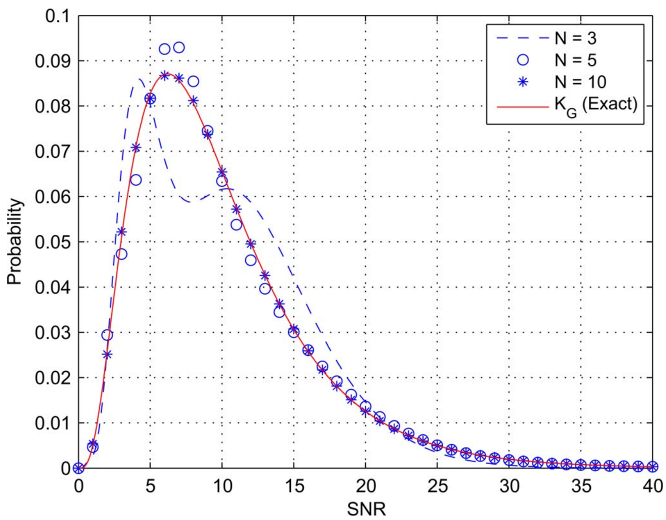

Fig. 1. Comparison of the K_G distribution and MG distribution with different N ($k = 5$ and $m = 7$).

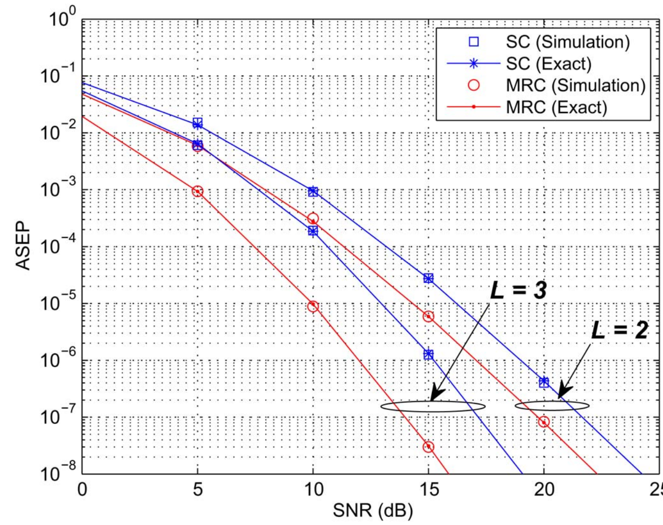

Fig. 2. ASEP comparison between analysis and simulation for the MRC and the SC ($k = 5$ and $m = 2$).

Then, for positive integer shaping parameters k and m, we can derive the ergodic capacity of the MRC (43), shown at the bottom of the page, and the SC (44) by utilizing the expression $I_n(\mu)$. Also, in (44), the integral term is simply calculated as (45), shown at the bottom of the page.

V. Numerical Results

In this section, we confirm the validity of our error probability and ergodic capacity analysis through Monte Carlo simulations. In Fig. 1, we evaluate the accuracy of the MG distribution with respect to the number of terms N. In this plot, we compare the PDF of the K_G distribution and the MG distribution with various N for $k = 5$, $m = 7$, and $\bar{\rho} = 10$ dB. From this plot, it is verified that when N equals 10, the MG distribution nearly corresponds to the exact PDF. As a result, we can conclude that the PDF of the K_G fading channel is well approximated by utilizing the MG distribution with a large N. Throughout the simulations, the number of terms N is set to 10.

For all simulations, we employ the following procedures. For each average SNR, we calculate the parameter of the MG distribution, i.e., α_i and ζ_i $(i = 1, \ldots, N)$. Then, the analysis curves with L receiver branches are calculated and plotted in figures. In numerical results by Monte Carlo simulations, we generate the instantaneous SNR value from the PDF of K_G fading channels [38]. From these values, the ASEP and the ergodic capacity are computed for each scheme.

Figs. 2 and 3 illustrate the ASEP for MRC and SC for the i.i.d. case with $L = 2$ and 3 as a function of $\bar{\rho}$ for BPSK with $m = 2$. Fig. 2 presents the comparison of the ASEP between the analysis and the numerical result. It is clear that, from this figure, the empirical results correspond to the exact ASEP analysis. In Fig. 3, as expected, we can see that the diversity gains for both schemes are the same, while the array gain of the MRC is greater than that of the SC. From (35) and (42), the diversity gains of both schemes for $L = 2$ and 3 are computed

$$
\begin{aligned}
C_{\rho_{MRC}} &= (\Gamma(m))^L \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \alpha_i^{k_i} \sum_{j=1}^{N} \sum_{l=0}^{k_j m} \frac{R_{jl}}{l!} \int_0^\infty \log_2(1+x) x^{(l+1)-1} e^{-\zeta_j x} dx \\
&= \frac{(\Gamma(m))^L}{\ln 2} \sum_{k_1+\ldots+k_N=L} \binom{L}{k_1,\ldots,k_N} \prod_{1\le i\le N} \alpha_i^{k_i} \sum_{j=1}^{N} \sum_{l=0}^{k_j m} \frac{R_{jl}}{l!} I_{l+1}(\zeta_j)
\end{aligned}
\tag{43}
$$

$$
C_{\rho_{SC}} = \frac{L}{\ln 2} \sum_{i=1}^{N} \alpha_i \sum_{k_1+\ldots+k_N=L-1} \binom{L-1}{k_1,\ldots,k_N} \prod_{1\le j\le N} \left(\alpha_j \zeta_j^{-m}\right)^{k_j} \int_0^\infty \ln(1+x) H(x,\mathcal{S},i) dx. \tag{44}
$$

$$
\int_0^\infty \ln(1+x) H(x,\mathcal{S},i) dx = \{(m-1)!\}^{L-1} \left(I_m(\zeta_i) + \sum_{r=1}^{L-1} (-1)^r \sum_{\substack{\mathcal{S} \\ p_k \in \mathcal{P}}} \sum_{l=0}^{r(m-1)} \sum_{h=1}^{B} \prod_{k=1}^{r} \frac{(\zeta_{\tilde{\mathcal{S}}_{hk}})^{p_k}}{p_k!} I_{l+m} \left(\sum_{k=1}^{r} \zeta_{\tilde{\mathcal{S}}_{hk}} + \zeta_i \right) \right) \tag{45}
$$

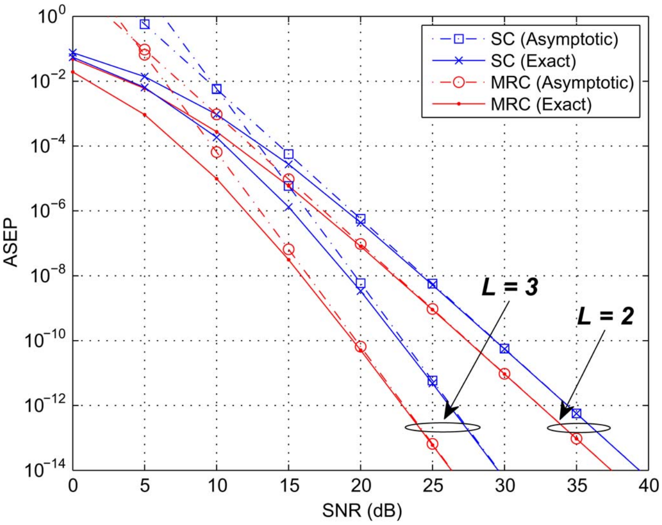

Fig. 3. Exact and asymptotic analysis of ASEP for the MRC and the SC ($k = 5$ and $m = 2$).

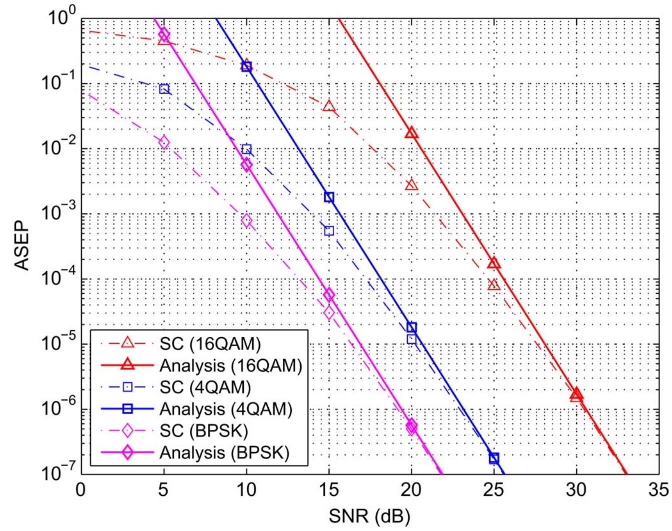

Fig. 5. ASEP comparison of SC for various constellations ($L = 2$).

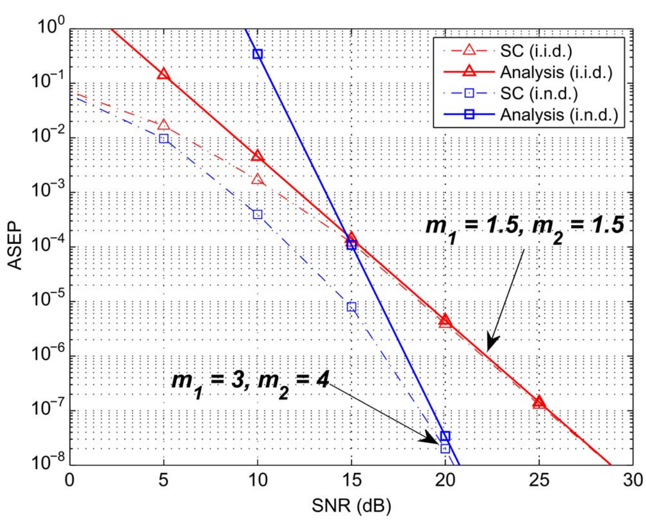

Fig. 4. ASEP for SC for i.i.d and i.n.d. cases with BPSK ($L = 2$).

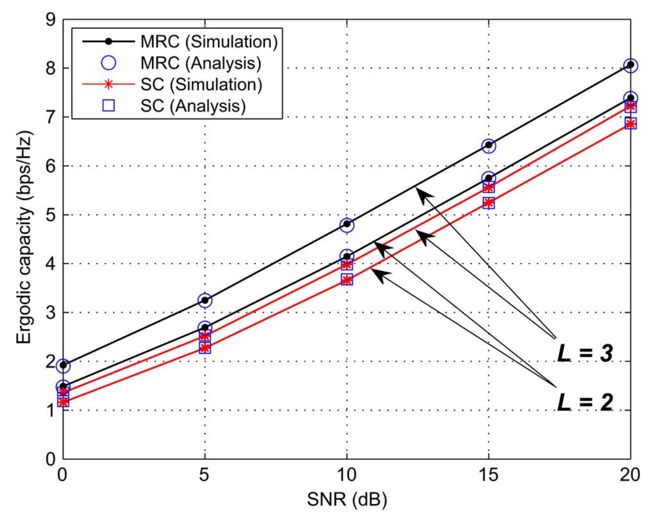

Fig. 6. Ergodic capacity of MRC and SC versus the average output SNR ($L = 2$ and 3).

as 4 and 6, respectively. From this figure, we observe that our derived diversity and array gains match well with the empirical results at the high SNR region. The performance gap between the MRC and the SC is obtained as $10 \log(G_{a,MRC}/G_{a,SC})$ dB where $G_{a,MRC}$ and $G_{a,SC}$ denote the array gain of the MRC and the SC by plugging (35) and (42) into (31), respectively, and the gap for $L = 2$ and 3 is calculated as 2.0 dB and 3.3 dB, respectively. Fig. 3 shows that the performance gap between MRC and SC curves agrees well with these analysis results at high SNR.

Next, we compare the performance of the SC in Fig. 4 for i.i.d. and i.n.d. cases by changing the multipath fading parameter m for BPSK. Here, the multipath parameters m_1 and m_2 are set to 1.5 and 1.5 for the i.i.d. case and 3 and 4 for the i.n.d. case, respectively. From (42), the diversity gains for the i.i.d. and i.n.d. cases are computed as 3 and 7, respectively, which correspond to the sum of multipath fading parameters. From this figure, we confirm that the proposed analysis accurately predicts the diversity and array gains for both cases.

In Fig. 5, we present the ASEP with various modulations. From this figure, it can be checked that our performance analysis matches well with the Monte Carlo simulation results regardless of the employed modulation levels. Also, we observe that the diversity gains are the same for all modulation levels, while only the shift of the ASEP occurs.

Next, we provide numerical results for the capacity of diversity reception schemes. As expected, the capacity of the MRC is higher than that of the SC. In Fig. 6, the ergodic capacities of the MRC and the SC are depicted separately with respect to SNR for different number of branches with the fading parameters $k = 5$ and $m = 2$. As L increases, the performance enhancement of the MRC is larger than that of the SC. As seen in this figure, the analysis results from equations in (43) and (44) are accurately matched with the actual results over all SNR ranges.

Finally, we present simulation results of the capacity for various shaping parameters k and m in the generalized fading channel. We set the average SNR as 20 dB. In Fig. 7, the

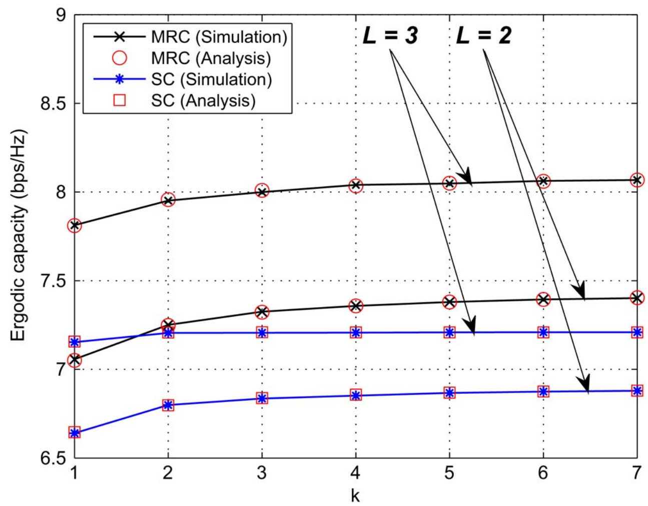

Fig. 7. Ergodic capacity of MRC and SC for different multipath fading factor k ($L = 2$ and 3).

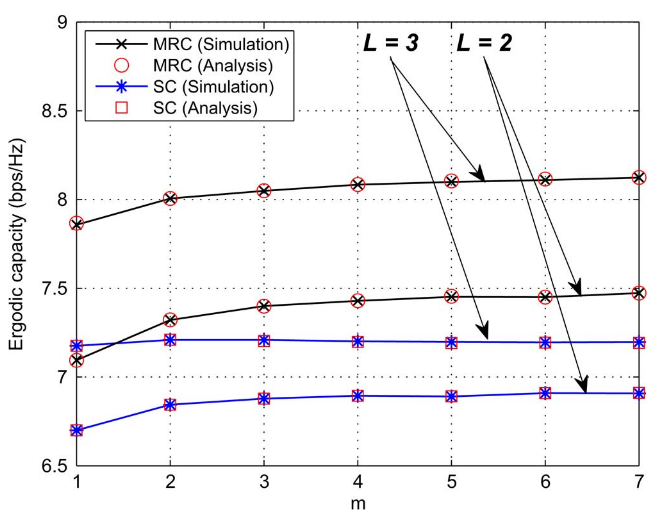

Fig. 8. Ergodic capacity of MRC and SC for different shadowing factor m ($L = 2$ and 3).

ergodic capacity is plotted with respect to different multipath fading factors k for $m = 2$. Also, Fig. 8 illustrates the ergodic capacity with various shadowing factors m for $k = 5$. From these two figures, it is shown that the capacity of each diversity reception scheme has a tendency to converge when the shaping parameter becomes large. For the SC, it can be seen that as the shaping parameter increases, the gain of the capacity with respect to L lessens, while the gain for the MRC is invariant. Since the variance of the output SNR for the K_G fading channel becomes smaller as the shaping parameter k or m increases, the selective gain for the PDF decreases. Numerical results reveal that this effect becomes more pronounced for the shadowing parameter m. Again, the proposed analysis exactly matches with the simulation results.

VI. CONCLUSION

In this paper, we have provided the diversity and the ergodic capacity analysis of diversity reception schemes over K_G fading channel models by employing the MG distribution. Exploiting the properties of the MG distribution, the PDFs of SNR for both MRC and SC are represented by the form of the weighted sum of gamma distributions. Compared to existing works, we have newly obtained the exact ASEP and simple closed-form expressions of diversity and array gains. Our derived expressions provide useful insights on the ASEP performance for the MRC and SC schemes using the parameters of the MG distribution. It is remarkable that our analytic results are applicable for general cases with any number of branches in both i.i.d. and i.n.d. cases. Moreover, we emphasize that in comparison to the MGF-based analysis, our PDF-based approach allows us to perform the capacity analysis for the MRC and the SC in a simpler manner. Numerical results confirm that our error probability and ergodic capacity analysis are well matched with the Monte Carlo simulation. The analysis on correlated channels and different diversity schemes such as generalized selection combing remains as an interesting future work.

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Jaehoon Jung (S’12) received the B.S. and M.S. degrees in electrical engineering from Korea University, Seoul, Korea, in 2011 and 2013, respectively. He is currently working toward the Ph.D. degree in the School of Electrical Engineering, Korea University. His research interests include communication theory and signal processing techniques for multicell MIMO wireless network systems. He was awarded the Student Travel Grant at the IEEE International Conference on Communications in 2013.

Sang-Rim Lee (S’06) received the B.S., M.S., and Ph.D. degrees from Korea University, Seoul, Korea, in 2005, 2007, and 2013, respectively, all in electrical engineering. From 2007 to 2010, he was a Research Engineer with Samsung Electronics, where he conducted research on the WiMAX system. Since September 2013, he has been a Postdoctoral Fellow with Korea University. His main research interests include communication theory and signal processing techniques for multiuser MIMO wireless networks and distributed antenna systems. His current research interests include convex optimization, random matrix theory, and stochastic geometry. He was awarded the Silver and Bronze Prizes in the Samsung Humantech Paper Contest in February 2012.

Haewook Park (S’10) received the B.S., M.S., and Ph.D. degrees from Korea University, Seoul, Korea, in 2008, 2010 and 2014, all in electrical engineering. During the Winter of 2010, he visited the University of Southern California, Los Angeles, CA, USA, to conduct a collaborative research. Since 2014, he has been with the Incujector Inc., Seoul, as a Chief Technical Officer. His research interests include signal processing techniques for wireless communication systems, with an emphasis on multiple-antenna techniques for throughput optimization in multicell and multitier networks. He was awarded the Bronze Prize in the Samsung Humantech Paper Contest in February 2013.

Sunho Lee received the B.S. degree (summa cum laude) in mathematics from Sogang University, Seoul, Korea, in 1983, the M.S. degree in statistics from Stanford University, Stanford, CA, USA, in 1984, and Ph.D. degree in applied statistics from Yonsei University, Seoul, Korea, in 1991. Since 1993, she joined the Faculty of Mathematics and Statistics in Sejong University, Seoul, Korea. From 2005 to 2007, she served as a Dean of the College of Natural Science in Sejong University. During 2012–2013, she was an editor for the Korean Jorunal of Applied Statistics. Her research interests include mathematical statistics and bioinformatics, specially in microarray analysis.

Inkyu Lee (S’92–M’95–SM’01) received the B.S. (Hon.) degree in control and instrumentation engineering from Seoul National University, Seoul, Korea, in 1990 and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, USA, in 1992 and 1995, respectively. From 1995 to 2001, he was a Member of Technical Staff with Bell Laboratories, Lucent Technologies, where he studied the high-speed wireless system design. From 2001 to 2002, he worked for Agere Systems (formerly Microelectronics Group of Lucent Technologies), Murray Hill, NJ, USA, as a Distinguished Member of Technical Staff. Since September 2002, he has been with Korea University, Seoul, Korea, where he is currently a Professor at the School of Electrical Engineering. During 2009, he visited the University of Southern California, Los Angeles, CA, USA, as a Visiting Professor. He has published over 100 journal papers in IEEE, and has 30 U.S. patents granted or pending. His research interests include digital communications, signal processing, and coding techniques applied for next-generation wireless systems. Dr. Lee has served as an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS from 2001 to 2011 and IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS from 2007 to 2011. Also, he has been a Chief Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (Special Issue on 4G Wireless Systems) in 2006. He received the IT Young Engineer Award as the IEEE/IEEK Joint Award in 2006, and was awarded the Best Paper Award at APCC in 2006, IEEE VTC in 2009, and ISPACS in 2013. He was also a recipient of the Hae-Dong Best Research Award of the Korea Information and Communications Society (KICS) in 2011 and the Best Young Engineer Award of the National Academy of Engineering in Korea (NAEK) in 2013.