Shaping-Power-Constrained Transceiver Designs for MIMO AF Relaying Systems With Direct Link

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***Abstract*—In this paper, we propose new relay transceiver designs based on the minimum mean square error (MMSE) criterion for amplify-and-forward multiple-input–multiple-output (MIMO) relaying systems with direct link. Since each antenna element is equipped with its own power amplifier, a norm power constraint, which restricts the transmit power with the expected norm of the transmit signal vector, is not suitable for practical systems. Therefore, we consider a shaping constraint (SC), which imposes a limit on the shape of the transmit covariance matrix. The SC includes several power constraints such as the peak power constraint and the per-antenna power constraint as special cases. To this end, we first derive the optimal structure of the MMSE relay transceiver under the SC. Then, by introducing an upper bound of the mean square error, we provide closed-form relay transceiver solutions. Due to limited bandwidth of the feedback channel, perfect channel knowledge at the transmitter may not be feasible. Thus, we also propose a quantized relay transceiver design based on Grassmannian codebooks for a limited-feedback scenario. From simulation results, it is confirmed that the proposed relay transceiver techniques demonstrate a significant performance improvement compared with conventional schemes.**

***Index Terms*—MIMO systems, transceiver design, relay, MMSE, direct link.**

I. Introduction

THERE have been intensive studies on multiple-input multiple-output (MIMO) wireless systems to improve spectral efficiency and communication reliability [1]–[4]. Recently, relaying systems have been considered as effective techniques to combat wireless fading and enhance link performance [5], [6]. These benefits have motivated many researches on MIMO relaying systems [7]–[10]. Among several relaying protocols, the amplify-and-forward (AF) protocol which forwards the amplified version of the received signal at the relay has attracted considerable attentions due to its simple implementation.

Over the past few years, many approaches have been investigated to study the optimum filter and analyze the performance of relaying systems assuming no direct link between the source and the destination. In [11] and [12], the optimal relay transceiver methods have been developed based on the minimum mean-squared error (MMSE) criterion and the capacity maximization, respectively. Also, closed-form source-relay joint transceiver techniques which minimize the mean-squared error (MSE) were introduced in [13].

Recently, it has been shown that when direct link between the source and the destination is non-negligible, a filter design considering the direct link can provide an enormous spectral efficiency improvement [8]. However, the non-negligible direct link makes the optimization problem even more complicated. The authors in [14] have studied a local optimal source-relay joint beamforming method under the end-to-end signal-to-noise ratio (SNR) maximization criteria for relaying systems with direct link. Also, a local optimal MMSE relay transceiver based on a projected gradient method was developed in [15].

One of the most widely considered power constraints in MIMO transmission is norm constraint (NC) which limits the sum of the transmit power over multiple antennas [11], [12], [15]. However, in practical implementations, shaping constraint (SC) which imposes a constraint on the shape of the transmit covariance matrix may be better suited to satisfy a system design requirement such as peak power constraint and per-antenna power constraint. The author in [16] presented a transceiver technique for point-to-point MIMO systems with the SC where the number of data streams is equal to that of transmit antennas. As a special case of the SC, in [17] and [13], maximum eigenvalue constraint (MVC) which ensures peak power constraint was considered for point-to-point MIMO systems and MIMO AF relaying systems without direct link, respectively. However, little works have been done in MIMO AF relaying with direct link.

In this paper, we provide MMSE transceiver solutions for the MIMO AF relaying systems with non-negligible direct link under the SC. In this case, due to non-convexity of the problem, identifying a closed-form solution is intractable. To circumvent this difficulty, we first derive the optimal structure of the MMSE relay transceiver under the SC as the multiplication of two matrices. Then, by exploiting the decomposition technique [18], the error covariance matrix is decomposed into a sum of two individual covariance matrices. Unfortunately, in the presence of the direct link, it is still hard to determine closed-form solutions even with decomposed error matrices. To arrive at tractable solutions, we obtain an upper bound of the MSE which allows us to develop closed-form solutions. Then, we provide the closed-form relay transceiver techniques which minimize

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the bound under the SC and the MVC for relaying systems with the direct link.

To the best of authors' knowledge, the optimal relay transceivers under the SC and the MVC have not been studied in the existing literature, and thus the optimal performance of systems with the SC and the MVC is not available. To validate the effectiveness of the proposed schemes, as an alternative approach, we impose a relaxation of the power constraint and introduce closed-form solutions for the relaxed problems with the SC and the MVC. Since feasible sets of the relaxed problems contain that of the original problems, the performance of the solutions for the relaxed problems can be considered as an upper bound of the optimal performance.

Also, we consider a relay transceiver method for a limited feedback scenario, since perfect channel state information (CSI) may not be available in practical wireless systems due to limited bandwidth of the feedback channel. In [14], adopting a single stream transmission scheme in MIMO AF relaying systems with direct link, a quantization scheme based on Grassmannian codebooks has been proposed. However, an extension of the work in [14] to the case of multiple streams is normally non-trivial, since the relay transceivers in [15] and [18] do not impose a specific structure.

Fortunately, the structure of the proposed relay transceiver consists of two matrices which are determined by the relay-to-destination link channel and the direct link channel. Therefore, we employ two individual codebooks which are designed to quantize each matrix, and show that the Grassmannian codebooks could also be efficient in minimizing the MSE of multiple-stream transmission schemes for relaying systems with the direct link. Through simulation results, we confirm that the proposed closed-form transceivers provide a substantial performance gain compared to conventional methods.

This paper is organized as follows. In Section II, we present the system model of MIMO AF relaying systems. The optimal structure of the relay transceiver is introduced in Section III. Section IV provides closed-form relay transceiver designs under MVC and SC. The relay transceiver solutions for the relaxed problems are presented in Section V. In Section VI, a quantization technique for the relay transceiver is addressed. Numerical results demonstrate the effectiveness of the proposed schemes in Section VII. Finally, the conclusions are made in Section VIII.

Throughout the paper, we will use the following notations. The bold uppercase, bold lowercase and normal letters denote matrices, vectors and scalars, respectively. We use the operators $(\cdot)^T$, $(\cdot)^H$, $\|\mathbf{x}\|$, $E[\cdot]$, $[\mathbf{A}]_{i,i}$ and $\mathrm{Tr}(\mathbf{A})$ to represent transpose, conjugate transpose, Euclidean 2-norm of a vector $\mathbf{x}$, expectation, the i-th diagonal element of a matrix $\mathbf{A}$ and trace of $\mathbf{A}$, respectively. $\mathbf{I}_N$ is defined as an $N \times N$ identity matrix. Also, $\mathbb{S}^N$ indicates a set of $N \times N$ positive semi-definite matrices. $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is a positive semi-definite matrix.

II. System Model

We consider MIMO AF relaying systems where direct link between the source and the destination is non-negligible. As shown in Fig. 1, the source, relay and destination nodes are equipped with N_S, N_R, and N_D antennas, respectively. In this paper, we assume that no CSI is available at the source. In this case, the number of spatial data streams N_S is bounded as $N_S \le N_D + \min(N_R, N_D)$ where $N_D + \min(N_R, N_D)$ is the effective channel rank of the relaying systems with direct link. A relay node operates in the half-duplex mode to assist data transmission from the source to the destination, and thus data transmission takes place over two time slots.

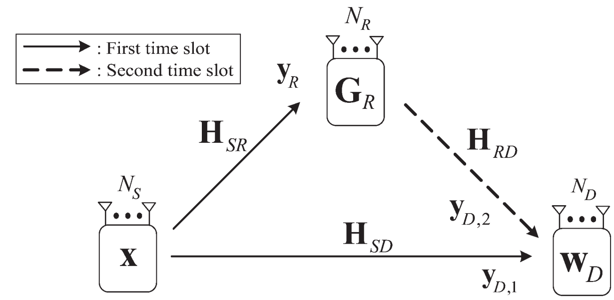

Fig. 1. System model for MIMO AF relaying systems with direct link.

In the first time slot, the source broadcasts the signal vector $\mathbf{x} \in \mathbb{C}^{N_S}$ to the relay and the destination. Then, the received signals at the relay and the destination $\mathbf{y}_R \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{y}_{D,1} \in \mathbb{C}^{N_D \times 1}$ are respectively written as

$$\mathbf{y}_R = \mathbf{H}_{SR}\mathbf{x} + \mathbf{n}_R \quad \text{and} \quad \mathbf{y}_{D,1} = \mathbf{H}_{SD}\mathbf{x} + \mathbf{n}_{D,1},$$

where $\mathbf{H}_{SR} \in \mathbb{C}^{N_R \times N_S}$ and $\mathbf{H}_{SD} \in \mathbb{C}^{N_D \times N_S}$ represent the channel matrices of the source-to-relay and source-to-destination link, respectively, and $\mathbf{n}_R \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{n}_{D,1} \in \mathbb{C}^{N_D \times 1}$ are the additive white Gaussian noise (AWGN) vectors with zero mean and unit variance at the relay and the destination, respectively. Here the power constraint at the source is given as $E[\mathbf{x}\mathbf{x}^H] = \rho \mathbf{I}_{N_S}$ where $\rho \triangleq P_S/N_S$ and P_S indicates the available transmit power at the source.[1]

In the second time slot, the relay multiplies the received signal by the relay transceiver $\mathbf{G}_R \in \mathbb{C}^{N_R \times N_R}$ and transmits the amplified signal to the destination. Then, the received signal at the destination $\mathbf{y}_{D,2} \in \mathbb{C}^{N_D \times 1}$ can be expressed as

$$\mathbf{y}_{D,2} = \mathbf{H}_{RD}\mathbf{G}_R\mathbf{y}_R + \mathbf{n}_{D,2},$$

where $\mathbf{H}_{RD} \in \mathbb{C}^{N_D \times N_R}$ and $\mathbf{n}_{D,2} \in \mathbb{C}^{N_D \times 1}$ denote the channel for the relay-to-destination link and the AWGN vector with zero mean and unit variance at the destination in the second time slot, respectively.

Now, we present the stacked received signal vector at the destination $\mathbf{y}_D \in \mathbb{C}^{2N_D \times 1}$ as

$$\mathbf{y}_D = \begin{bmatrix} \mathbf{y}_{D,1} \\ \mathbf{y}_{D,2} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{G}_R\mathbf{H}_{SR} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{n}_D \end{bmatrix},$$

where $\mathbf{n}_D = \mathbf{H}_{RD}\mathbf{G}_R\mathbf{n}_R + \mathbf{n}_{D,2}$ equals the effective noise vector at the second time slot with covariance matrix $\mathbf{R}_{n_D} = \mathbf{H}_{RD}\mathbf{G}_R\mathbf{G}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_{N_D}$. Finally, by employing the receive filter $\mathbf{W}_D \in \mathbb{C}^{N_S \times 2N_D}$, the destination estimates the transmitted data as $\hat{\mathbf{x}} = \mathbf{W}_D\mathbf{y}_D$.

[1] It is also possible to consider peak power constraint or per-antenna power constraint at the source node by simply multiplying the scaling factors to each element of the signal vector $\mathbf{x}$.

Next, let us look at the power constraint at the relay. One may consider NC which restricts the expected norm of the transmit signal vector as $E[\|\mathbf{G}_R\mathbf{y}_R\|^2] = \mathrm{Tr}(\mathbf{G}_R(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\mathbf{G}_R^H) \leq P_R$ where P_R indicates the transmit power budget at the relay. However, since the NC may not be suitable for practical implementation, we consider SC which imposes a constraint on the shape of the transmit covariance matrix as

$$\mathbf{G}_R\left(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R}\right)\mathbf{G}_R^H \preceq \mathbf{S},$$

where $\mathbf{S} \in \mathbb{S}^{N_R}$ accounts for the shaping bound. Here, any positive semi-definite matrix $\mathbf{S}$ can be adopted according to the system requirement. For example, the peak power constraint and the per-antenna power constraint can be employed by invoking $\mathbf{S}$ as $\mathbf{S} = P_{peak}\mathbf{I}_{N_R}$ and $\mathbf{S} = \mathrm{diag}\{P_1, \ldots, P_{N_R}\}$, respectively, where P_{peak} is the maximum output power at each antenna and P_i denotes the available power budget of the i-th antenna.

Also, MVC is a special case of the SC which imposes a limit on the maximum eigenvalue of the transmit covariance matrix as

$$\lambda_{\max}\left(\mathbf{G}_R\left(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R}\right)\mathbf{G}_R^H\right) \leq P_{peak},$$

where $\lambda_{\max}(\mathbf{A})$ represents the maximum eigenvalue of a matrix $\mathbf{A}$. Note that the MVC satisfies the peak power constraint $\max_i\,[\mathbf{G}_R(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\mathbf{G}_R^H]_{i,i} \leq P_{peak}$ due to the fact that $\max_i[\mathbf{A}]_{i,i} \leq \lambda_{\max}(\mathbf{A})$ for $\mathbf{A} \in \mathbb{S}^N$ [19].

III. Optimal Relay Transceiver Structure

In this section, we provide the optimal structure of the relay transceiver for MMSE based MIMO AF relaying systems. Defining the error vector as $\mathbf{e} \triangleq \hat{\mathbf{x}} - \mathbf{x}$, the MSE minimization problem under the SC can be formulated as

$$\begin{aligned} \min_{\mathbf{G}_R, \mathbf{W}_D} \quad & \mathrm{Tr}\left(\mathbf{R}_e(\mathbf{G}_R, \mathbf{W}_D)\right) \\ \text{subject to} \quad & \mathbf{R}_c(\mathbf{G}_R) \preceq \mathbf{S}, \end{aligned} \tag{1}$$

where the error covariance matrix $\mathbf{R}_e(\mathbf{G}_R, \mathbf{W}_D)$ is defined as $\mathbf{R}_e(\mathbf{G}_R, \mathbf{W}_D) \triangleq E[\mathbf{e}\mathbf{e}^H]$ and $\mathbf{R}_c(\mathbf{G}_R) \triangleq \mathbf{G}_R(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\mathbf{G}_R^H$.

Note that this joint optimization problem is non-convex and it is hard to find closed-form expressions for $\mathbf{G}_R$ and $\mathbf{W}_D$. However, for a given relay transceiver $\mathbf{G}_R$, the problem is convex with respect to $\mathbf{W}_D$, and thus the MSE minimizing destination receive filter can be obtained as [20]

$$\mathbf{W}_D = \left(\mathbf{H}_{SR}^H\mathbf{G}_R^H\mathbf{H}_{RD}^H\mathbf{R}_{n_D}^{-1}\mathbf{H}_{RD}\mathbf{G}_R\mathbf{H}_{SR} + \mathbf{\Psi}^{-1}\right)^{-1}\mathbf{H}_w^H, \tag{2}$$

where $\mathbf{\Psi} \triangleq (\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \rho^{-1}\mathbf{I}_{N_S})^{-1}$ and $\mathbf{H}_w \triangleq [\mathbf{H}_{SD}^T(\mathbf{R}_{n_D}^{-1}\mathbf{H}_{RD}\mathbf{G}_R\mathbf{H}_{SR})^T]^T$. Throughout this paper, we assume that the optimal MMSE receiver $\mathbf{W}_D$ in (2) is employed at the destination. Then, the corresponding error covariance matrix is given by

$$\mathbf{R}_e(\mathbf{G}_R) = \left(\mathbf{H}_{SR}^H\mathbf{G}_R^H\mathbf{H}_{RD}^H\mathbf{R}_{n_D}^{-1}\mathbf{H}_{RD}\mathbf{G}_R\mathbf{H}_{SR} + \mathbf{\Psi}^{-1}\right)^{-1}. \tag{3}$$

Now, we focus on the relay transceiver $\mathbf{G}_R$. In [18], by employing a Lagrangian method, the optimal structure of $\mathbf{G}_R$ was presented under the assumption of NC at the relay node. Since this structure of $\mathbf{G}_R$ decomposes the error covariance matrix into two individual covariance matrices and makes the problem more tractable, it is important to investigate the optimality of the structure under SC. However, it is not immediate when it comes to the problem with the SC because the SC makes it difficult to derive the optimal structure of $\mathbf{G}_R$. In the following Lemma, we introduce the optimal structure of $\mathbf{G}_R$ which minimizes the MSE while satisfying the SC.

Lemma 1: The structure of the optimal MMSE relay transceiver $\mathbf{G}_R$ for the problem in (1) can be expressed as

$$\mathbf{G}_R = \mathbf{F}_R\mathbf{W}_R, \tag{4}$$

where $\mathbf{F}_R \in \mathbb{C}^{N_R \times N_S}$ is an arbitrary matrix and $\mathbf{W}_R$ is given as $\mathbf{W}_R = (\mathbf{H}_{SR}^H\mathbf{H}_{SR} + \mathbf{H}_{SD}^H\mathbf{H}_{SD} + \rho^{-1}\mathbf{I}_{N_S})^{-1}\mathbf{H}_{SR}^H \in \mathbb{C}^{N_S \times N_R}$.

Proof: See Appendix A. ■

Here, since $\mathbf{W}_R$ is applied to the received signal at the relay $\mathbf{y}_R$ and the relay transmits the signal by multiplying $\mathbf{F}_R$ to $\mathbf{W}_R\mathbf{y}_R$, we call $\mathbf{W}_R$ and $\mathbf{F}_R$ as the relay receiver and the relay transmitter, respectively. It is important to note that the derived optimal structure of $\mathbf{G}_R$ in (4) can be adopted to relaying systems with any power constraints. Therefore, this result is a generalization of the previous work in [18].

Let us define $\mathbf{\Omega} \triangleq \mathbf{W}_R\mathbf{H}_{SR}\mathbf{\Psi}$ and its eigenvalue decomposition (EVD) as $\mathbf{\Omega} = \mathbf{U}_\omega\mathbf{\Lambda}_\omega\mathbf{U}_\omega^H$ where $\mathbf{U}_\omega \in \mathbb{C}^{N_S \times N_S}$ represents a unitary matrix and $\mathbf{\Lambda}_\omega \in \mathbb{C}^{N_S \times N_S}$ denotes a diagonal matrix with eigenvalues $\lambda_{\omega,i}$ for $i = 1, \ldots, N_S$ in descending order on the main diagonal. Then, following the relay filter structure in (4), it has been shown in [18] that the error covariance matrix in (3) can be decomposed into a sum of two individual covariance matrices as

$$\begin{aligned} \mathbf{R}_e(\mathbf{F}_R) = {} & \left(\mathbf{H}_{SR}^H\mathbf{H}_{SR} + \mathbf{H}_{SD}^H\mathbf{H}_{SD} + \rho^{-1}\mathbf{I}_{N_S}\right)^{-1} \\ & + \tilde{\mathbf{U}}_\omega\left(\tilde{\mathbf{U}}_\omega^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{F}_R\tilde{\mathbf{U}}_\omega + \tilde{\mathbf{\Lambda}}_\omega^{-1}\right)^{-1}\tilde{\mathbf{U}}_\omega^H, \end{aligned} \tag{5}$$

where $\tilde{\mathbf{U}}_\omega \in \mathbb{C}^{N_S \times M}$ indicates a matrix constructed by the first M columns of $\mathbf{U}_\omega$, $\tilde{\mathbf{\Lambda}}_\omega \in \mathbb{C}^{M \times M}$ is an upper-left submatrix of $\mathbf{\Lambda}_\omega$ and M equals the rank of $\mathbf{\Omega}$, i.e., $M = \min(N_S, N_R)$. Note that while the second term in (5) is related to the relay transmitter $\mathbf{F}_R$, the first term depends only on the channel matrices. Therefore, in the following, we will focus on the MSE minimization problem of the second error covariance matrix in (5) under the SC.

IV. Proposed Relay Transceiver Designs

In this section, we introduce the MSE minimizing relay transceiver techniques for MIMO AF relaying systems with direct link. Although SC is a more general metric than MVC, identifying the relay transceiver which satisfies the SC for an arbitrary number of antennas is intractable. Thus, we will first derive the MVC based relay transceiver which is applicable to the arbitrary number of relay antennas. Then, a relay transceiver technique under the SC will be provided for the case of $N_R = N_S$.

A. Relay Transceiver Design Under the MVC

In this subsection, we propose a closed-form relay transceiver solution which minimizes the MSE. By exploiting the results in (4) and (5), the MSE minimization problem under the MVC can be formulated as

$$\min_{\mathbf{F}_R} \quad \mathrm{Tr}\left(\left(\tilde{\mathbf{U}}_\omega^H \mathbf{F}_R^H \mathbf{H}_{RD}^H \mathbf{H}_{RD} \mathbf{F}_R \tilde{\mathbf{U}}_\omega + \tilde{\mathbf{\Lambda}}_\omega^{-1}\right)^{-1}\right)$$
$$\text{subject to} \quad \lambda_{\max}\left(\mathbf{F}_R \mathbf{\Upsilon} \mathbf{F}_R^H\right) \leq P_{peak}, \tag{6}$$

where $\mathbf{\Upsilon} \triangleq \mathbf{W}_R(\rho \mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\mathbf{W}_R^H$.

Let us introduce EVD of $\mathbf{H}_{RD}^H\mathbf{H}_{RD}$ as $\mathbf{H}_{RD}^H\mathbf{H}_{RD} = \mathbf{U}_{RD}\mathbf{\Lambda}_{RD}\mathbf{U}_{RD}^H$ where $\mathbf{U}_{RD} \in \mathbb{C}^{N_R\times N_R}$ represents a unitary matrix and $\mathbf{\Lambda}_{RD} \in \mathbb{C}^{N_R\times N_R}$ denotes a diagonal matrix with eigenvalues $\lambda_{RD,i}$ for $i = 1,\ldots,N_R$ in descending order. Then, without loss of generality, one can write $\mathbf{F}_R$ as

$$\mathbf{F}_R = \mathbf{U}_{RD}\begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{\Phi}_3 & \mathbf{\Phi}_4 \end{bmatrix}\mathbf{U}_\omega^H, \tag{7}$$

where $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$, $\mathbf{\Phi}_3$, and $\mathbf{\Phi}_4$ are arbitrary matrices with dimensions of $M \times M$, $M \times (N_S - M)$, $(N_R - M) \times M$, and $(N_R - M) \times (N_S - M)$, respectively.

Plugging (7) into the object function in (6) and after some manipulations, we can see that the object function is independent of $\mathbf{\Phi}_2$ and $\mathbf{\Phi}_4$. Also, non-zero $\mathbf{\Phi}_2$, $\mathbf{\Phi}_3$, and $\mathbf{\Phi}_4$ always result in the increase of $\lambda_{\max}(\mathbf{F}_R\mathbf{\Upsilon}\mathbf{F}_R^H)$. Thus, without loss of generality, we set $\mathbf{\Phi}_i = \mathbf{0}$ for $i = 2$, 3, 4. Then, $\mathbf{F}_R$ in (7) becomes

$$\mathbf{F}_R = \tilde{\mathbf{U}}_{RD}\mathbf{\Phi}_1\tilde{\mathbf{U}}_\omega^H, \tag{8}$$

where $\tilde{\mathbf{U}}_{RD} \in \mathbb{C}^{N_R\times M}$ stands for a matrix constructed by the first M columns of $\mathbf{U}_{RD}$.

Substituting $\mathbf{F}_R$ in (8) into (6), the problem can be reformulated as

$$\min_{\mathbf{\Phi}_1} \quad \mathrm{Tr}\left(\left(\mathbf{\Phi}_1^H \tilde{\mathbf{\Lambda}}_{RD}\mathbf{\Phi}_1 + \tilde{\mathbf{\Lambda}}_\omega^{-1}\right)^{-1}\right)$$
$$\text{subject to} \quad \lambda_{\max}\left(\mathbf{\Phi}_1\mathbf{\Upsilon}_\omega\mathbf{\Phi}_1^H\right) \leq P_{peak}, \tag{9}$$

where $\tilde{\mathbf{\Lambda}}_{RD} \in \mathbb{C}^{M\times M}$ is an upper-left submatrix of $\mathbf{\Lambda}_{RD}$ and $\mathbf{\Upsilon}_\omega \triangleq \tilde{\mathbf{U}}_\omega^H\mathbf{\Upsilon}\tilde{\mathbf{U}}_\omega$. Note that for $\mathbf{A} \in \mathbb{S}^N$, we have $\mathrm{Tr}(\mathbf{A}^{-1}) \geq \sum_{i=1}^M 1/[\mathbf{A}]_{i,i}$ and $\lambda_{\max}(\mathbf{A}) \geq \max_i[\mathbf{A}]_{i,i}$ where equalities hold when $\mathbf{A}$ is a diagonal matrix. Hence, if both $\mathbf{\Phi}_1^H\tilde{\mathbf{\Lambda}}_{RD}\mathbf{\Phi}_1$ and $\mathbf{\Phi}_1\mathbf{\Upsilon}_\omega\mathbf{\Phi}_1^H$ can be simultaneously diagonalized, we can simply obtain the optimal solution by finding $\mathbf{\Phi}_1$ which diagonalizes both matrices. However, owing to the non-diagonal structure of $\mathbf{\Upsilon}_\omega$, there exists no such a case, and thus it is difficult to identify the optimal solution for the problem in (9) in a closed-form.

To circumvent such a difficulty, we consider an upper bound of the object function in (9). First, we examine the relationship between $\tilde{\mathbf{\Lambda}}_\omega$ and $\mathbf{\Upsilon}_\omega$ in the following lemma.

Lemma 2: The upper-left submatrix of $\mathbf{\Lambda}_\omega$, $\tilde{\mathbf{\Lambda}}_\omega \in \mathbb{C}^{M\times M}$ and $\mathbf{\Upsilon}_\omega = \tilde{\mathbf{U}}_\omega^H\mathbf{\Upsilon}\tilde{\mathbf{U}}_\omega$ satisfy the following inequality:

$$\tilde{\mathbf{\Lambda}}_\omega \preceq \mathbf{\Upsilon}_\omega. \tag{10}$$

Proof: See Appendix B. ■

From the result in Lemma 2, an upper bound of the MSE is given by

$$\mathrm{Tr}\left(\left(\mathbf{\Phi}_1^H\tilde{\mathbf{\Lambda}}_{RD}\mathbf{\Phi}_1 + \tilde{\mathbf{\Lambda}}_\omega^{-1}\right)^{-1}\right) \leq \mathrm{Tr}\left(\left(\mathbf{\Phi}_1^H\tilde{\mathbf{\Lambda}}_{RD}\mathbf{\Phi}_1 + \mathbf{\Upsilon}_\omega^{-1}\right)^{-1}\right). \tag{11}$$

In the sequel, we concentrate on the minimization of the upper bound of the MSE in (11). Let us define EVD of $\mathbf{\Upsilon}_\omega$ as $\mathbf{\Upsilon}_\omega = \mathbf{U}_\upsilon\mathbf{\Lambda}_\upsilon\mathbf{U}_\upsilon^H$ where $\mathbf{U}_\upsilon \in \mathbb{C}^{M\times M}$ and $\mathbf{\Lambda}_\upsilon \in \mathbb{C}^{M\times M}$ represent a unitary matrix and a diagonal matrix with eigenvalues $\lambda_{\upsilon,i}$ for $i = 1,\ldots,M$ in descending order. Then, without loss of generality, we can express $\mathbf{\Phi}_1$ as

$$\mathbf{\Phi}_1 = \tilde{\mathbf{\Phi}}\mathbf{U}_\upsilon^H, \tag{12}$$

where $\tilde{\mathbf{\Phi}} \in \mathbb{C}^{M\times M}$ is an arbitrary matrix.

By substituting (11) and (12) into (9), the problem is rewritten as

$$\min_{\tilde{\mathbf{\Phi}}} \quad \mathrm{Tr}\left(\left(\tilde{\mathbf{\Phi}}^H\tilde{\mathbf{\Lambda}}_{RD}\tilde{\mathbf{\Phi}} + \mathbf{\Lambda}_\upsilon^{-1}\right)^{-1}\right)$$
$$\text{subject to} \quad \lambda_{\max}\left(\tilde{\mathbf{\Phi}}\mathbf{\Lambda}_\upsilon\tilde{\mathbf{\Phi}}^H\right) \leq P_{peak}. \tag{13}$$

As mentioned before, the object function in (13) is minimized when both $\tilde{\mathbf{\Phi}}^H\tilde{\mathbf{\Lambda}}_{RD}\tilde{\mathbf{\Phi}}$ and $\tilde{\mathbf{\Phi}}\mathbf{\Lambda}_\upsilon\tilde{\mathbf{\Phi}}^H$ are diagonalized. Hence, without loss of generality, we can assume that $\tilde{\mathbf{\Phi}}$ is a diagonal matrix with diagonal elements ϕ_i for $i = 1,\ldots,M$.

Then, the object function in (13) becomes

$$\mathrm{Tr}\left(\left(\tilde{\mathbf{\Phi}}^H\tilde{\mathbf{\Lambda}}_{RD}\tilde{\mathbf{\Phi}} + \mathbf{\Lambda}_\upsilon^{-1}\right)^{-1}\right) = \sum_{i=1}^M \frac{\lambda_{\upsilon,i}}{1 + \lambda_{RD,i}\lambda_{\upsilon,i}|\phi_i|^2} \geq \sum_{i=1}^M \frac{\lambda_{\upsilon,i}}{1 + P_{peak}\lambda_{RD,i}},$$

where the inequality follows from the fact $\lambda_{\max}(\tilde{\mathbf{\Phi}}\mathbf{\Lambda}_\upsilon\tilde{\mathbf{\Phi}}^H) = \max_{i=1,\ldots,M}(\lambda_{\upsilon,i}|\phi_i|^2) \leq P_{peak}$. Note that the equality for the object function holds when $\lambda_{\upsilon,i}|\phi_i|^2 = P_{peak}$ for $i = 1,\ldots,M$, and thus we can obtain $\tilde{\mathbf{\Phi}}$ as

$$\tilde{\mathbf{\Phi}} = \sqrt{P_{peak}}\mathbf{\Lambda}_\upsilon^{-1/2}. \tag{14}$$

Finally, exploiting the results in (8), (12), and (14), the relay transmitter is determined in closed-form as

$$\mathbf{F}_R = \sqrt{P_{peak}}\tilde{\mathbf{U}}_{RD}\mathbf{\Lambda}_\upsilon^{-1/2}\mathbf{U}_\upsilon^H\tilde{\mathbf{U}}_\omega^H. \tag{15}$$

In general, due to the non-diagonal structure of the transmit covariance matrix, the peak power is not equal to the maximum eigenvalue of the transmit covariance matrix. However, in our case, the covariance matrix with $\mathbf{F}_R$ in (15) and $\mathbf{G}_R$ in (4) is $\mathbf{G}_R(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\mathbf{G}_R^H = P_{peak}\tilde{\mathbf{U}}_{RD}\tilde{\mathbf{U}}_{RD}^H$. Therefore, when $N_S \geq N_R$, the peak power becomes equal to the maximum eigenvalue of the transmit covariance matrix as $\max_i[P_{peak}\tilde{\mathbf{U}}_{RD}\tilde{\mathbf{U}}_{RD}^H]_{i,i} = \lambda_{\max}(P_{peak}\tilde{\mathbf{U}}_{RD}\tilde{\mathbf{U}}_{RD}^H) = P_{peak}$.

B. Relay Transceiver Design Under the SC

In this subsection, we focus on the MMSE relay transceiver solution under the SC for the case of $N_R = N_S$. From the results in (4) and (5), the MSE minimization problem under the SC can be formulated as

$$\min_{\mathbf{F}_R} \quad \mathrm{Tr}\left(\left(\mathbf{U}_\omega^H \mathbf{F}_R^H \mathbf{H}_{RD}^H \mathbf{H}_{RD} \mathbf{F}_R \mathbf{U}_\omega + \boldsymbol{\Lambda}_\omega^{-1}\right)^{-1}\right)$$
$$\text{subject to} \quad \mathbf{F}_R \boldsymbol{\Upsilon} \mathbf{F}_R^H \preceq \mathbf{S}. \tag{16}$$

Since $\mathbf{U}_\upsilon$ and $\mathbf{U}_\omega$ are non-singular matrices, without loss of generality, we can represent the relay transmitter $\mathbf{F}_R$ as

$$\mathbf{F}_R = \tilde{\boldsymbol{\Phi}} \mathbf{U}_\upsilon^H \mathbf{U}_\omega^H, \tag{17}$$

where $\tilde{\boldsymbol{\Phi}} \in \mathbb{C}^{M\times M}$ is an arbitrary matrix. By plugging $\mathbf{F}_R$ in (17) and the upper bound of $\boldsymbol{\Lambda}_\omega$ in (10) into the problem in (16) and introducing a matrix $\tilde{\mathbf{S}} \in \mathbb{S}^M$, the problem in (16) is transformed to

$$\min_{\tilde{\boldsymbol{\Phi}}} \quad \mathrm{Tr}\left(\left(\tilde{\boldsymbol{\Phi}}^H \mathbf{H}_{RD}^H \mathbf{H}_{RD} \tilde{\boldsymbol{\Phi}} + \boldsymbol{\Lambda}_\upsilon^{-1}\right)^{-1}\right)$$
$$\text{subject to} \quad \tilde{\boldsymbol{\Phi}} \boldsymbol{\Lambda}_\upsilon \tilde{\boldsymbol{\Phi}}^H = \tilde{\mathbf{S}}$$
$$\tilde{\mathbf{S}} \preceq \mathbf{S}. \tag{18}$$

Since $\tilde{\mathbf{S}}$ can always be factorized as $\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{1/2}\mathbf{V}\mathbf{V}^H\tilde{\mathbf{S}}^{H/2}$ where $\mathbf{V} \in \mathbb{C}^{M\times M}$ is an arbitrary unitary matrix and $\tilde{\mathbf{S}}^{1/2} \in \mathbb{C}^{M\times M}$ denotes a matrix satisfying $\tilde{\mathbf{S}}^{1/2}\tilde{\mathbf{S}}^{H/2} = \tilde{\mathbf{S}}$, $\tilde{\boldsymbol{\Phi}}$ can be expressed as

$$\tilde{\boldsymbol{\Phi}} = \tilde{\mathbf{S}}^{1/2}\mathbf{V}\boldsymbol{\Lambda}_\upsilon^{-1/2}. \tag{19}$$

Substituting (19) into the problem in (18) yields

$$\min_{\tilde{\mathbf{S}}} \quad \epsilon(\tilde{\mathbf{S}})$$
$$\text{subject to} \quad \tilde{\mathbf{S}} \preceq \mathbf{S}, \tag{20}$$

where we define the function $\epsilon(\tilde{\mathbf{S}})$ as

$$\epsilon(\tilde{\mathbf{S}}) \triangleq \min_{\mathbf{V}\in\mathcal{U}(M)} \mathrm{Tr}\Big(\boldsymbol{\Lambda}_\upsilon^{H/2}\mathbf{V}^H\Big(\tilde{\mathbf{S}}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\tilde{\mathbf{S}}^{1/2} + \mathbf{I}_M\Big)^{-1}\mathbf{V}\boldsymbol{\Lambda}_\upsilon^{1/2}\Big), \tag{21}$$

and denote $\mathcal{U}(M)$ as the set of $M \times M$ unitary matrices.

To solve (20), let us look at $\epsilon(\tilde{\mathbf{S}})$. First, we define EVD as $\tilde{\mathbf{S}}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\tilde{\mathbf{S}}^{1/2} = \mathbf{U}_{\tilde{S}}\boldsymbol{\Lambda}_{\tilde{S}}\mathbf{U}_{\tilde{S}}^H$ where $\mathbf{U}_{\tilde{S}} \in \mathbb{C}^{M\times M}$ indicates a unitary matrix and $\boldsymbol{\Lambda}_{\tilde{S}} \in \mathbb{C}^{M\times M}$ is a diagonal matrix with eigenvalues. Then, $\epsilon(\tilde{\mathbf{S}})$ can be represented as

$$\epsilon(\tilde{\mathbf{S}}) = \min_{\bar{\mathbf{V}}\in\mathcal{U}(M)} \mathrm{Tr}\left(\boldsymbol{\Lambda}_\upsilon^{H/2}\bar{\mathbf{V}}^H(\boldsymbol{\Lambda}_{\tilde{S}} + \mathbf{I}_M)^{-1}\bar{\mathbf{V}}\boldsymbol{\Lambda}_\upsilon^{1/2}\right).$$

For a matrix $\hat{\mathbf{S}} \in \mathbb{S}^M$ which satisfies $\hat{\mathbf{S}} \succeq \tilde{\mathbf{S}}$, we have $\mathbf{C}\hat{\mathbf{S}}\mathbf{C}^H \succeq \mathbf{C}\tilde{\mathbf{S}}\mathbf{C}^H$ for an arbitrary matrix $\mathbf{C} \in \mathbb{C}^{N\times M}$, and thus we obtain $\lambda_i(\mathbf{C}\hat{\mathbf{S}}\mathbf{C}^H) \geq \lambda_i(\mathbf{C}\tilde{\mathbf{S}}\mathbf{C}^H)$ for $i = 1, \ldots, N$, where $\lambda_i(\mathbf{A})$ stands for the i-th largest eigenvalue of a matrix $\mathbf{A}$ [21]. Since $\lambda_i(\mathbf{AB}) = \lambda_i(\mathbf{BA})$, it follows that $\lambda_i(\hat{\mathbf{S}}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\hat{\mathbf{S}}^{1/2}) \geq \lambda_i(\tilde{\mathbf{S}}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\tilde{\mathbf{S}}^{1/2})$, i.e., $\boldsymbol{\Lambda}_{\hat{S}} \succeq \boldsymbol{\Lambda}_{\tilde{S}}$ where $\boldsymbol{\Lambda}_{\hat{S}}$ equals a diagonal matrix with eigenvalues of $\hat{\mathbf{S}}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\hat{\mathbf{S}}^{1/2}$. Also, $\mathbf{A} \succeq \mathbf{B}$ implies $\mathrm{Tr}(\mathbf{A}^{-1}) \leq \mathrm{Tr}(\mathbf{B}^{-1})$. Therefore, we can conclude that $\epsilon(\tilde{\mathbf{S}})$ is minimized when $\tilde{\mathbf{S}}$ is equal to $\mathbf{S}$, since $\epsilon(\hat{\mathbf{S}}) \leq \epsilon(\tilde{\mathbf{S}})$ for $\hat{\mathbf{S}} \succeq \tilde{\mathbf{S}}$.

Substituting $\tilde{\mathbf{S}}$ with $\mathbf{S}$ in (21), the problem in (20) is reformulated as

$$\min_{\mathbf{V}\in\mathcal{U}(M)} \mathrm{Tr}\Big(\boldsymbol{\Lambda}_\upsilon^{H/2}\mathbf{V}^H\Big(\mathbf{S}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{S}^{1/2}+\mathbf{I}_M\Big)^{-1}\mathbf{V}\boldsymbol{\Lambda}_\upsilon^{1/2}\Big), \tag{22}$$

where $\mathbf{S}^{1/2} \in \mathbb{C}^{M\times M}$ is a matrix satisfying $\mathbf{S}^{1/2}\mathbf{S}^{H/2} = \mathbf{S}$. It is well-known that the trace function is a Schur-concave function, and thus the object function in (22) is minimized when $\boldsymbol{\Lambda}_\upsilon^{H/2}\mathbf{V}^H(\mathbf{S}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{S}^{1/2} + \mathbf{I}_M)^{-1}\mathbf{V}\boldsymbol{\Lambda}_\upsilon^{1/2}$ becomes a diagonal matrix.

Defining EVD as $\mathbf{S}^{H/2}\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{S}^{1/2} = \mathbf{U}_S\boldsymbol{\Lambda}_S\mathbf{U}_S^H$ where $\mathbf{U}_S \in \mathbb{C}^{M\times M}$ and $\boldsymbol{\Lambda}_S \in \mathbb{C}^{M\times M}$ represent a unitary matrix and a diagonal matrix with eigenvalues, respectively, $\mathbf{V}$ becomes

$$\mathbf{V} = \mathbf{U}_S. \tag{23}$$

Finally, from the results in (17), (19), and (23), the relay transmitter is given by

$$\mathbf{F}_R = \mathbf{S}^{1/2}\mathbf{U}_S\boldsymbol{\Lambda}_\upsilon^{-1/2}\mathbf{U}_\upsilon^H\mathbf{U}_\omega^H. \tag{24}$$

Note that when $\mathbf{S} = P_{peak}\mathbf{I}_M$, the relay transmitter in (24) is identical to the solution in (15).

Now, we investigate the tightness of the upper bound in (10). Since the bound follows from the fact that $\boldsymbol{\Psi} = (\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \rho^{-1}\mathbf{I}_{N_S})^{-1} \preceq \rho\mathbf{I}_{N_S}$, the gap between $\tilde{\boldsymbol{\Lambda}}_\omega$ and $\boldsymbol{\Upsilon}_\omega$ decreases as the strength of direct link becomes worse. Also, if the source-to-relay link is much weaker than the direct link, both $\tilde{\boldsymbol{\Lambda}}_\omega$ and $\boldsymbol{\Upsilon}_\omega$ approach $\mathbf{0}$. Therefore, the bound in (10) is tight when the direct link is severely degraded or is much stronger than the source-to-relay link. Although showing the tightness of the bound for general cases are intractable, from the simulation results in Section VII, it will be shown that the proposed schemes based on the upper bound in (10) exhibit a negligible performance loss compared to the upper bound of the optimal performance.

Note that the proposed schemes in (15) and (24) minimize an upper bound of the MSE which follows from the bound in (10). Also, as mentioned above, the bound in (10) is tight when direct link is negligible. Thus, by setting $\mathbf{H}_{SD} = \mathbf{0}$, the proposed techniques in (15) and (24) become equivalent to the optimal solutions for the relaying systems without the direct link. For example, the solution in (15) with $\mathbf{H}_{SD} = \mathbf{0}$ is the same as the relay transceiver in [13].

V. Relay Transceiver Designs for the Relaxed Problems

Unfortunately, the optimal relay transceivers under MVC and SC have not been studied in the literature. As an alternative way, to verify the effectiveness of the proposed schemes in

Section IV, we introduce relaxed problems for the problems (6) and (16) and their solutions. First, we focus on a relay transmitter with the MVC. Note that the problem in (6) can be rewritten as the problem in (9) by employing the relay transmitter $\mathbf{F}_R = \tilde{\mathbf{U}}_{RD}\mathbf{\Phi}_1\tilde{\mathbf{U}}_\omega^H$ in (8). Also, from the result in Lemma 2, we obtain a lower bound of $\lambda_{\max}(\mathbf{\Phi}_1\mathbf{\Upsilon}_\omega\mathbf{\Phi}_1^H)$ in problem (9) as

$$\lambda_{\max}\left(\mathbf{\Phi}_1\mathbf{\Upsilon}_\omega\mathbf{\Phi}_1^H\right) \geq \lambda_{\max}\left(\mathbf{\Phi}_1\tilde{\mathbf{\Lambda}}_\omega\mathbf{\Phi}_1^H\right). \tag{25}$$

By considering the lower bound in (25), the optimization problem can be written as

$$\begin{aligned} \min_{\mathbf{\Phi}_1} \quad & \mathrm{Tr}\left(\left(\mathbf{\Phi}_1^H\tilde{\mathbf{\Lambda}}_{RD}\mathbf{\Phi}_1 + \tilde{\mathbf{\Lambda}}_\omega^{-1}\right)^{-1}\right) \\ \text{subject to} \quad & \lambda_{\max}\left(\mathbf{\Phi}_1\tilde{\mathbf{\Lambda}}_\omega\mathbf{\Phi}_1^H\right) \leq P_{peak}. \end{aligned} \tag{26}$$

Since we impose a relaxation of the power constraint compared to the problem in (6), a solution for the problem in (26) exhibits a lower MSE than the optimal solution in (6). Here the solution becomes $\mathbf{\Phi}_1 = \sqrt{P_{peak}}\tilde{\mathbf{\Lambda}}_\omega^{-1/2}$ which can be easily derived in a similar fashion as in (13) and (14). Therefore, the relay transmitter solution for the problem in (26) is given by

$$\mathbf{F}_R = \sqrt{P_{peak}}\tilde{\mathbf{U}}_{RD}\tilde{\mathbf{\Lambda}}_\omega^{-1/2}\tilde{\mathbf{U}}_\omega^H. \tag{27}$$

Now, we consider a relay transmitter design with the SC. Since $\mathbf{U}_\omega$ is a non-singular matrix for the case of $N_R = N_S$, the relay transmitter $\mathbf{F}_R$ can be written as $\mathbf{F}_R = \mathbf{\Phi}_1\mathbf{U}_\omega^H$. Then, the problem in (16) becomes

$$\begin{aligned} \min_{\mathbf{\Phi}_1} \quad & \mathrm{Tr}\left(\left(\mathbf{\Phi}_1^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{\Phi}_1 + \mathbf{\Lambda}_\omega^{-1}\right)^{-1}\right) \\ \text{subject to} \quad & \mathbf{\Phi}_1\mathbf{\Upsilon}_\omega\mathbf{\Phi}_1^H \preceq \mathbf{S}. \end{aligned} \tag{28}$$

Also, from the result in Lemma 2, we obtain the following inequality as

$$\mathbf{\Phi}_1\mathbf{\Upsilon}_\omega\mathbf{\Phi}_1^H \succeq \mathbf{\Phi}_1\mathbf{\Lambda}_\omega\mathbf{\Phi}_1^H. \tag{29}$$

By substituting (29) into the problem in (28) and introducing a matrix $\tilde{\mathbf{S}} \in \mathbb{S}^M$, the problem in (28) can be reformulated as

$$\begin{aligned} \min_{\mathbf{\Phi}_1} \quad & \mathrm{Tr}\left(\left(\mathbf{\Phi}_1^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{\Phi}_1 + \mathbf{\Lambda}_\omega^{-1}\right)^{-1}\right) \\ \text{subject to} \quad & \mathbf{\Phi}_1\mathbf{\Lambda}_\omega\mathbf{\Phi}_1^H = \tilde{\mathbf{S}} \\ & \tilde{\mathbf{S}} \preceq \mathbf{S}. \end{aligned} \tag{30}$$

Since the constraint on the transmit covariance matrix is relaxed, the performance of a solution for the problem in (30) can be considered as an upper bound of the performance of the optimal solution for the problem in (16). Using the similar procedure in (18)–(23), we compute $\mathbf{\Phi}_1$ as $\mathbf{\Phi}_1 = \mathbf{S}^{1/2}\mathbf{U}_S\mathbf{\Lambda}_\omega^{-1/2}$. Then, the relay transmitter for the problem in (30) is determined as

$$\mathbf{F}_R = \mathbf{S}^{1/2}\mathbf{U}_S\mathbf{\Lambda}_\omega^{-1/2}\mathbf{U}_\omega^H. \tag{31}$$

In this paper, we will compare the performance of the proposed methods with the solutions in (27) and (31). It is confirmed from the simulation results that the proposed schemes show a negligible performance loss compared to (27) and (31).

VI. Quantized Relay Transceiver Design

So far, we have assumed that perfect CSI for all links between nodes is available at the relay node. However, obtaining the perfect CSI may not be possible in practical systems due to limited feedback channel bandwidth. In this section, we introduce a quantized relay transceiver technique for the limited feedback scenario. We assume that the receiver sides of each link have knowledge of its CSI by utilizing the training sequences. In other words, the relay knows $\mathbf{H}_{SR}$ and the destination knows $\mathbf{H}_{SD}$ and $\mathbf{H}_{RD}$. Also, it is assumed that the CSI for $\mathbf{H}_{SR}$ is available at the destination, since the relay can append the received training symbols of the source-to-relay channel to the relay-to-destination training symbols and forward them to the destination [22]. Therefore, additional information required at the relay is $\mathbf{H}_{SD}$ and $\mathbf{H}_{RD}$ that can be conveyed from the destination using a limited bandwidth feedback channel.

To reduce the feedback burden, we employ codebooks which are known to both the relay and the destination. Then, the relay can compute the relay transceivers based on the received codeword indices from the destination. Unlike conventional codebook methods in [13], [14], and [23], when direct link is non-negligible, a codebook design normally becomes nontrivial, since the relay transceiver does not impose a specific structure which allows a tractable codebook design.

Now, let us have a look at the relay transceiver in (4) and (15). We can check that only $\tilde{\mathbf{U}}_{RD}$ comes from $\mathbf{H}_{RD}$ while the rest of terms are relevant to $\mathbf{H}_{SD}$. Motivated by this observation, we employ two codebooks which correspond to the direct link channel matrix $\mathbf{H}_{SD}$ and the orthonormal matrix $\tilde{\mathbf{U}}_{RD}$. First, since $\mathbf{H}_{SD}$ is involved in the relay transceiver as the form of $\mathbf{H}_{SD}^H\mathbf{H}_{SD}$, we focus on the quantization of $\mathbf{D} \triangleq \mathbf{H}_{SD}^H\mathbf{H}_{SD}$ which can be represented as $\mathbf{D} = \sum_{i=1}^{N_S}\lambda_{D,i}\mathbf{u}_{D,i}\mathbf{u}_{D,i}^H$ where $\lambda_{D,i}$ and $\mathbf{u}_{D,i} \in \mathbb{C}^{N_S\times 1}$ equal the i-th largest eigenvalue of $\mathbf{D}$ and its corresponding eigenvector, respectively. Note that $\mathbf{u}_{D,i}$ is uniformly distributed on the unit sphere [24], and thus a Grassmannian codebook is efficient for quantization.[2] Therefore, we consider the Grassmannian codebook consisting of N_1 distinct unit vectors $\mathbf{C}_1(N_S, N_1) = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{N_1}\}$.

Then, the eigenvectors quantized at the destination are expressed by

$$\hat{\mathbf{u}}_{D,i} = \underset{\mathbf{w}_j\in\mathbf{C}_1(N_S,N_1)}{\arg\min}\; d(\mathbf{u}_{D,i}, \mathbf{w}_j) \quad \text{for} \quad i = 1, \ldots, N_S,$$

where $d(\mathbf{w}_i, \mathbf{w}_j) \triangleq \sqrt{1 - |\mathbf{w}_i^H\mathbf{w}_j|^2}$ indicates the distance between two unit vectors which is known as the chordal distance. Based on the codeword indices which are fed back to the relay from the destination, the relay can identify $\mathbf{D}$ as $\hat{\mathbf{D}} \triangleq \sum_{i=1}^{N_S}\lambda_{SD,i}\hat{\mathbf{u}}_{SD,i}\hat{\mathbf{u}}_{SD,i}^H$.

[2] Here we consider the quantization of eigenvectors only, because eigenvalues can be efficiently quantized with conventional scalar quantizers.

For a given $\hat{\mathbf{D}}$, the relay computes $\hat{\mathbf{\Psi}} \triangleq (\hat{\mathbf{D}} + \rho^{-1}\mathbf{I}_{N_S})^{-1}$, $\hat{\mathbf{W}}_R \triangleq (\mathbf{H}_{SR}^H\mathbf{H}_{SR} + \hat{\mathbf{D}} + \rho^{-1}\mathbf{I}_{N_S})^{-1}\mathbf{H}_{SR}^H$, $\hat{\mathbf{\Omega}} \triangleq \hat{\mathbf{W}}_R\mathbf{H}_{SR} \times \hat{\mathbf{\Psi}}$ and its EVD as $\hat{\mathbf{\Omega}} = \hat{\mathbf{U}}_\omega\hat{\mathbf{\Lambda}}_\omega\hat{\mathbf{U}}_\omega^H$ where $\hat{\mathbf{U}}_\omega \in \mathbb{C}^{N_S\times N_S}$ and $\hat{\mathbf{\Lambda}}_\omega \in \mathbb{C}^{N_S\times N_S}$ represent a unitary matrix and a diagonal matrix with eigenvalues, respectively. Defining the quantized relay transceiver $\hat{\mathbf{G}}_R$ as $\hat{\mathbf{G}}_R = \hat{\mathbf{F}}_R\hat{\mathbf{W}}_R$, the error covariance matrix with $\hat{\mathbf{D}}$ can be decomposed in a similar fashion as described in (5) as

$$\mathbf{R}_e(\hat{\mathbf{F}}_R) = \left(\mathbf{H}_{SR}^H\mathbf{H}_{SR} + \hat{\mathbf{D}} + \rho^{-1}\mathbf{I}_{N_S}\right)^{-1} + \hat{\mathbf{V}}_\omega\left(\hat{\mathbf{V}}_\omega^H\hat{\mathbf{F}}_R^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\hat{\mathbf{F}}_R\hat{\mathbf{V}}_\omega + \hat{\mathbf{\Sigma}}_\omega^{-1}\right)^{-1}\hat{\mathbf{V}}_\omega^H, \quad (32)$$

where $\hat{\mathbf{V}}_\omega \in \mathbb{C}^{N_S\times M}$ is a matrix constructed by the first M columns of $\hat{\mathbf{U}}_\omega$ and $\hat{\mathbf{\Sigma}}_\omega$ indicates an $M \times M$ upper-left submatrix of $\hat{\mathbf{\Lambda}}_\omega$.

Let us define $\hat{\mathbf{\Upsilon}} \triangleq \hat{\mathbf{W}}_R(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\hat{\mathbf{W}}_R^H$ and $\hat{\mathbf{\Upsilon}}_\omega \triangleq \hat{\mathbf{V}}_\omega^H\hat{\mathbf{\Upsilon}}\hat{\mathbf{V}}_\omega$. Here EVD of $\hat{\mathbf{\Upsilon}}_\omega$ can be represented as $\hat{\mathbf{\Upsilon}}_\omega = \hat{\mathbf{U}}_\upsilon\hat{\mathbf{\Lambda}}_\upsilon\hat{\mathbf{U}}_\upsilon^H$ where $\hat{\mathbf{U}}_\upsilon \in \mathbb{C}^{M\times M}$ denotes a unitary matrix and $\hat{\mathbf{\Lambda}}_\upsilon \in \mathbb{C}^{M\times M}$ stands for a diagonal matrix with eigenvalues $\lambda_{\upsilon,i}$ for $i = 1, \ldots, M$ in descending order on the main diagonal. Since the relay transceiver in (15) minimizes an upper bound of the MSE under the MVC, the structure of the quantized relay transmitter can be given by

$$\hat{\mathbf{F}}_R = \sqrt{P_{peak}}\mathbf{U}_Q\hat{\mathbf{\Lambda}}_\upsilon^{-1/2}\hat{\mathbf{U}}_\upsilon^H\hat{\mathbf{V}}_\omega^H, \quad (33)$$

where $\mathbf{U}_Q \in \mathbb{C}^{N_R\times M}$ equals an arbitrary matrix with orthonormal columns.

From now on, we focus on the quantization technique for $\mathbf{U}_Q$. We consider a codebook consisting of N_2 codewords as $\mathbf{C}_2(N_R, M, N_2) = \{\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_{N_2}\}$ where $\mathbf{U}_i \in \mathbb{C}^{N_R\times M}$ has orthonormal column vectors. Plugging the quantized relay transmitter in (33) into (32), the MSE of the second term in (32) becomes

$$\epsilon(\mathbf{U}_Q) \triangleq \mathrm{Tr}\Big(\Big(P_{peak}\hat{\mathbf{U}}_\upsilon\hat{\mathbf{\Lambda}}_\upsilon^{-H/2}\mathbf{U}_Q^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{U}_Q \times \hat{\mathbf{\Lambda}}_\upsilon^{-1/2}\hat{\mathbf{U}}_\upsilon^H + \hat{\mathbf{\Sigma}}_\omega^{-1}\Big)^{-1}\Big).$$

Since the first term in (32) is irrelevant to $\mathbf{U}_Q$, we quantize $\mathbf{U}_Q$ as

$$\hat{\mathbf{U}}_Q = \underset{\mathbf{U}_j\in\mathbf{C}_2(N_R,M,N_2)}{\arg\min}\ \epsilon(\mathbf{U}_j). \quad (34)$$

Unfortunately, the codebook which directly minimizes $\epsilon(\hat{\mathbf{U}}_Q)$ is hard to obtain due to the complicated form of $\epsilon(\mathbf{U}_Q)$. To solve this difficulty, we derive an upper bound of $\epsilon(\mathbf{U}_Q)$ as

$$\begin{aligned}\epsilon(\mathbf{U}_Q) &\leq \mathrm{Tr}\left(\hat{\mathbf{\Lambda}}_\upsilon\left(P_{peak}\mathbf{U}_Q^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{U}_Q + \mathbf{I}_{N_M}\right)^{-1}\right)\\ &\leq \mathrm{Tr}\left(\lambda_{\upsilon,1}\left(P_{peak}\mathbf{U}_Q^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{U}_Q + \mathbf{I}_{N_M}\right)^{-1}\right)\\ &\leq \mathrm{Tr}\left(\lambda_{\upsilon,1}\left(P_{peak}\mathbf{U}_Q^H\mathbf{H}_{RD}^H\mathbf{H}_{RD}\mathbf{U}_Q\right)^{-1}\right)\\ &\leq \frac{M\lambda_{\upsilon,1}}{P_{peak}\sigma_{\min}^2(\mathbf{H}_{RD}\mathbf{U}_Q)}, \end{aligned} \quad (35)$$

where the inequality in (35) follows from the fact $\hat{\mathbf{\Sigma}}_\omega \preceq \hat{\mathbf{\Upsilon}}_\omega$ which is readily computed in a similar fashion as Lemma 2, and $\sigma_{\min}(\mathbf{A})$ denotes the minimum singular value of a matrix $\mathbf{A}$. In fact, a Grassmannian codebook which is designed for maximizing the minimum projection two-norm distance between any pair of codewords maximizes $\sigma_{\min}^2(\mathbf{H}_{RD}\mathbf{U}_Q)$ [25]. Thus, we employ the Grassmannian codebook for quantizing $\mathbf{U}_Q$ to minimize the upper bound of $\epsilon(\hat{\mathbf{U}}_Q)$.

As a result, in our scheme, the number of total feedback bits equals $N_S\log_2 N_1 + \log_2 N_2$. Compared to conventional relaying systems without direct link, our scheme requires additional $N_S\log_2 N_1$ feedback bits which account for the channel quantization of the direct link. However, it is confirmed from the simulation results that the proposed quantization technique outperforms the conventional schemes. Also, it will be shown that the proposed quantization method approaches the performance of systems with full CSI with much reduced feedback overhead.

VII. Simulation Results

In this section, we provide the numerical results to evaluate the performance of the proposed schemes. For a system with N_S source, N_R relay and N_D destination antennas, we use the notation $(N_S \times N_R \times N_D)$. We assume that all channel matrices have independent and identically distributed complex Gaussian entries with zero mean and variances σ_{SR}^2, σ_{RD}^2, and σ_{SD}^2 for $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$, and $\mathbf{H}_{SD}$, respectively. The total transmit power at the source and the relay are denoted as P_S and P_R, respectively, and the peak power at the relay node is set to $P_{peak} = P_R/M$. Also, we employ the SC as $\mathbf{S} = \mathrm{diag}\{P_1, \ldots, P_{N_R}\}$ to impose per-antenna power constraint at the relay. The SNR in each link is defined as $\mathrm{SNR}_{\mathrm{SR}} \triangleq \sigma_{SR}^2P_S$, $\mathrm{SNR}_{\mathrm{RD}} \triangleq \sigma_{RD}^2P_R$, and $\mathrm{SNR}_{\mathrm{SD}} \triangleq \sigma_{SD}^2P_S$ for the source-to-relay, relay-to-destination and source-to-destination link, respectively. We compare the performance of the proposed techniques with the following relay transceiver methods.

- *Naive AF*: only transmit power normalization is performed with the relay transceiver, i.e., $\mathbf{G}_R = \sqrt{P_R/\mathrm{Tr}(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})}\mathbf{I}_{N_R}$.
- *MVC without DL*: a MVC based relay transceiver which minimizes the MSE for relaying systems without direct link is employed [13].
- *R-MVC*: a relay transceiver solution (27) for the relaxed problem with the MVC in (26) is applied.
- *R-SC*: a SC based relay transceiver (31) which optimizes the relaxed problem in (30) is adopted.
- *PG-NC*: a relay filter with the NC based on the projected gradient (PG) method is employed [15].
- *JSR-NC*: an iterative joint source and relay filter optimization with the NC is performed [26].

Since the feasible set of the problem with the NC includes that of the problem with the MVC, the optimal performance under the NC can be considered as a performance upper bound of the systems with the MVC. However, since the problem with the NC is non-convex, the PG-NC scheme cannot guarantee the

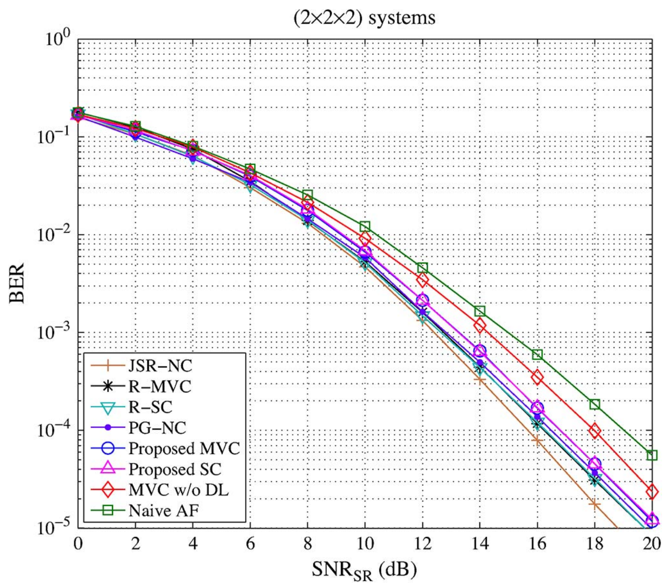

Fig. 2. BER performance comparison as a function of $\mathrm{SNR}_{\mathrm{SR}}$ with $\mathrm{SNR}_{\mathrm{RD}} = \mathrm{SNR}_{\mathrm{SD}} = \mathrm{SNR}_{\mathrm{SR}}$.

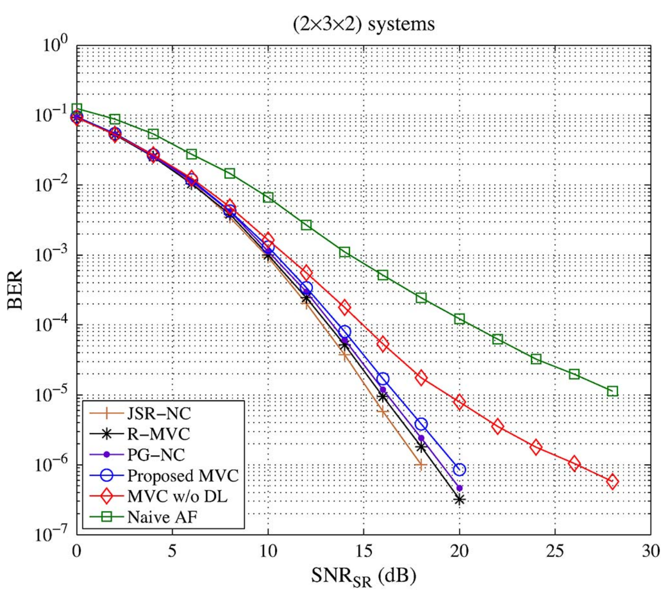

Fig. 3. BER performance comparison as a function of $\mathrm{SNR}_{\mathrm{SR}}$ with $\mathrm{SNR}_{\mathrm{RD}} = 15$ dB.

optimal performance. Note that the feasible sets of the relaxed problems in (26) and (30) contain that of the original problems in (6) and (16). Also, the R-MVC scheme and the R-SC scheme are the optimal solutions for the problems in (26) and (30), respectively. Therefore, we consider the performance of the R-MVC scheme and the R-SC scheme as an upper bound of the optimal schemes for the problems in (6) and (16), respectively.

In Figs. 2–4, we illustrate the average bit error rate (BER) performance for AF MIMO relaying systems with QPSK constellation. In Fig. 2, the performance for $(2 \times 2 \times 2)$ systems is exhibited as a function of $\mathrm{SNR}_{\mathrm{SR}}$ with $\mathrm{SNR}_{\mathrm{RD}} = \mathrm{SNR}_{\mathrm{SD}} = \mathrm{SNR}_{\mathrm{SR}}$. Here, we employ the SC as $\mathbf{S} = \mathrm{diag}\{P_1, P_2\}$ with $P_1 = (1/3)P_R$ and $P_2 = (2/3)P_R$. We can see that the proposed closed-form solutions under the MVC and the SC provide the performance almost identical to the R-MVC scheme and the

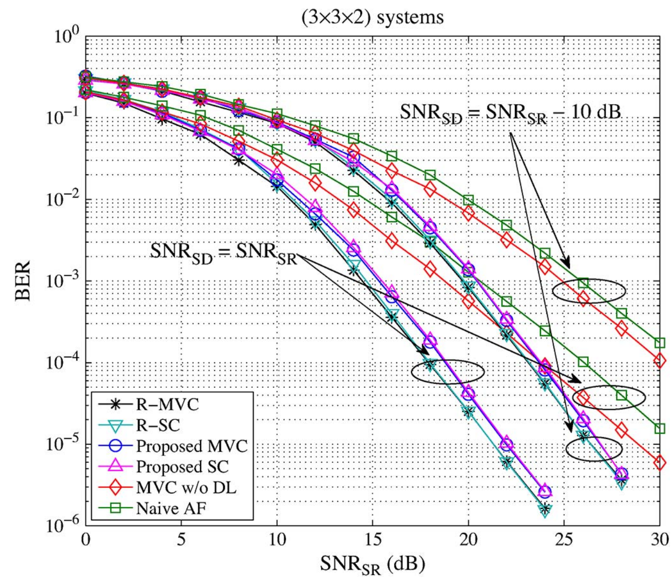

Fig. 4. BER performance comparison as a function of $\mathrm{SNR}_{\mathrm{SR}}$ with $\mathrm{SNR}_{\mathrm{SR}} = \mathrm{SNR}_{\mathrm{RD}}$.

R-SC scheme, respectively. Also, it is confirmed that considering not only the source-relay-destination link but also the direct link enhances the performance of relaying systems. Note that in spite of strict practical power constraints, the proposed schemes under the MVC and the SC exhibit only negligible performance loss compared to the PG-NC scheme.

Fig. 3 presents the average BER performance for $(2 \times 3 \times 2)$ systems as a function of $\mathrm{SNR}_{\mathrm{SR}}$ when $\mathrm{SNR}_{\mathrm{SD}} = \mathrm{SNR}_{\mathrm{SR}}$ and $\mathrm{SNR}_{\mathrm{RD}}$ is fixed at 15 dB. We can observe that the performance of the proposed relay transceiver with MVC shows a quite small performance loss compared to the R-MVC scheme. Also, it is shown that the proposed scheme outperforms the MVC without DL scheme at overall SNR regime. Since the fixed $\mathrm{SNR}_{\mathrm{RD}}$ incurs the bottleneck effect on the source-relay-destination link, the performance of the MVC without DL scheme which does not take the direct link into account becomes deteriorated. As shown in Fig. 3, we observe gain of about 6 dB for the proposed scheme over the MVC without DL scheme at BER $= 10^{-6}$. Meanwhile, the JSR-NC scheme which requires heavy computational burden and global CSI at the source obtains only less than 2 dB gain at BER $= 10^{-6}$ over the proposed MVC scheme. From these results, we can see that compared to the joint source and relay filter design, a proper relay transceiver design without optimizing the source filter may be more efficient considering the computational complexity and feedback overhead.

In Fig. 4, we illustrate the average BER performance for $(3 \times 3 \times 2)$ systems with respect to $\mathrm{SNR}_{\mathrm{SR}}$ when $\mathrm{SNR}_{\mathrm{SR}} = \mathrm{SNR}_{\mathrm{RD}}$. Here, we adopt the SC as $\mathbf{S} = \mathrm{diag}\{P_1, P_2, P_3\}$ with $P_1 = (1/6)P_R$, $P_2 = (1/3)P_R$ and $P_3 = (1/2)P_R$. It is observed that the proposed schemes under the MVC and the SC show almost identical performance with the R-MVC scheme and the R-SC scheme. On the other hand, the performance of the MVC without DL scheme is significantly degraded since a diversity gain from direct link is not exploited. Note that the additional diversity gain from utilizing the direct link is achieved even when the direct link experiences severe path loss compared to the source-to-relay link.

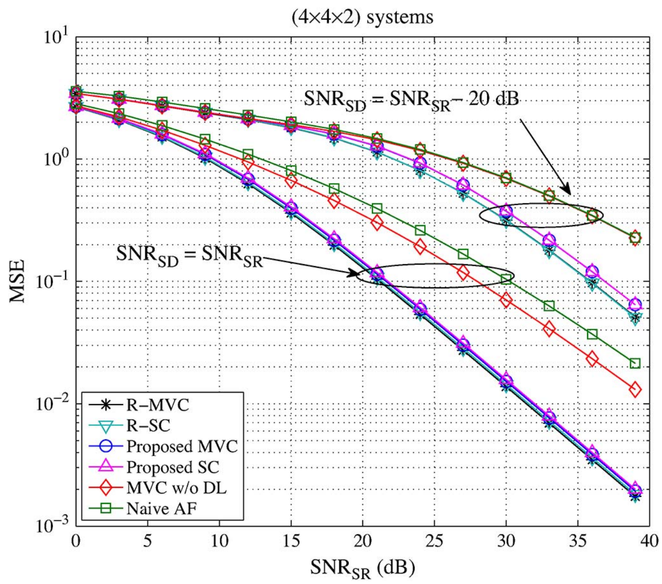

Fig. 5. MSE performance comparison as a function of $\mathrm{SNR_{SR}}$ with $\mathrm{SNR_{SR}} = \mathrm{SNR_{RD}}$.

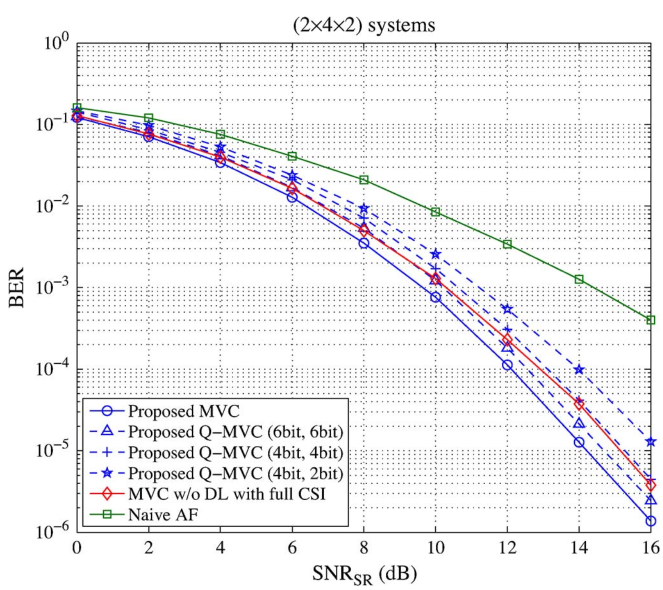

Fig. 6. BER performance comparison as a function of $\mathrm{SNR_{SR}}$ with $\mathrm{SNR_{RD}} = \mathrm{SNR_{SD}} = \mathrm{SNR_{SR}}$.

Fig. 5 plots the average MSE performance for $(4 \times 4 \times 2)$ systems with QPSK constellation when $\mathrm{SNR_{SR}} = \mathrm{SNR_{RD}}$. Here, the SC is chosen as $\mathbf{S} = \mathrm{diag}\{P_1, P_2, P_3, P_4\}$ with $P_1 = (4/10)P_R$, $P_2 = (3/10)P_R$, $P_3 = (2/10)P_R$, and $P_4 = (1/10)P_R$. It is shown in the plot that the MVC without DL scheme suffers a significant performance loss. Meanwhile, the proposed schemes with MVC and SC obtain almost identical performance with the R-MVC scheme and the R-SC scheme in all SNR region. From this result, we can confirm that a performance loss from the bound in (10) is relatively small and the proposed schemes are efficient in minimizing the MSE.

In Figs. 6 and 7, we demonstrate the average BER performance of the proposed quantized relay transceiver technique for $(2 \times 4 \times 2)$ systems with QPSK constellation. Fig. 6 depicts the performance for relaying systems as a function of $\mathrm{SNR_{SR}}$

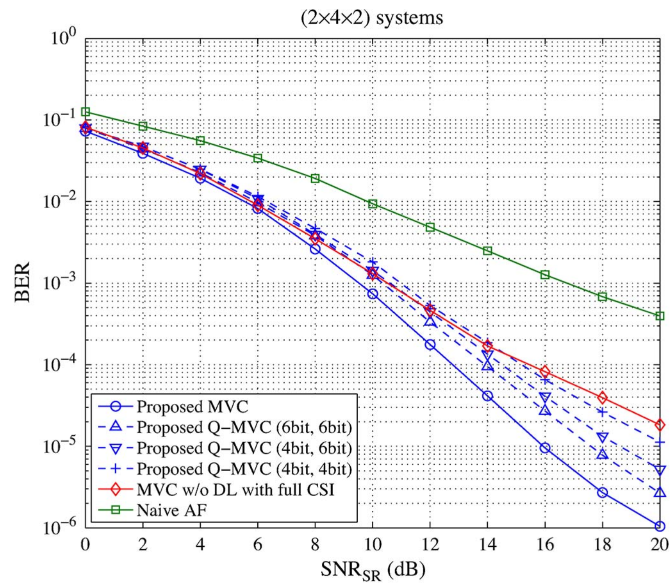

Fig. 7. BER performance comparison as a function of $\mathrm{SNR_{SR}}$ with $\mathrm{SNR_{RD}} = 10$ dB.

with $\mathrm{SNR_{RD}} = \mathrm{SNR_{SD}} = \mathrm{SNR_{SR}}$. We use the notation ($b_1$ bit, b_2 bit) to denote the quantized relay transceiver technique (Proposed Q-MVC) with $b_1 = N_S \log_2 N_1$ and $b_2 = \log_2 N_2$ feedback bits. We can see that the proposed scheme exhibits a performance loss compared to the proposed MVC scheme with full CSI when 6 feedback bits are employed. This is due to the fact that the allocated feedback bits for the codebook $\mathbf{C}_2$ in (34) are insufficient to quantize $\mathbf{U}_Q$ in (33). However, the performance is improved by applying more feedback bits for the proposed quantization scheme. For example, the proposed scheme with 12 feedback bits outperforms the MVC without DL scheme with full CSI and shows a very small performance loss compared to the proposed MVC scheme with perfect CSI.

In Fig. 7, we illustrate the average BER performance of relaying systems with respect to $\mathrm{SNR_{SR}}$ when $\mathrm{SNR_{SD}} = \mathrm{SNR_{SR}}$ and $\mathrm{SNR_{RD}}$ is fixed at 10 dB. The proposed quantized scheme provides a performance gain over the MVC without DL scheme with full CSI by employing only 8 feedback bits. It is seen that the proposed method with 12 feedback bits gives about a 3.5 dB gain at $\mathrm{BER} = 2 \times 10^{-5}$ compared to the MVC without DL scheme with full CSI.

From simulation results, we conclude that the proposed schemes outperform the conventional schemes.

VIII. Conclusion

In this paper, we have proposed new relay transceiver designs for the MSE minimization in MIMO AF relaying systems with non-negligible direct link. Since NC at the relay node may not be suitable for practical systems, we have considered SC which imposes a constraint on the shape of the transmit covariance matrix. We have first determined the optimal structure of the relay transceiver under the SC and derived an upper bound expression for the MSE. Then, we have presented closed-form relay transceiver solutions under the SC and the MVC which minimize the bound. We have also introduced relay transceiver methods for the relaxed problems whose feasible sets contain the optimal solutions of the original problems. In addition, we have provided a Grassmannian codebook based relay transceiver quantization technique for the limited feedback scenario. From numerical simulations, we have verified that the proposed schemes outperform conventional schemes for all simulated configurations and show a negligible performance loss compared to the upper bound of the optimal performance.

Appendix A
Proof of Lemma 1

Without loss of generality, we can write $\mathbf{G}_R$ in a general form as $\mathbf{G}_R = \mathbf{G}_{R_\parallel} + \mathbf{G}_{R_\perp}$ where $\mathbf{G}_{R_\parallel}$ and $\mathbf{G}_{R_\perp}$ denote the components of $\mathbf{G}_R$ such that the row space of $\mathbf{G}_{R_\parallel}$ and $\mathbf{G}_{R_\perp}$ are parallel and orthogonal to the column space of $\mathbf{H}_{SR}$, respectively. Then, the MSE is written as

$$\begin{aligned}
&\mathrm{Tr}\left(\mathbf{R}_e(\mathbf{G}_R)\right)\\
&= \mathrm{Tr}\Bigg(\bigg(\mathbf{H}_{SR}^H\mathbf{G}_{R_\parallel}^H\mathbf{H}_{RD}^H\\
&\qquad\times\Big(\mathbf{H}_{RD}\Big(\mathbf{G}_{R_\parallel}\mathbf{G}_{R_\parallel}^H+\mathbf{G}_{R_\perp}\mathbf{G}_{R_\perp}^H\Big)\mathbf{H}_{RD}^H+\mathbf{I}_{N_D}\Big)^{-1}\\
&\qquad\times\mathbf{H}_{RD}\mathbf{G}_{R_\parallel}\mathbf{H}_{SR}+\boldsymbol{\Psi}^{-1}\bigg)^{-1}\Bigg)\\
&\geq \mathrm{Tr}\Bigg(\bigg(\mathbf{H}_{SR}^H\mathbf{G}_{R_\parallel}^H\mathbf{H}_{RD}^H\left(\mathbf{H}_{RD}\mathbf{G}_{R_\parallel}\mathbf{G}_{R_\parallel}^H\mathbf{H}_{RD}^H+\mathbf{I}_{N_D}\right)^{-1}\\
&\qquad\quad\times\mathbf{H}_{RD}\mathbf{G}_{R_\parallel}\mathbf{H}_{SR}+\boldsymbol{\Psi}^{-1}\bigg)^{-1}\Bigg)\\
&= \mathrm{Tr}\left(\mathbf{R}_e\left(\mathbf{G}_{R_\parallel}\right)\right),
\end{aligned}\tag{36}$$

where the inequality follows from the fact $\mathrm{Tr}(\mathbf{A}) \geq \mathrm{Tr}(\mathbf{B})$ if $\mathbf{A} \succeq \mathbf{B}$ for positive semidefinite matrices $\mathbf{A}$ and $\mathbf{B}$.

Also, the transmit covariance matrix becomes

$$\begin{aligned}
&\mathbf{R}_c(\mathbf{G}_R)\\
&\quad= \left(\mathbf{G}_{R_\parallel}+\mathbf{G}_{R_\perp}\right)\left(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H+\mathbf{I}_{N_R}\right)\left(\mathbf{G}_{R_\parallel}+\mathbf{G}_{R_\perp}\right)^H\\
&\quad= \mathbf{G}_{R_\parallel}\left(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H+\mathbf{I}_{N_R}\right)\mathbf{G}_{R_\parallel}^H+\mathbf{G}_{R_\perp}\mathbf{G}_{R_\perp}^H\\
&\quad\succeq \mathbf{G}_{R_\parallel}\left(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H+\mathbf{I}_{N_R}\right)\mathbf{G}_{R_\parallel}^H\\
&\quad= \mathbf{R}_c\left(\mathbf{G}_{R_\parallel}\right).
\end{aligned}\tag{37}$$

It is seen from the results in (36) and (37) that a non-zero $\mathbf{G}_{R_\perp}$ always increases both the MSE and the transmit power, i.e., $\mathbf{R}_c(\mathbf{G}_{R_\parallel}+\mathbf{G}_{R_\perp}) \succeq \mathbf{R}_c(\mathbf{G}_{R_\parallel})$. Thus, setting $\mathbf{G}_{R_\perp} = \mathbf{0}$ incurs no loss of optimality.[3] Then, we have $\mathbf{G} = \mathbf{G}_{R_\parallel} = \mathbf{B}\mathbf{H}_{SR}^H$ where $\mathbf{B}$ is an arbitrary matrix and can be expressed as $\mathbf{B} = \hat{\mathbf{B}}\mathbf{P}$ with $\mathbf{P}$ being an arbitrary square invertible matrix. Here, without loss of generality, $\mathbf{P}$ can be chosen as $\mathbf{P} = (\mathbf{H}_{SR}^H\mathbf{H}_{SR} + \mathbf{H}_{SD}^H\mathbf{H}_{SD} + \rho^{-1}\mathbf{I}_{N_S})^{-1}$. ■

Appendix B
Proof of Lemma 2

Invoking the matrix inversion lemma [27], the relay receiver in (4) is given by

$$\begin{aligned}
\mathbf{W}_R &= \left(\mathbf{H}_{SR}^H\mathbf{H}_{SR}+\boldsymbol{\Psi}^{-1}\right)^{-1}\mathbf{H}_{SR}^H\\
&= \boldsymbol{\Psi}\mathbf{H}_{SR}^H\left(\mathbf{H}_{SR}\boldsymbol{\Psi}\mathbf{H}_{SR}^H+\mathbf{I}_{N_R}\right)^{-1}.
\end{aligned}$$

[3]Since $\mathbf{A} \succeq \mathbf{B}$ implies $\lambda_{\max}(\mathbf{A}) \geq \lambda_{\max}(\mathbf{B})$, setting $\mathbf{G}_{R_\perp} = \mathbf{0}$ does not lose the optimality under the MVC either.

Post-multiplying both sides of the equation by $\mathbf{H}_{SR}\boldsymbol{\Psi}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R}$, we get $\mathbf{W}_R(\mathbf{H}_{SR}\boldsymbol{\Psi}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R}) = \boldsymbol{\Psi}\mathbf{H}_{SR}^H$. Therefore, $\boldsymbol{\Omega} = \mathbf{W}_R\mathbf{H}_{SR}\boldsymbol{\Psi}$ can be rewritten as

$$\boldsymbol{\Omega} = \mathbf{W}_R\left(\mathbf{H}_{SR}\boldsymbol{\Psi}\mathbf{H}_{SR}^H+\mathbf{I}_{N_R}\right)\mathbf{W}_R^H.$$

Note that $\boldsymbol{\Upsilon} = \mathbf{W}_R(\rho\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \mathbf{I}_{N_R})\mathbf{W}_R^H$ and $\boldsymbol{\Psi} = (\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \rho^{-1}\mathbf{I}_{N_S})^{-1} \preceq \rho\mathbf{I}_{N_S}$ due to the fact that $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A}^{-1} \preceq \mathbf{B}^{-1}$. Therefore, the following inequality holds as

$$\boldsymbol{\Omega} \preceq \boldsymbol{\Upsilon}.$$

Finally, multiplying the above inequality by $\tilde{\mathbf{U}}_\omega^H$ on the left and $\tilde{\mathbf{U}}_\omega$ on the right, we have

$$\tilde{\boldsymbol{\Lambda}}_\omega \preceq \boldsymbol{\Upsilon}_\omega.$$

This concludes the proof. ■

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