MMSE-Based Filter Designs for Cognitive Multiuser Two-Way Relay Networks

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***Abstract*—This paper considers cognitive multiuser two-way relay networks (MU-TRNs) where multiple secondary users (SUs) concurrently exchange their data with other SUs with an aid of a two-way relay in licensed primary networks. In this paper, we particularly focus on linear relay filter designs that effectively minimize the mean square error of SUs while providing a certain level of quality of service for the primary network. To this end, we first propose an iterative algorithm that identifies a solution based on an alternating optimization technique. Then, a closed-form relay filter design is presented, which employs a high signal-to-noise ratio approximation to reduce the computational complexity. Through the simulation results, we confirm that the proposed closed-form relay filter design outperforms conventional approaches and provides the performance close to the proposed iterative algorithm with much reduced complexity.**

***Index Terms*—Cognitive radio, convex optimization, two-way relay.**

I. Introduction

Recently, technologies for cognitive radio (CR) have been studied as a promising and cost-effective solution for efficient spectrum utilization in wireless communication systems [1], [2]. The key idea of the CR is to allow an unlicensed secondary user (SU) to opportunistically or concurrently utilize the frequency band allocated to the licensed primary network. For CR systems, the major challenge is to ensure quality of service of the primary network while maximizing the utility functions of SUs. To efficiently handle this issue, multiple-antenna transmission [2]–[4] is employed to improve performance and reliability for the CR systems. Meanwhile, two-way relay networks [5]–[9] have been widely studied and considered a good candidate for the next-generation communication system since they can provide better spectral efficiency compared with one-way relaying schemes. In the two-way relay networks, each source node exchanges its message with an aid of a relay in two orthogonal channel uses. Regarding the forwarding strategy, there are two well-known protocols, namely decode-and-forward (DF) and amplify-and-forward (AF) protocols [10]. Compared with the DF relaying system that applies the decoding process at the relay, the AF relay is more suitable for practical relaying systems due to its simple implementation. For this reason, we focus on the AF relay protocol in this paper.

Combining the two-way AF relay and the CR can provide a promising solution that improves spectrum utilization [11]–[13]. In [11], power control and relay filter designs were introduced to maximize the sum rate of the cognitive two-way relay systems, and in [12], the optimal relay selection and power control algorithm was proposed for cognitive multipair two-way relay networks. For cognitive multiuser multiway relay systems, in [13], the MSE-based source–relay joint designs were introduced.

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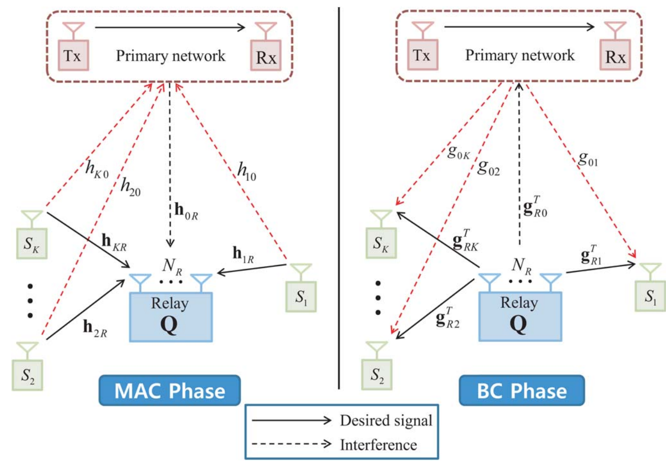

Fig. 1. Schematic of cognitive MU-TRN.

In this paper, we consider cognitive multiuser two-way relay networks (MU-TRNs) where multiple SUs exchange their data in an arbitrary unicast manner. Here, each SU sends its data to another user and could receive data from a different SU with the help of a two-way AF relay. Note that the arbitrary unicast has interesting features since any traffic patterns such as unicast, multicast, and broadcast can be realized by scheduling a sequence of unicast flows [14], and thereby, it can be widely employed to various wireless communication systems. For this cognitive MU-TRN, we first propose an iterative relay filter design that minimizes the sum MSE using an alternating optimization method [15]. Since an iterative algorithm requires high computation complexity, we also present a closed-form relay filter design utilizing a high SNR approximation. Through simulation results, it is confirmed that the closed-from design outperforms the conventional approaches and provides almost identical performance to the iterative scheme with much reduced complexity.

The following notations are used throughout this paper. We employ uppercase boldface letters for matrices, lowercase boldface for vectors, and normal letters for scalar quantities. Conjugate, transpose, and conjugate transpose of a matrix or a vector are represented by $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$, respectively. $\mathbf{I}_d$ indicates an identity matrix of size d, $E[\cdot]$ accounts for expectation, $\|\cdot\|$ denotes the Euclidean two-norm of a vector, and $|\cdot|$ represents the absolute value. In addition, $\mathrm{Tr}(\mathbf{A})$ equals the trace of a matrix $\mathbf{A}$, and $\mathrm{diag}\{\mathbf{a}\}$ indicates a diagonal matrix whose diagonal elements consist of a vector $\mathbf{a}$.

II. System Model

Here, we present a general description of a cognitive MU-TRN. As shown in Fig. 1, K SUs equipped with a single antenna communicate with each other with an aid of a single relay equipped with N_R antennas. In this cognitive MU-TRN, the unlicensed secondary network concurrently operates in the same spectrum band of the licensed primary network with a single antenna, i.e., we consider the underlay cognitive system [1]; thus, it may cause interference to the primary network.

Assuming narrow-band flat fading, the channel vectors from the kth SU S_k $(k = 1, 2, \ldots, K)$ to the relay and from the relay to S_k are represented by $\mathbf{h}_{kR} \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{g}_{Rk} \in \mathbb{C}^{N_R \times 1}$, respectively.

Moreover, let us denote $h_{k0} \in \mathbb{C}^1$ and $g_{0k} \in \mathbb{C}^1$ as the scalar channel coefficients from S_k to the primary receiver and from the primary transmitter to S_k, respectively, and $\mathbf{h}_{0R} \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{g}_{R0} \in \mathbb{C}^{N_R \times 1}$ as the channel vectors from the primary transmitter to the relay and from the relay to the primary receiver, respectively. We focus on the pure unicast case where each SU transmits the data to its corresponding one user only, and it is assumed that direct link among SUs is ignored as in [14].

Since the relay is assumed to operate in the half-duplex mode, the information exchange among all SUs occurs in two orthogonal phases. As shown in Fig. 1, in the multiple access control (MAC) phase, all S_k's transmit independent data signal x_k for $k = 1, 2, \ldots, K$ to the relay. In this case, the transmit power P_k at S_k should satisfy the following power constraints as

$$E|x_k|^2 = P_k \leq P_{\max}, \qquad \sum_{k=1}^{K} |h_{k0}|^2 P_k \leq \eta \tag{1}$$

where we define $P_{\max}$ and η as the maximum transmit power at the SUs and the allowed interference power at the primary network, respectively.

Since this paper concerns the relay filter design, the following power control algorithm for SUs is employed throughout this paper:

$$P_k = \min\left\{P_{\max}, \frac{\eta}{\sum_k |h_{k0}|^2}\right\}, \text{ for } k = 1, 2, \ldots, K. \tag{2}$$

Then, the received signal at the relay can be expressed as

$$\mathbf{y}_R = \mathbf{H}\mathbf{x} + \mathbf{h}_{0R}x_{01} + \mathbf{n}_R \tag{3}$$

where we have $\mathbf{H} \triangleq [\mathbf{h}_{1R}\mathbf{h}_{2R} \cdots \mathbf{h}_{KR}] \in \mathbb{C}^{N_R \times K}$ and $\mathbf{x} = [x_1, x_2, \ldots, x_K]^T$, and x_{01} denotes the transmit data signal from the primary transmitter with $E|x_{01}|^2 = P_0$ at the MAC phase, and $\mathbf{n}_R \sim CN(0, \mathbf{I}_{N_R})$ represents the additive complex Gaussian noise vector at the relay.

During the broadcast (BC) phase, the received signal $\mathbf{y}_R$ at the relay is multiplied by the linear relay filter $\mathbf{Q} \in \mathbb{C}^{N_R \times N_R}$ and retransmitted to SUs. We define covariance matrices $\mathbf{R}_{\mathbf{y}_R}$, $\mathbf{R}_n$, and $\mathbf{R}_\mathbf{x}$ as $\mathbf{R}_{\mathbf{y}_R} = \mathbf{H}\mathbf{R}_\mathbf{x}\mathbf{H}^H + \mathbf{R}_n$, $\mathbf{R}_n = P_0\mathbf{h}_{0R}\mathbf{h}_{0R}^H + \mathbf{I}_{N_R}$, and $\mathbf{R}_\mathbf{x} = \text{diag}\{P_1, P_2, \ldots, P_K\}$, respectively. Then, similar to the MAC phase, the relay filter $\mathbf{Q}$ needs to satisfy the following power constraints:

$$\text{Tr}(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H) \leq P_R, \quad \text{Tr}\left(\mathbf{g}_{R0}^T\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{g}_{R0}^*\right) \leq \eta. \tag{4}$$

Let us define $\mathbf{F}$ as a permutation matrix to represent the traffic flow pattern among SUs as in [14]. For instance, when $K = 2$, $\mathbf{F} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Then, the received signal y_k at S_k can be expressed as

$$\begin{aligned} y_k &= \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{y}_R + g_{0k}x_{02} + n_k \\ &= \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{H}\mathbf{x} + \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{0R}x_0 + \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{n}_R + g_{0k}x_{02} + n_k \\ &= \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{\bar{k}R}x_{\bar{k}} + \sum_{i \neq \bar{k}} \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{iR}x_i + \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{0R}x_0 \\ &\quad + \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{n}_R + g_{0k}x_{02} + n_k \end{aligned} \tag{5}$$

where x_{02} denotes the transmit data signal from the primary transmitter with $E|x_{02}|^2 = P_0$ at the BC phase, $\bar{k}$ indicates the index of the SU that transmits the desired signal for S_k, and n_k represents the independent complex Gaussian noise at S_k with zero mean and unit variance. Note that the first and second terms in (5) account for the desired signal and interuser interference, respectively. If S_k knows $\mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{kR}$ perfectly, a considerable performance gain is achieved by self-interference cancelation (SIC) from the received signal, i.e., $y_k - \mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{kR}x_k$ [7], [16].

III. Proposed Iterative Relay Filter Design

In this section, we present an iterative relay filter design for the cognitive MU-TRN. To this end, we first establish the minimum MSE (MMSE) problem considering SIC, and then based on Karush–Khun–Tucker (KKT) conditions of the formulated problem and the alternating optimization, we propose a relay filter design.

A. Problem Formulation

By defining the stacked received signal vector of (5) as $\mathbf{y}_S \triangleq [y_1, y_2, \ldots, y_K]^T$ and employing a receive combining matrix $\mathbf{C}$, the postprocessing signal vector $\hat{\mathbf{y}}$ can be given as

$$\begin{aligned} \hat{\mathbf{y}} &= \mathbf{C}\mathbf{y}_S \\ &= \mathbf{C}\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{x} + \mathbf{C}\mathbf{G}\mathbf{Q}\mathbf{h}_{0R}x_{01} + \mathbf{C}\mathbf{G}\mathbf{Q}\mathbf{n}_R \\ &\quad + \mathbf{C}\mathbf{g}_0x_{02} + \mathbf{C}\mathbf{n}_S \end{aligned} \tag{6}$$

where $\mathbf{G} = [\mathbf{g}_{R1}, \mathbf{g}_{R2}, \ldots, \mathbf{g}_{RK}]^T$, $\mathbf{g}_0 = [g_{01}, g_{02}, \ldots, g_{0K}]^T$, $\mathbf{n}_S = [n_1, n_2, \ldots, n_K]^T$, $\mathbf{C} = \text{diag}\{c_1, c_2, \ldots, c_K\}$, and c_k denotes the scalar combining coefficient for S_k for $k = 1, 2, \ldots, K$. Then, the sum MSE is written as $E\|\gamma^{-1}\hat{\mathbf{y}} - \mathbf{F}\mathbf{x}\|^2$ where γ is a scaling factor.

In the case where SIC is allowed, the goal is to minimize the difference between the estimated signal $\hat{y}_k$ and y_k after performing the SIC. Then, denoting u_k as a scalar value corresponding to the SIC at S_k, the MSE metric can be slightly modified as

$$\text{MSE}_\Sigma = E\left\|\frac{1}{\gamma}\hat{\mathbf{y}} - (\mathbf{U} + \mathbf{F})\mathbf{x}\right\|^2 \tag{7}$$

where $\mathbf{U}$ represents the matrix corresponding to the SIC operation as $\mathbf{U} = \text{diag}\{u_1, u_2, \ldots, u_K\}$. Note that, with the perfect SIC, u_k equals $\mathbf{g}_{Rk}^T\mathbf{Q}\mathbf{h}_{kR}x_k$.

Under relay power constraints in (4) and by employing the sum MSE expression in (7), the problem of minimizing the sum MSE can be constructed as

$$\begin{aligned} &\min_{\gamma, \mathbf{Q}, \mathbf{C}, \mathbf{U}} \text{MSE}_\Sigma \\ &\text{subject to} \quad \text{Tr}\left(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\right) \leq P_R \\ &\qquad\qquad\quad \text{Tr}\left(\mathbf{g}_{R0}^T\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{g}_{R0}^*\right) \leq \eta \\ &\qquad\qquad\quad \mathbf{C}, \mathbf{U} \in \mathcal{D}_K \end{aligned} \tag{8}$$

where $\mathcal{D}_K$ denotes the set of $K \times K$ diagonal matrices. Note that, to effectively support K users and satisfy the interference constraint at the primary network, we focus on the system with $N_R \geq K + 1$. Since (8) is not jointly convex with respect to γ, $\mathbf{Q}$, $\mathbf{C}$, and $\mathbf{U}$, it is hard to identify a solution analytically. To solve this issue, we first propose an iterative optimization method in the following.

B. Iterative Optimization Algorithm

Here, we propose an alternating optimization method that finds a solution iteratively. First, we compute the optimal γ and $\mathbf{Q}$ for given $\mathbf{C}$ and $\mathbf{U}$. Then, the problem (8) becomes

$$\begin{aligned} &\min_{\gamma, \mathbf{Q}} \text{MSE}_\Sigma \\ &\text{subject to} \quad \text{Tr}\left(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\right) \leq P_R \\ &\qquad\qquad\quad \text{Tr}\left(\mathbf{g}_{R0}^T\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{g}_{R0}^*\right) \leq \eta. \end{aligned} \tag{9}$$

To efficiently solve (9), we construct the Lagrangian function given as

$$\begin{aligned}\mathcal{L} = &\ \mathrm{Tr}\left(\frac{1}{\gamma^2}\mathbf{C}\mathbf{R}_{\mathbf{y}_S}\mathbf{C}^H + (\mathbf{U}+\mathbf{F})\mathbf{R}_{\mathbf{x}}(\mathbf{U}+\mathbf{F})^H\right)\\ &- \mathrm{Tr}\left(\frac{1}{\gamma}\mathbf{C}\boldsymbol{\Psi}(\mathbf{U}+\mathbf{F})^H - \frac{1}{\gamma}(\mathbf{U}+\mathbf{F})\boldsymbol{\Psi}^H\mathbf{C}^H\right)\\ &+ \lambda_1\left(\mathrm{Tr}\left(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\right) - P_R\right)\\ &+ \lambda_2\mathrm{Tr}\left(\left(\mathbf{g}_{R0}^T\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{g}_{R0}^*\right) - \eta\right) \qquad (10)\end{aligned}$$

where $\mathbf{R}_{\mathbf{y}_S} = \mathbf{G}\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{G}^H + P_0\mathbf{g}_0\mathbf{g}_0^H + \mathbf{I}_K$, $\boldsymbol{\Psi} = \mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{R}_{\mathbf{x}}$, and λ_i $(i = 1, 2)$ represents the Lagrangian multiplier. Note that the first term in (10) is obtained from the average operation of transmit signal and noise power. Then, the optimal γ and $\mathbf{Q}$ defined as $\hat{\gamma}$ and $\hat{\mathbf{Q}}$ are determined as follows.

Lemma 1: For given $\mathbf{C}$ and $\mathbf{U}$, the optimal relay filter $\hat{\mathbf{Q}}$ and the scaling factor $\hat{\gamma}$ are expressed, respectively, as

$$\hat{\mathbf{Q}} = \hat{\gamma}\bar{\mathbf{Q}} \quad \hat{\gamma} = \sqrt{\frac{P_R}{\mathrm{Tr}\left(\bar{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\bar{\mathbf{Q}}^H\right)}} \qquad (11)$$

where $\bar{\mathbf{Q}} \triangleq (\mathbf{G}^H\mathbf{C}^H\mathbf{C}\mathbf{G} + \mu\mathbf{g}_{R0}^*\mathbf{g}_{R0}^T + (\Omega - \mu\eta/P_R)\mathbf{I}_{N_R})^{-1} \times \mathbf{G}^H\mathbf{C}^H(\mathbf{U}+\mathbf{F})\mathbf{R}_{\mathbf{x}}\mathbf{H}^H\mathbf{R}_{\mathbf{y}_R}^{-1}$, $\Omega \triangleq P_0\mathrm{Tr}(\mathbf{C}\mathbf{g}_0\mathbf{g}_0^H\mathbf{C}^H) + \mathrm{Tr}(\mathbf{C}\mathbf{C}^H)$, and $\mu \triangleq \gamma^2\lambda_2$.

Proof: See Appendix. ■

Note that, by defining $\bar{\mathbf{Q}} \triangleq \mathbf{B}\mathbf{L}$, where $\mathbf{B} = (\mathbf{G}^H\mathbf{C}^H\mathbf{C}\mathbf{G} + \mu\mathbf{g}_{R0}^*\mathbf{g}_{R0}^T + (\Omega - \mu\eta/P_R)\mathbf{I}_{N_R})^{-1}\mathbf{G}^H\mathbf{C}^H$ and $\mathbf{L} = (\mathbf{U}+\mathbf{F})\mathbf{R}_{\mathbf{x}}\mathbf{H}^H\mathbf{R}_{\mathbf{y}_R}^{-1}$, $\hat{\gamma}\mathbf{B}$ and $\mathbf{L}$ follow the conventional transmit and receive Wiener filter structures, respectively [17]–[19]. Moreover, if the SIC is not allowed, we can obtain a solution for the sum MSE minimization problem by simply setting $\mathbf{U} = \mathbf{0}$.

Next, we derive the optimal $\mathbf{C}$ and $\mathbf{U}$ for given γ and $\mathbf{Q}$. With the given γ and $\mathbf{Q}$, the original problem (8) is transformed to an unconstrained problem as

$$\min_{\mathbf{C},\mathbf{U}} \mathrm{MSE}_\Sigma. \qquad (12)$$

Due to the diagonal structure of $\mathbf{C}$, $\mathbf{U}$, and $\mathbf{R}_{\mathbf{x}}$, MSE_Σ can be equivalently rewritten as

$$\begin{aligned}\mathrm{MSE}_\Sigma = &\ \mathrm{Tr}(\mathbf{R}_{\mathbf{x}}) + \mathbf{u}^H\mathbf{R}_{\mathbf{x}}\mathbf{u} - \mathbf{c}^H\mathbf{s} - \mathbf{s}^H\mathbf{c}\\ &- \mathbf{c}^H\tilde{\boldsymbol{\Psi}}\mathbf{u} - \mathbf{u}^H\tilde{\boldsymbol{\Psi}}^H\mathbf{c} + \mathbf{c}^H\tilde{\mathbf{R}}_{\mathbf{y}_S}\mathbf{c} \qquad (13)\end{aligned}$$

where $\mathbf{u} = [u_1, u_2, \ldots, u_K]^T$, $\mathbf{c} = [c_1, c_2, \ldots, c_K]^T$, $\mathbf{s}$ is a column vector whose elements consist of the diagonal elements of $\gamma^{-1}\mathbf{F}\boldsymbol{\Psi}$, and $\tilde{\boldsymbol{\Psi}}$ and $\tilde{\mathbf{R}}_{\mathbf{y}_S}$ are diagonal matrices with the diagonal elements of $\gamma^{-1}\boldsymbol{\Psi}^H$ and $\gamma^{-2}\mathbf{R}_{\mathbf{y}_S}$, respectively. When computing (13), we utilize the fact that $\mathbf{F}\mathbf{F}^H = \mathbf{I}_K$ and $\mathrm{Tr}(\mathbf{U}\mathbf{R}_x\mathbf{F}^H) = 0$. Then, employing two zero-gradient conditions $(\partial\mathrm{MSE}_\Sigma/\partial\mathbf{u}^*) = 0$ and $(\partial\mathrm{MSE}_\Sigma/\partial\mathbf{c}^*) = 0$, we obtain the solution as

$$\mathbf{u} = \mathbf{R}_{\mathbf{x}}^{-1}\tilde{\boldsymbol{\Psi}}^H\mathbf{c} \quad \text{and} \quad \mathbf{c} = \left(\tilde{\mathbf{R}}_{\mathbf{y}_S} - \tilde{\boldsymbol{\Psi}}\mathbf{R}_{\mathbf{x}}^{-1}\tilde{\boldsymbol{\Psi}}^H\right)^{-1}\mathbf{s}. \qquad (14)$$

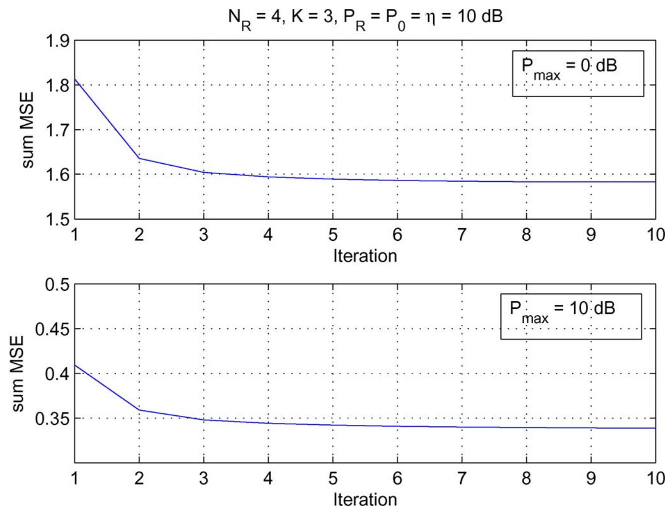

Fig. 2. Convergence trend for the cognitive MU-TRN with a sample channel realization, $N_R = 4$, $K = 3$, and $P_R = P_0 = \eta = 10$ dB.

The overall algorithm is summarized as follows.

1) Initialize $\mathbf{U}$ and $\mathbf{C}$ with arbitrary diagonal matrices.
Main Loop
2) Compute $\mathbf{Q}$ and γ using (11).
3) Compute $\mathbf{U}$ and $\mathbf{C}$ using (14).
4) Go back to step 2 until convergence.

It is worthwhile to note that our proposed algorithm guarantees convergence to a local minimum solution since the sum MSE is lower bounded by zero and decreases monotonically in each step of the algorithm as in [14] and [15]. Fig. 2 shows the convergence trend of the proposed iterative scheme with a sample channel realization.

IV. Closed-Form Relay Filter Design

Although the proposed iterative relay filter design given earlier provides good performance, as will be shown later, it requires high computational complexity. To alleviate this issue, here, we also propose a closed-form relay filter design. In the closed-form design, we assume the receive combining filter as an identity matrix, i.e., $\mathbf{C} = \mathbf{I}_K$, to avoid an iterative process. Then, for a given SIC matrix of a closed-form design $\mathbf{U}_c$, the closed-form relay filter $\mathbf{Q}_c$ can be similarly computed from Lemma 1 as

$$\begin{aligned}\mathbf{Q}_c &= \gamma_c\bar{\mathbf{Q}}_c\\ &= \gamma_c(\mathbf{G}^H\mathbf{G} + \mathbf{R}_g)^{-1}\mathbf{G}^H(\mathbf{U}_c + \mathbf{F})\mathbf{R}_{\mathbf{x}}\mathbf{H}^H\mathbf{R}_{\mathbf{y}_R}^{-1} \qquad (15)\end{aligned}$$

where $\gamma_c = \sqrt{P_R/\mathrm{Tr}(\bar{\mathbf{Q}}_c\mathbf{R}_{\mathbf{y}_R}\bar{\mathbf{Q}}_c^H)}$, $\mathbf{R}_g \triangleq \mu\mathbf{g}_{R0}^*\mathbf{g}_{R0}^T + (\Omega_c - \mu\eta/P_R)\mathbf{I}_{N_R}$, and $\Omega_c \triangleq P_0\mathrm{Tr}(\mathbf{g}_0\mathbf{g}_0^H) + K$.

To efficiently derive the SIC matrix $\mathbf{U}_c$, we modify the sum MSE by plugging $\mathbf{Q}_c$ and $\mathbf{C} = \mathbf{I}_K$ as (16) in the following:

$$\begin{aligned}\mathrm{MSE}_\Sigma = &\ \mathrm{Tr}\Big((\mathbf{U}_c + \mathbf{F})\mathbf{R}_{\mathbf{x}}(\mathbf{U}_c + \mathbf{F})^H\\ &\quad - (\mathbf{U}_c + \mathbf{F})\tilde{\mathbf{R}}_{\mathbf{y}_R}(\mathbf{U}+\mathbf{F})^H\mathbf{G}\left(\mathbf{G}^H\mathbf{G} + \mathbf{R}_g\right)^{-1}\mathbf{G}^H\Big)\\ &+ \mathrm{Tr}\Big((\mathbf{U}_c + \mathbf{F})\tilde{\mathbf{R}}_{\mathbf{y}_R}(\mathbf{U}_c + \mathbf{F})^H\\ &\quad - (\mathbf{U}_c + \mathbf{F})\tilde{\mathbf{R}}_{\mathbf{y}_R}(\mathbf{U}_c + \mathbf{F})^H\Big) \qquad (16)\\ = &\ \mathrm{Tr}\Big((\mathbf{U}_c + \mathbf{F})\left(\mathbf{R}_{\mathbf{x}}^{-1} + \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\right)^{-1}(\mathbf{U}_c + \mathbf{F})^H\\ &\quad + (\mathbf{U}_c + \mathbf{F})\tilde{\mathbf{R}}_{\mathbf{y}_R}(\mathbf{U}_c + \mathbf{F})^H\left(\mathbf{I}_K + \mathbf{G}\mathbf{R}_g^{-1}\mathbf{G}^H\right)^{-1}\Big) \qquad (17)\end{aligned}$$

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON BETWEEN THE PROPOSED ALGORITHMS

Algorithm	$\mathbf{Q}$	$\mathbf{C}/\mathbf{U}$
Iterative	$J \cdot \mathcal{O}\left(N_R^3 + K^3 + \delta\right)$	$J \cdot \mathcal{O}\left(2K\right)$
Closed-form	$\mathcal{O}\left(N_R^3 + K^3 + \delta_c\right)$	$\mathcal{O}\left(N_R^3 + 2K^3 + 3K\right)$

where $\tilde{\mathbf{R}}_{\mathbf{y}_R}$ is defined by $\tilde{\mathbf{R}}_{\mathbf{y}_R} \triangleq \mathbf{R}_{\mathbf{x}}\mathbf{H}^H\mathbf{R}_{\mathbf{y}_R}^{-1}\mathbf{H}\mathbf{R}_{\mathbf{x}}$, (16) follows from (26), and (17) is computed by invoking the matrix inversion lemma [20].

Using matrix inversion lemma again and applying a high SNR approximation, $\tilde{\mathbf{R}}_{\mathbf{y}_R}$ can be approximated as $\mathbf{R}_{\mathbf{x}}$ since $\tilde{\mathbf{R}}_{\mathbf{y}_R} = (\mathbf{R}_{\mathbf{x}}^{-1} + \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H})^{-1}\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\mathbf{R}_{\mathbf{x}} \approx \mathbf{R}_{\mathbf{x}}$. Then, denoting $\boldsymbol{\Upsilon}$ and $\boldsymbol{\Phi}$ as $\boldsymbol{\Upsilon} \triangleq \mathbf{R}_{\mathbf{x}}^{-1/2}(\mathbf{R}_x^{-1} + \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H})^{-1}\mathbf{R}_{\mathbf{x}}^{-1/2}$ and $\boldsymbol{\Phi} \triangleq (\mathbf{I}_K + \mathbf{G}\mathbf{R}_g^{-1}\mathbf{G}^H)^{-1}$, respectively, the approximated sum MSE can be expressed as

$$\overline{\mathrm{MSE}}_{\Sigma} = \mathrm{Tr}\left((\mathbf{U}_c + \mathbf{F})\mathbf{R}_{\mathbf{x}}^{1/2}\boldsymbol{\Upsilon}\mathbf{R}_{\mathbf{x}}^{1/2}(\mathbf{U}_c + \mathbf{F})^H + (\mathbf{U}_c + \mathbf{F})\mathbf{R}_{\mathbf{x}}(\mathbf{U}_c + \mathbf{F})^H\boldsymbol{\Phi}\right). \quad (18)$$

Employing the fact that $\mathbf{U}_c$ and $\mathbf{R}_{\mathbf{x}}$ are diagonal matrices, (18) is equivalently expressed as

$$\overline{\mathrm{MSE}}_{\Sigma} = \tilde{\mathbf{u}}^H(\tilde{\boldsymbol{\Upsilon}} + \tilde{\boldsymbol{\Phi}})\tilde{\mathbf{u}} + \tilde{\mathbf{u}}^H\tilde{\mathbf{s}} + \tilde{\mathbf{s}}^H\tilde{\mathbf{u}} + \mathrm{Tr}(\mathbf{R}_x\mathbf{F}^H\boldsymbol{\Phi}\mathbf{F} + \mathbf{R}_x\boldsymbol{\Upsilon}) \quad (19)$$

where $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{s}}$ are column vectors that consist of diagonal elements of $\mathbf{U}_c\mathbf{R}_{\mathbf{x}}^{1/2}$ and $\boldsymbol{\Upsilon}\mathbf{F}\mathbf{R}_{\mathbf{x}}^{1/2} + \mathbf{F}\mathbf{R}_{\mathbf{x}}^{1/2}\boldsymbol{\Phi}$, respectively, and $\tilde{\boldsymbol{\Upsilon}}$ and $\tilde{\boldsymbol{\Phi}}$ indicate diagonal matrices with the diagonal elements of $\boldsymbol{\Upsilon}$ and $\boldsymbol{\Phi}$, respectively.

Then, by taking a derivative with respect to $\tilde{\mathbf{u}}$ and setting it to zero, we simply obtain the optimal $\tilde{\mathbf{u}}$, which minimizes $\overline{\mathrm{MSE}}_{\Sigma}$ as

$$\tilde{\mathbf{u}} = (\tilde{\boldsymbol{\Upsilon}} + \tilde{\boldsymbol{\Phi}})^{-1}\tilde{\mathbf{s}}. \quad (20)$$

Finally, we have the SIC matrix $\mathbf{U}_c$ for a closed-form design as

$$\mathbf{U}_c = \mathrm{diag}\left\{\mathbf{R}_{\mathbf{x}}^{-1/2}\tilde{\mathbf{u}}\right\}. \quad (21)$$

By plugging the obtained SIC matrix in (21) back to (15), the relay filter $\mathbf{Q}_c$ can be computed in closed form.

Table I lists the computational complexity between the proposed algorithms where J represents the number of iterations for reaching the stationary point of the iterative algorithm. The major complexity of the proposed schemes results from the calculation of the inverse operation[1] and the bisection method, which is denoted by δ and δ_c for iterative and closed-form designs, respectively. From the intensive Monte Carlo simulations, we have checked that the iterative scheme requires at least ten iterations ($J = 10$), as shown in Fig. 2. As a result, the computational complexity of the closed-form design is significantly reduced compared with the iterative algorithm.

V. SIMULATION RESULTS

Here, we present numerical results to confirm the effectiveness of the proposed algorithms by running 10 000 Monte Carlo simulations using the MATLAB software. In our simulation, all channel elements are set to be independent complex Gaussian random variables with zero mean and unit variance, i.e., Rayleigh fading is assumed. It is assumed that perfect SIC is applied at each S_k. We compare our proposed scheme with the following schemes.

- *Naive AF*: Only the power normalization operation is performed with $\mathbf{Q} = \gamma_{\mathrm{naive}}\mathbf{I}_{N_R}$ where $\gamma_{\mathrm{naive}} = \min g\{\sqrt{P_R/\mathrm{Tr}(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H)}, \sqrt{\eta/\mathrm{Tr}(\mathbf{g}_{R0}^T\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{g}_{R0}^*)}\}$.
- *Maximum ratio transmission (MRT)* [11]: Relay filters are obtained to conduct orthogonal projection to the primary network and MRT for the SUs.
- *Zero Forcing (ZF)* [11]: Relay filters are computed to apply orthogonal projection to the primary network and ZF beamforming for the SUs.

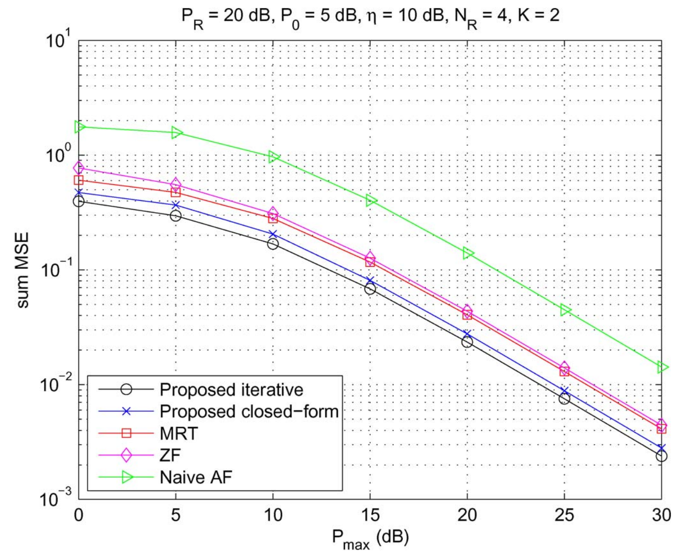

Fig. 3. Average sum MSE performance of SUs with respect to $P_{\max}$.

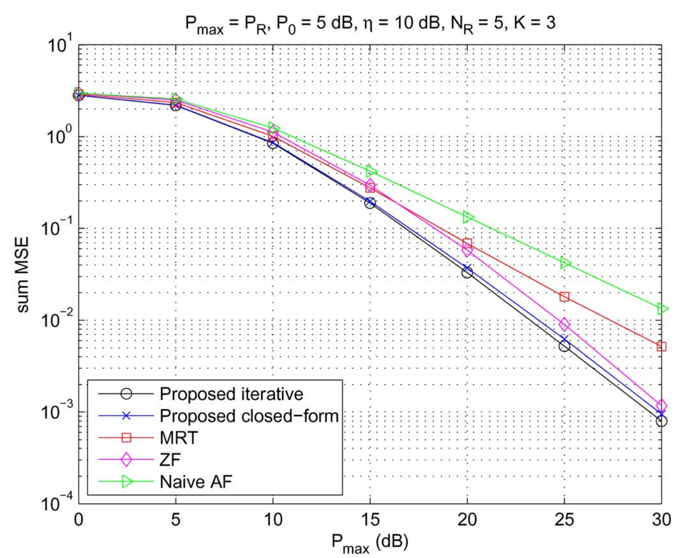

Fig. 4. Average sum MSE performance of SUs with respect to $P_{\max}$.

Figs. 3 and 4 compare the sum MSE performance of our schemes with various relay filter designs with $K = 2$ and $K = 3$, respectively, as a function of $P_{\max}$. Note that since we assume the noise variance is one, $P_{\max}$ can be considered the transmit SNR at SUs. In these simulation, the permutation matrices $\mathbf{F}$ for $K = 2, 3$ are set to $\mathbf{F}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{F}_3 = g\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, respectively. We assume that the Wiener filter [17] is employed at each SU. From these plot, we first see that

[1]For a complex matrix $\mathbf{A} \in \mathbb{C}^{N\times N}$, the number of multiplications for computing $\mathbf{A}^{-1}$ is $\mathcal{O}(N^3)$ [20]. In addition, $\mathcal{O}(N)$ is required for the case of the diagonal matrix.

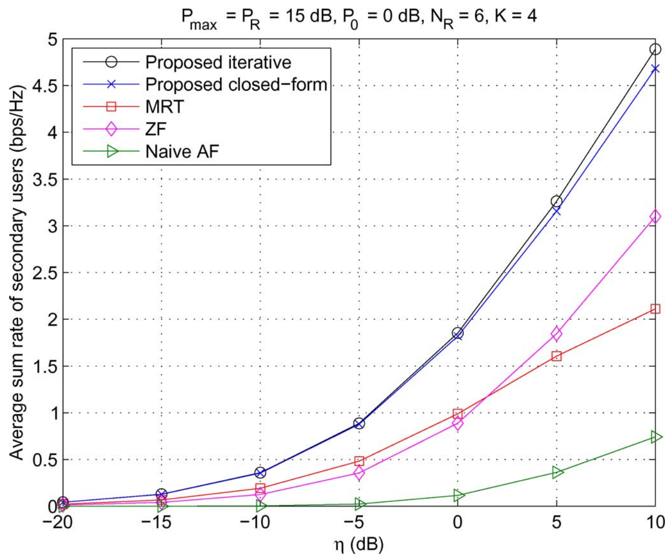

Fig. 5. Average sum rate performance of SUs with respect to η.

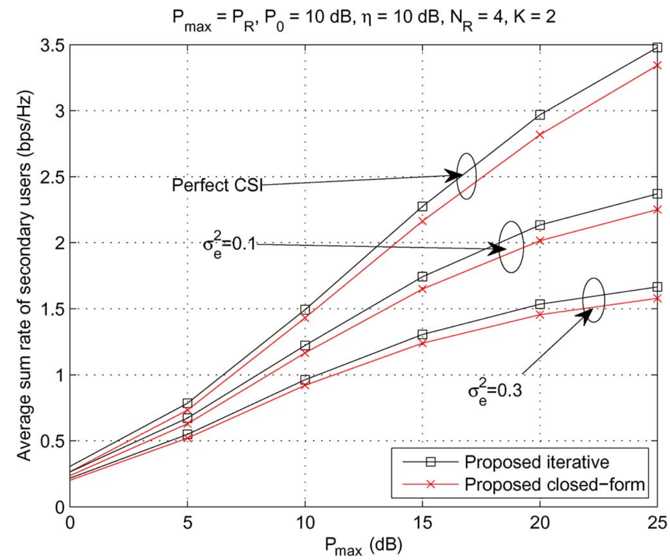

Fig. 6. Average sum rate performance of SUs with respect to SNR for the MU-TRN with imperfect CSI.

the proposed iterative filter design outperforms other schemes, and the closed-form design provides almost identical sum MSE performance to the iterative filter design with much reduced complexity. Although the perfect SIC is applied, there still exists interuser interference at each S_k when $K \geq 3$. Therefore, the ZF-based scheme yields better performance than the MRT-based scheme as SNR increases.

In Fig. 5, we present the sum-rate performance with $K = 4$ as a function of η. In this case, we set the permutation matrix as $\mathbf{F} = \begin{bmatrix} \mathbf{F}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{F}_2 \end{bmatrix}$. First, the sum rate R_Σ is defined as $R_\Sigma = (1/2)$ $\sum_{k=1}^{K} \log_2(1+(|\mathbf{g}_{Rk}^T \mathbf{Q}\mathbf{h}_{\bar{k}R}|^2 P_{\bar{k}}/\Gamma_k))$ where $\Gamma_k \triangleq \|\mathbf{g}_{Rk}^T \mathbf{Q}\mathbf{H}\mathbf{R}^{1/2}\|^2 - |\mathbf{g}_{Rk}^T \mathbf{Q}\mathbf{h}_{\bar{k}R}|^2 P_{\bar{k}} + \|\mathbf{g}_{Rk}^T \mathbf{Q}\|^2 + P_0(|\mathbf{g}_{Rk}^T \mathbf{Q}\mathbf{h}_{0R}|^2 + |g_{0k}|^2) + 1$, and the pre-log factor $1/2$ is introduced due to the half-duplex protocol. With the SIC operation, $|\mathbf{g}_{Rk}^T \mathbf{Q}\mathbf{h}_{kR}|^2 P_k$ is additionally subtracted from Γ_k. In the plot, a similar trend is observed that the closed-form design provides the sum rate performance close to the iterative scheme and outperforms conventional schemes. Moreover, we can see that the sum rate increases as η grows due to the increased allowable power at each SU in (2). Comparing ZF and the proposed MMSE schemes, we confirm that managing both interuser and primary network interference is an important issue to improve the performance for the cognitive MU-TRN.

Fig. 6 compares the sum rate performance with our proposed scheme as a function of $P_{\max}$ for the MU-TRN with imperfect CSI. For the simulation, we assume that the estimated channel $\hat{\mathbf{h}}$ is related to the true channel $\mathbf{h}$ as $\hat{\mathbf{h}} = \mathbf{h} + \mathbf{e}$ where the elements of $\mathbf{e}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and variance σ_e^2 as in [15]. It is confirmed from this plot that the performance drastically deteriorates as σ_e^2 grows. It would be an interesting future work to analyze the performance and its robust design under imperfect CSI scenario.

VI. Conclusion

In this paper, we have proposed linear relay filter designs that minimize the MSE of SUs for the cognitive MU-TRN. Through the simulation results, we have confirmed that the proposed closed-form relay filter design outperforms conventional approaches and provides the performance close to the proposed iterative algorithm with much reduced complexity.

Appendix

Proof of Lemma 1: Using (10), the KKT conditions [21] for problem (9) are written as

$$\frac{\partial \mathcal{L}}{\partial \mathbf{Q}^*} = 0 \quad \frac{\partial \mathcal{L}}{\partial \gamma} = 0 \tag{22}$$

$$\lambda_1 \geq 0, \quad \mathrm{Tr}\left(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\right) - P_R \leq 0$$
$$\lambda_1\left(\mathrm{Tr}\left(\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\right) - P_R\right) = 0 \tag{23}$$

$$\lambda_2 \geq 0, \quad \mathrm{Tr}\left(\mathbf{g}_{R0}^T \mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H \mathbf{g}_{R0}^*\right) - \eta \leq 0$$
$$\lambda_2 \mathrm{Tr}\left(\left(\mathbf{g}_{R0}^T \mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H \mathbf{g}_{R0}^*\right) - \eta\right) = 0. \tag{24}$$

From the two conditions in (22), we have

$$\hat{\mathbf{Q}} = \gamma\left(\mathbf{G}^H\mathbf{C}^H\mathbf{C}\mathbf{G} + \mu_1 \mathbf{I}_{N_R} + \mu_2 \mathbf{g}_{R0}^* \mathbf{g}_{R0}^T\right)^{-1} \times \mathbf{G}^H\mathbf{C}^H(\mathbf{U}+\mathbf{F})\mathbf{R}_\mathbf{x}\mathbf{H}^H\mathbf{R}_{\mathbf{y}_R}^{-1}, \tag{25}$$

$$\mathrm{Tr}\left(\mathbf{C}\left(\mathbf{G}\mathbf{Q}\mathbf{R}_{\mathbf{y}_R}\mathbf{Q}^H\mathbf{G}^H + P_0 \mathbf{g}_0\mathbf{g}_0^H + \mathbf{I}_K\right)\mathbf{C}^H\right) = \hat{\gamma}\mathrm{Tr}\left((\mathbf{U}+\mathbf{F})\mathbf{R}_\mathbf{x}\mathbf{H}^H\mathbf{Q}^H\mathbf{G}^H\mathbf{C}^H\right) \tag{26}$$

where $\mathrm{Tr}((\mathbf{U}+\mathbf{F})\mathbf{R}_\mathbf{x}\mathbf{H}^H\mathbf{Q}^H\mathbf{G}^H\mathbf{C}^H) = \mathrm{Tr}(\mathbf{C}\mathbf{G}\mathbf{Q}\mathbf{H}\mathbf{R}_\mathbf{x}(\mathbf{U}+\mathbf{F})^H)$, and the constant values μ_1 and μ_2 are defined as $\mu_1 \triangleq \gamma^2\lambda_1$ and $\mu_2 \triangleq \gamma^2\lambda_2$, respectively, to avoid interconnected variables as in [2] and [17].

Then, by plugging (25) to (26), the right-hand side of (26) can be equivalently given as

$$\begin{aligned} &\hat{\gamma}\mathrm{Tr}\left((\mathbf{U}+\mathbf{F})\mathbf{R}_\mathbf{x}\mathbf{H}^H\hat{\mathbf{Q}}^H\mathbf{G}^H\mathbf{C}^H\right) \\ &= \mathrm{Tr}\left(\hat{\gamma}\mathbf{G}^H\mathbf{C}^H(\mathbf{U}+\mathbf{F})\mathbf{R}_\mathbf{x}\mathbf{H}^H\hat{\mathbf{Q}}^H\right) \\ &= \mathrm{Tr}\left(\left(\mathbf{G}^H\mathbf{C}^H\mathbf{C}\mathbf{G} + \mu_1\mathbf{I}_{N_R} + \mu_2\mathbf{g}_{R0}^*\mathbf{g}_{R0}^T\right)\hat{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}^H\right). \end{aligned} \tag{27}$$

By combining (26) and (27), and by exploiting KKT conditions in (23) and (24), it follows that

$$\begin{aligned} \Omega &= P_0 \mathrm{Tr}\left(\mathbf{C}\mathbf{g}_0\mathbf{g}_0^H\mathbf{C}^H\right) + \mathrm{Tr}(\mathbf{C}\mathbf{C}^H) \\ &= \mu_1 \mathrm{Tr}\left(\hat{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}^H\right) + \mu_2 \mathrm{Tr}\left(\mathbf{g}_{R0}^T\hat{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}^H\mathbf{g}_{R0}^*\right) \\ &= \mu_1 P_R + \mu_2 \eta. \end{aligned} \tag{28}$$

Rearranging (28) with respect to μ_1, i.e., $\mu_1 = (\Omega - \mu_2\eta/P_R)$, the optimal relay filter can be calculated as in (11) by defining μ as the optimal μ_2. Since $\mu_1, \mu_2 \geq 0$, μ should be a positive value $0 \leq \mu \leq (\Omega/\eta)$.

Now, the remaining work is to find $\hat{\gamma}$. By substituting $\hat{\mathbf{Q}}$ into two power constraints in (9), we obtain $\hat{\gamma}^2 \geq (P_R/\mathrm{Tr}(\hat{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}^H))$ and $\hat{\gamma}^2 \geq (\eta/\mathrm{Tr}(\mathbf{g}_{R0}^T\hat{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}^H\mathbf{g}_{R0}^*))$. Let us define $f(\mu) \triangleq \mathrm{Tr}((P_R\mathbf{g}_{R0}^*\mathbf{g}_{R0}^T - \eta\mathbf{I}_{N_R})\hat{\mathbf{Q}}(\mu)\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}(\mu)^H)$. Then, if $f(\mu) \leq 0$, $\hat{\gamma}$ becomes $\hat{\gamma}^2 = (P_R/\mathrm{Tr}(\hat{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\hat{\mathbf{Q}}^H))$ to satisfy both power constraints. In this case, according to (24), μ should be set to zero when $f(0) < 0$ or a nonzero value computed from the bisection method over $0 < \mu \leq (\Omega/\eta)$ if $f(\mu) = 0$. For $f(\mu) > 0$, μ_1 equals 0 due to the KKT conditions in (23), and then it follows $\mu = (\Omega/\eta)$. However, for $N_R \geq K + 1$, $\mathrm{Tr}(\mathbf{g}_{R0}^T\hat{\mathbf{Q}}(\Omega/\eta))$ is equal to 0 since $\mathbf{B}$ in $\hat{\mathbf{Q}}$ acts like a ZF transmit filter. Then, we have $f(\Omega/\eta) < 0$, which contradicts the assumption of $f(\mu) > 0$. For this reason, we only consider the case $f(\mu) \leq 0$ and thereby $\hat{\gamma} = \sqrt{P_R/\mathrm{Tr}(\bar{\mathbf{Q}}\mathbf{R}_{\mathbf{y}_R}\bar{\mathbf{Q}}^H)}$. This completes the proof. ■

References

[1] A. J. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[2] K.-J. Lee and I. Lee, "MMSE based block diagonalization for cognitive radio MIMO broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3139–3144, Oct. 2011.

[3] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[4] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 38–51, Jan. 2008.

[5] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, Jun. 2009.

[6] K.-J. Lee, H. Sung, E. Park, and I. Lee, "Joint optimization for one and two-way MIMO AF multiple-relay systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3671–3681, Dec. 2010.

[7] R. Vaze and R. W. Heath, Jr., "On the capacity and diversity-multiplexing tradeoff of the two-way relay channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4219–4234, Jul. 2011.

[8] C. W. R. Chiong, Y. Rong, and Y. Xiang, "Channel training algorithms for two-way MIMO relay systems," *IEEE Trans. Signal Process.*, vol. 61, no. 16, pp. 3988–3998, Aug. 2013.

[9] C. Wang, X. Dong, and Y. Shi, "Robust relay design for two-way multi-antenna relay systems with imperfect CSI," *IEEE J. Commun. Netw.*, vol. 16, no. 1, pp. 45–55, Feb. 2014.

[10] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[11] K. Jitvanichphaibool, Y.-C. Liang, and R. Zhang, "Beamforming and power control for multi-antenna cognitive two-way relaying," in *Proc. IEEE WCNC*, Apr. 2009, pp. 1–6.

[12] P. Ubaidulla and S. Aïssa, "Optimal relay selection and power allocation for cognitive two-way relaying networks," *IEEE Wireless Commun. Lett.*, vol. 1, no. 3, pp. 225–228, Jun. 2012.

[13] H. Mu and J. K. Tugnait, "MSE-based source and relay precoder design for cognitive multiuser multi-way relay systems," *IEEE Trans. Signal Process.*, vol. 61, no. 7, pp. 1770–1785, Apr. 2013.

[14] F. Wang, X. Yuan, S. C. Liew, and D. Guo, "Wireless MIMO switching: Weighted sum mean square error and sum rate optimization," *IEEE Trans. Inf. Theory*, vol. 59, no. 9, pp. 5297–5312, Sep. 2013.

[15] H. Park, S.-H. Park, H.-B. Kong, and I. Lee, "Weighted sum MSE minimization under per-BS power constraint for network MIMO systmes," *IEEE Commun. Lett.*, vol. 16, no. 3, pp. 360–363, Mar. 2012.

[16] J. Zhang, R. Chen, J. G. Andrews, A. Ghosh, and R. W. Heath, "Networked MIMO with clusterd linear precoding," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1910–1921, Apr. 2009.

[17] M. Joham, W. Utschick, and J. A. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.

[18] C. Song, K.-J. Lee, and I. Lee, "MMSE based transceiver designs in closed-loop non-regenerative MIMO relaying systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 7, pp. 2310–2319, Jul. 2010.

[19] C. Song, K.-J. Lee, and I. Lee, "MMSE-based MIMO cooperative relaying systems: Closed-form designs and outage behavior," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 8, pp. 1390–1401, Sep. 2012.

[20] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd ed. Baltimore, MD, USA: The Johns Hopkins Univ. Press, 1996.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004, The Edinburgh Building.

Efficient Solutions to Distributed Beamforming for Two-Way Relay Networks Under Individual Relay Power Constraints

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***Abstract*—This paper investigates distributed beamforming (DBF) designs for two-way relay networks (TWRNs), where two terminal nodes exchange information through a set of amplify-and-forward (AF) relays. We assume individual relay power constraints and study two important design problems, namely, the max–min fairness (MMF) problem and the weighted sum-rate maximization (WSRM) problem. For the MMF problem, unlike the existing works that usually solve the problem by a bisection method, we propose an efficient optimal solution by solving one convex second-order cone program (SOCP) only. For the WSRM problem, we develop an efficient iterative algorithm based on SOCP reformulation and the successive convex approximation (SCA) technique. For both the MMF and WSRM designs, we further propose a distributed implementation framework where each of the relays can independently compute its beamforming weight using its local channel state information (CSI) and some common parameters broadcasted by a control center. Simulation results are presented to demonstrate the performance advantages of the proposed solutions.**

***Index Terms*—Distributed beamforming (DBF), max–min fairness (MMF), two-way relaying, weighted sum-rate maximization (WSRM).**

I. Introduction

Two-way relay networks (TWRNs) have received much attention in recent years owing to their capability of supporting two-way communication with improved spectral efficiency [1]. In particular, the

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