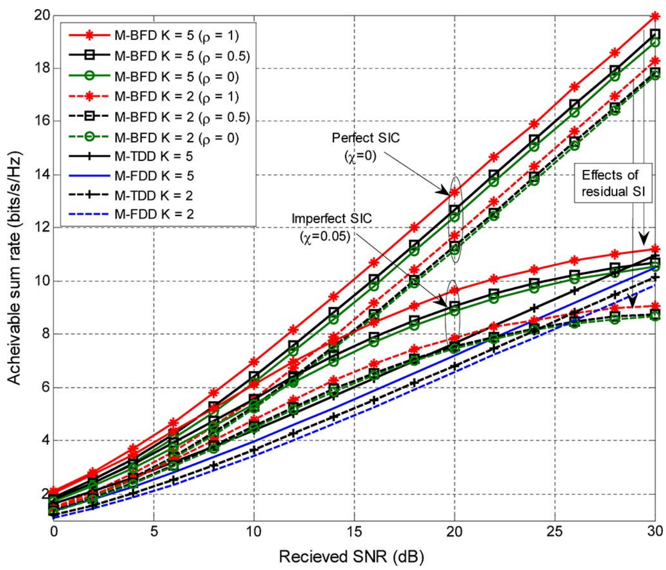

Fig. 5. ASRs for M-BFD and M-BHD (M-TDD and M-FDD) systems with various channel correlation and SIC coefficients when $K = 2$ and $K = 5$.

remaining power of the residual SI, whereas the ASR for the M-BHD system increases according to the SNR. The analysis shows that the M-BFD system is still advantageous for achieving higher ASRs, even when SI is present.

VI. Conclusion

This paper has investigated a BFD system in a multispectrum environment and proposed a spectrum selection strategy for use with it. Our spectrum selection strategy selects the best spectrum considering the CSI of the M-BFD system. This strategy is an efficient way for an M-BFD system to maximize the ASR. After applying the selection strategy, we compared M-BFD with M-BHD systems. Analytical and numerical results show that the M-BFD system can achieve the highest ASR although the M-TDD system has the highest spectrum selection gain. We also compared the ASRs for the M-BFD and M-BHD systems in practical environments such as imperfect SIC and channel correlation. Our analysis indicates that the M-BFD system is still an advantageous approach for achieving higher ASRs in multispectrum environments. Possible topics for the future include extending our work to cover multiple antennas with antenna selections, advanced spectrum selection scheme considering power of residual SI, and ad hoc networks with multiple nodes.

References

[1] H. Ju, X. Shang, H. V. Poor, and D. Hong, "Bi-directional use of spatial resources and effects of spatial correlation," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3368–3379, Oct. 2011.
[2] H. Ju, D. Kim, H. V. Poor, and D. Hong, "Bi-directional beamforming and its capacity scaling in pairwise two-way communications," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 346–357, Jan. 2012.
[3] D. Kim, S. Park, H. Ju, and D. Hong, "Transmission capacity of full-duplex based two-way ad-hoc networks with ARQ protocol," *IEEE Trans. Veh. Technol.*, vol. 63, no. 7, pp. 3167–3183, Sep. 2014.
[4] D. Kim, H. Ju, S. Park, and D. Hong, "Effects of channel estimation error on full-duplex two-way networks," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4666–4672, Nov. 2013.
[5] M. Duarte *et al.*, "Design and characterization of a full-duplex multi-antenna system for WiFi networks," *IEEE Trans. Veh. Technol.*, vol. 63, no. 3, pp. 1160–1177, Mar. 2014.
[6] D. Bharadia, E. McMilin, and S. Katti, "Full duplex radios," in *Proc. SIGCOMM*, 2013, pp. 375–386.
[7] H. Wang, J. Lee, S. Kim, and D. Hong, "Capacity of secondary users exploiting multispectrum and multiuser diversity in spectrum-sharing environments," *IEEE Trans. Veh. Technol.*, vol. 59, no. 2, pp. 1030–1036, Feb. 2010.
[8] G. Ozcan and M. Gursoy, "Cognitive radio transmissions exploiting multiuser diversity under channel and sensing uncertainty," *IEEE Commun. Lett.*, vol. 17, no. 9, pp. 1714–1717, Sep. 2013.
[9] D. Kim, H. Ju, S. Kim, H. Lee, and D. Hong, "Transmit antenna-switched receive diversity for bi-directional beamforming in two-way communications," in *Proc. 47th Asilomar Conf. Signals, Syst. Comput.*, 2013, pp. 19–23.
[10] *Reciprocity modeling for TDD CoMP evaluation*, 3GPP R1-111983, May 9–13, 2011, Alcatel-Lucent Shanghai Bell, Shanghai, China.
[11] A. Sabharwal *et al.*, "In-band Full-duplex Wireless: Challenges and Opportunities," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1637–1652, Sep. 2014.
[12] D. Chizhik, G. Foschini, M. Gans, and R. Valenzuela, "Keyholes, correlations, capacities of multielement transmit and receive antennas," *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 361–368, Apr. 2002.
[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, Products*, 7th ed. San Diego, CA, USA: Academic, 2007.
[14] A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*, 4th ed. New York, NY, USA: McGraw-Hill, 2002.

Diversity of Coded Beamforming in MIMO-OFDM AF Relaying Systems With Direct Link

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***Abstract*—This paper investigates the diversity order of beamforming schemes in frequency-selective multiple-input multiple-output (MIMO) amplify-and-forward (AF) relaying channels with a nonnegligible direct link between the source and the destination. In this system, orthogonal frequency-division multiplexing and bit-interleaved coded modulation schemes are adopted to exploit multipath diversity. Due to a nonconvex property of the problem, the optimal performance of flat-fading MIMO relaying channels, which is imperative to derive the diversity order, is still unknown. To this end, we first identify upper and lower bounds of the signal-to-noise ratio (SNR) of the flat-fading channels, which lead to upper and lower bounds of diversity order of coded beamforming in frequency-selective channels. Then, we establish the diversity order as a closed form by showing that these bounds are tight. In addition, our analysis provides an insightful guideline for code constructions to achieve a proper diversity gain of the system. The accuracy of our analysis is validated through simulation results.**

***Index Terms*—Channel coded systems, direct link, diversity, multiple-input multiple-output (MIMO) systems, relay.**

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I. Introduction

Over the past decade, multiple-input multiple-output (MIMO) relaying systems have been intensively investigated to extend transmission coverage and improve the throughput of wireless communication systems [1], [2]. Among various relaying protocols, the amplify-and-forward (AF) protocol, which forwards the amplified version of the received signal at the relay to the destination, has drawn considerable attention due to its simple implementation and low processing delay. Assuming flat-fading channels, a large amount of research efforts have been devoted to designing linear precoders and analyzing the performance of MIMO AF relaying systems in [3]–[6] and references therein. In addition, for wideband frequency-selective channels, the incorporation of orthogonal frequency-division multiplexing (OFDM) for MIMO AF relaying systems has been studied in [7]–[9].

In addition, most communication systems adopt channel coding schemes to enhance the system performance [10]. Nevertheless, the previous works in [4]–[9] are limited to uncoded systems. To address coding systems, we examined the diversity order of coded beamforming in MIMO-OFDM relaying systems without a direct link [11]. Lately, it has been shown that exploiting the direct link can provide considerable performance improvements in terms of the diversity order and the multiplexing gain [12]. Therefore, if the direct link is strong enough, we need to optimize the system by taking the direct link into account.

In this paper, we derive the diversity order of coded beamforming in MIMO-OFDM AF relaying systems with a nonnegligible direct link. Unfortunately, an extension of the work in [11] to the case of systems with a direct link is nontrivial. In particular, with the direct link, the beamforming strategy at the source, which maximizes the end-to-end signal-to-noise ratio (SNR), is still unknown, and thus, the exact derivation of the maximum achievable SNR, which is necessary to derive the diversity order, remains open. To overcome this difficulty, we first obtain an upper bound of the achievable SNR, which leads to an upper bound of diversity order, and then show that this diversity order is actually achievable.

Thereby, we characterize the diversity order of coded beamforming schemes as a closed form, which is given by a function of the free distance of a code, the channel profile of each link, and the number of antennas at each node. It is expected that the derived simple expression of the diversity order will be helpful in finding operating points of the system and predicting its performance. In addition, the analysis provides useful code construction criteria by identifying the minimum requirement for the free distance of the code to achieve full diversity. Finally, numerical results are provided to verify the analysis.

The notations adopted in this paper are as follows: The uppercase boldface, lowercase boldface, and normal letters stand for matrices, vectors, and scalars, respectively. The operators $(\cdot)^T$, $(\cdot)^H$, $\lceil x \rceil$, $\|\mathbf{x}\|$, and $E[\cdot]$ indicate the transpose, the conjugate transpose, the smallest integer not less than x, the Euclidian 2-norm of a vector $\mathbf{x}$, and expectation, respectively. $\mathrm{Tr}(\mathbf{A})$ and $\|\mathbf{A}\|_F$ represent the trace and Frobenius norm of a matrix $\mathbf{A}$, respectively. We denote $f(\rho) \doteq g(\rho)$, when two functions $f(\rho)$ and $g(\rho)$ are exponentially equal as $\lim_{\rho\to\infty}(\log f(\rho)/\log\rho) = \lim_{\rho\to\infty}(\log g(\rho)/\log\rho)$. Inequalities $\dot{\leq}$ and $\dot{\geq}$ are similarly defined.

II. System Model

We consider frequency-selective MIMO AF relaying systems with a nonnegligible direct link. We assume that OFDM modulation is employed at each node so that the wideband frequency-selective channel is converted into a set of K narrowband flat-fading channels. The source, relay, and destination nodes are equipped with N_S, N_R, and N_D antennas, respectively, which is denoted by $(N_S \times N_R \times N_D)$ systems. Moreover, $\mathbf{H}_{0,k} \in \mathbb{C}^{N_D\times N_S}$, $\mathbf{H}_{1,k} \in \mathbb{C}^{N_R\times N_S}$, and $\mathbf{H}_{2,k} \in \mathbb{C}^{N_D\times N_R}$ represent the channel matrices for the source-to-destination, source-to-relay, and relay-to-destination links for the kth subchannel, respectively. We define L_l and τ_n as the number of channel taps and the nth tap delay for $\mathbf{H}_{l,k}$ $(l = 0, 1, 2)$, respectively.

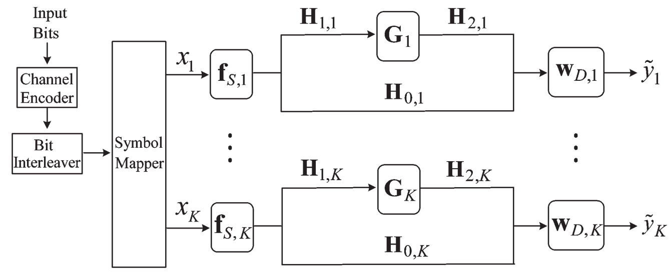

Fig. 1. System description for K parallel OFDM subchannels in coded MIMO AF relaying systems.

Then, by denoting the sampling period and the number of subcarriers as T and K, respectively, the (i,j)th element of $\mathbf{H}_{l,k}$ can be expressed as

$$H_{l,k}^{(i,j)} = \sum_{n=1}^{L_l} h_{l,n}^{(i,j)} e^{-j\frac{2\pi k\tau_n}{KT}} = \mathbf{h}_{l,i,j}^H \mathbf{w}_{l,k}$$

where $h_{l,n}^{(i,j)}$ is a complex Gaussian random variable with zero mean and unit variance, and we have $\mathbf{h}_{l,i,j} \triangleq [h_{l,1}^{(i,j)}, \ldots, h_{l,L_l}^{(i,j)}]^H$ and $\mathbf{w}_{l,k} \triangleq [e^{-j(2\pi k\tau_1/KT)}, \ldots, e^{-j(2\pi k\tau_{L_l}/KT)}]^T$.

As shown in Fig. 1, the information bits are encoded with a single channel encoder and then interleaved by a bit interleaver and mapped to K data symbols $x_1, \ldots, x_K$ that will be assigned to K different subcarriers. We assume that the relay operates in the half-duplex mode. In the first time slot, the source broadcasts x_k to the relay and the destination. Then, the received signal at relay $\mathbf{y}_{R,k}$ and destination $\mathbf{y}_{D,1,k}$ is written as $\mathbf{y}_{R,k} = \mathbf{H}_{1,k}\mathbf{f}_{S,k}x_k + \mathbf{n}_{R,k}$ and $\mathbf{y}_{D,1,k} = \mathbf{H}_{0,k}\mathbf{f}_{S,k}x_k + \mathbf{n}_{D,1,k}$, respectively, where $\mathbf{f}_{S,k}$ indicates the source beamforming vector, and $\mathbf{n}_{R,k}$ and $\mathbf{n}_{D,1,k}$ represent the noise vector at the relay and the destination, respectively.

In the second time slot, the relay amplifies the received signal from the source and forwards it to the destination. Therefore, the received signal at the destination in the kth subcarrier is given by $\mathbf{y}_{D,2,k} = \mathbf{H}_{2,k}\mathbf{G}_k\mathbf{y}_{R,k} + \mathbf{n}_{D,2,k}$, where $\mathbf{G}_k$ and $\mathbf{n}_{D,2,k}$ denote the relay transceiver and the noise vector at the destination, respectively. Here, all noise elements are assumed to be independent and identically distributed complex Gaussian random variables with zero mean and unit variance.

By stacking two signals that have arrived at the destination over two time slots, we obtain

$$\mathbf{y}_{D,k} \triangleq \begin{bmatrix} \mathbf{y}_{D,1,k} \\ \mathbf{y}_{D,2,k} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{0,k}\mathbf{f}_{S,k} \\ \mathbf{H}_{2,k}\mathbf{G}_k\mathbf{H}_{1,k}\mathbf{f}_{S,k} \end{bmatrix} x_k + \begin{bmatrix} \mathbf{n}_{D,1,k} \\ \mathbf{n}_{D,k} \end{bmatrix} \tag{1}$$

where $\mathbf{n}_{D,k} \triangleq \mathbf{H}_{2,k}\mathbf{G}_k\mathbf{n}_{R,k} + \mathbf{n}_{D,2,k}$. Then, after applying the receive combining vector $\mathbf{w}_{D,k}$, the destination estimates the data symbol at the kth subcarrier as

$$\tilde{y}_k = \mathbf{w}_{D,k}^H \mathbf{y}_{D,k} = c_k x_k + \tilde{n}_k \tag{2}$$

where c_k stands for the effective channel gain, and $\tilde{n}_k$ is equal to the effective noise with variance $\tilde{\sigma}_k^2$. Finally, by aggregating all $\tilde{y}_k$ for $k = 1, \ldots, K$, the destination restores the information bits through a maximum likelihood (ML) decoder, e.g., Viterbi decoder for a convolutional code.

We assume $E[|x_k|^2] = P_S$ for $k = 1, \ldots, K$. Moreover, for simplicity, it is assumed to have a per-carrier power constraint at the source and the relay as $\|\mathbf{f}_{S,k}\|^2 \leq 1$ and $\|\mathbf{G}_k\|_F^2 + P_S\|\mathbf{G}_k\mathbf{H}_{1,k}\mathbf{f}_{S,k}\|^2 \leq P_R$, respectively, where P_S and P_R are the available power budget at the source and the relay, respectively. Let us define the eigenvalue decomposition (EVD) of $\mathbf{H}_{l,k}^H\mathbf{H}_{l,k} = \mathbf{U}_{l,k}\mathbf{\Lambda}_{l,k}\mathbf{U}_{l,k}^H$, where $\mathbf{U}_{l,k}$ represents a unitary matrix, and $\mathbf{\Lambda}_{l,k}$ indicates a diagonal matrix with eigenvalues. Moreover, we denote the largest eigenvalue of $\mathbf{H}_{l,k}^H\mathbf{H}_{l,k}$ and its corresponding eigenvector as $\lambda_{l,k}$ and $\mathbf{u}_{l,k}$, respectively.

Employing the optimal structure of $\mathbf{G}_k$ and $\mathbf{w}_{D,k}$ in [5], the end-to-end SNR for the kth subchannel $\gamma(\mathbf{f}_{S,k})$ can be expressed as a function of $\mathbf{f}_{S,k}$ as $\gamma(\mathbf{f}_{S,k}) = \gamma_r(\mathbf{f}_{S,k}) + \gamma_0(\mathbf{f}_{S,k})$, where $\gamma_r(\mathbf{f}_{S,k}) \triangleq \lambda_{2,k}P_S P_R\|\mathbf{H}_{1,k}\mathbf{f}_{S,k}\|^2/(1 + P_S\|\mathbf{H}_{1,k}\mathbf{f}_{S,k}\|^2 + \lambda_{2,k}P_R)$, and $\gamma_0(\mathbf{f}_{S,k}) \triangleq P_S\|\mathbf{H}_{0,k}\mathbf{f}_{S,k}\|^2$. Then, the end-to-end SNR maximization problem is formulated as

$$\mathbf{f}_{S,k}^* = \arg\max_{\mathbf{f}_{S,k}} \gamma(\mathbf{f}_{S,k}) \text{ s.t. } \|\mathbf{f}_{S,k}\|^2 \le 1. \tag{3}$$

Note that, due to the nonconvexity of the problem, the optimal solution $\mathbf{f}_{S,k}^*$ is still unknown. Thus, it is difficult to derive the diversity order of the systems. In the following section, we will examine the diversity order by analyzing the upper and lower bounds of $\gamma_k^* \triangleq \gamma(\mathbf{f}_{S,k}^*)$. Throughout this paper, we assume that $P_S = P_R = P_0/2$, where P_0 represents the total transmit power; however, the result can be easily extended to general cases.

III. Pairwise Error Probability Analysis

Here, we analyze the average pairwise error probability (PEP) of coded MIMO-OFDM AF relaying systems with a direct link. Given the observation in (2), an ML decoder makes a decision according to the rule as $\hat{\mathbf{c}} = \arg\min_{\mathbf{c}\in\mathcal{C}} \sum_{k=1}^K (1/\sigma_k^2)|(1/c_k)\tilde{y}_k - x_k|^2$, where $\sigma_k^2 \triangleq \tilde{\sigma}_k^2/c_k^2$, and $\mathcal{C}$ indicates a codebook. Then, given $\mathbf{H}_{l,k}$ for all l and k, the conditional PEP that the ML decoder chooses the erroneous sequence $\bar{\mathbf{c}}$ instead of the corrected sequence $\mathbf{c}$ is given by

$$\begin{aligned} P(\mathbf{c} \to \bar{\mathbf{c}}|\mathbf{H}_{l,k},\ \forall l,k) &= P\left(\sum_{k=1}^K \frac{1}{\sigma_k^2}\left(\left|\frac{1}{c_k}\tilde{y}_k - x_k\right|^2 - \left|\frac{1}{c_k}\tilde{y}_k - \bar{x}_k\right|^2\right) > 0\right) \\ &= Q\left(\sqrt{\frac{1}{2}\sum_{k=1}^K \frac{1}{\sigma_k^2}|x_k - \bar{x}_k|^2}\right) = Q\left(\sqrt{\frac{P_S}{2}\sum_{k=1}^K \frac{d_k^2}{\sigma_k^2}}\right) \end{aligned} \tag{4}$$

where $Q(\cdot)$ is the Q-function, and $d_k^2 \triangleq |x_k - \bar{x}_k|^2/P_S$ indicates the normalized Euclidean distance.

Let us define d_f and $d_{\min}$ as the minimum Hamming distance (or free distance) for an employed channel code and the minimum of all possible nonzero d_k, respectively. Note that the diversity order is determined by the worst case performance, and conditions for the worst case are as follows: 1) The Hamming distance between $\mathbf{c}$ and $\bar{\mathbf{c}}$ is d_f, i.e., $d_k = 0$ for all k except those d_f terms inside the summation in (4). 2) All nonzeros d_k are equal to $d_{\min}$, i.e., $\sum_{k=1}^K (d_k^2/\sigma_k^2) = d_{\min}^2 \sum_{k\in S_k}(1/\sigma_k^2)$, where $S_k \triangleq \{k|d_k \ne 0\}$, and $|S_k| = d_f$. 3) $\sum_{k\in S_k}(1/\sigma_k^2) = \sum_{k,d_f}^K (1/\sigma_k^2)$, where $\sum_{k,d_f}^K$ represents a summation taken with index k over d_f smallest components among K different terms. Therefore, the conditional PEP in (4) can be written as

$$\begin{aligned} P(\mathbf{c} \to \bar{\mathbf{c}}|\mathbf{H}_{l,k}\ \forall l,k) &\doteq Q\left(\sqrt{\frac{P_S d_{\min}^2}{2}\sum_{k,d_f}^K \frac{1}{\sigma_k^2}}\right) \\ &\overset{(a)}{\doteq} \exp\left(-\frac{P_S d_{\min}^2}{4}\sum_{k,d_f}^K \frac{1}{\sigma_k^2}\right) = \exp\left(-\frac{d_{\min}^2}{4}\sum_{k,d_f}^K \gamma_k^*\right) \end{aligned} \tag{5}$$

where (a) is due to the Chernoff bound, which is known to be exponentially tight at a high SNR, and $\gamma_k^* = P_S c_k^2/\tilde{\sigma}_k^2 = P_S/\sigma_k^2$. Note that, as previously mentioned, since the expression for γ_k^* is unavailable, it is hard to derive the diversity order of the coded beamforming.

To circumvent this difficulty, we first identify the upper and lower bounds of γ_k^* and then characterize the diversity order by showing that these bounds are tight in terms of the diversity order. Now, we focus on an upper bound of γ_k^*. Let us divide problem (3) into two separate problems as follows:

$$\mathbf{f}_{S,r,k}^* = \arg\max_{\mathbf{f}_{S,k}} \gamma_r(\mathbf{f}_{S,k}) \quad \text{s.t.} \quad \|\mathbf{f}_{S,k}\|^2 \le 1 \tag{6}$$

$$\mathbf{f}_{S,0,k}^* = \arg\max_{\mathbf{f}_{S,k}} \gamma_0(\mathbf{f}_{S,k}) \quad \text{s.t.} \quad \|\mathbf{f}_{S,k}\|^2 \le 1. \tag{7}$$

Then, it immediately follows that $\gamma_k^* \le \gamma_r(\mathbf{f}_{S,r,k}^*) + \gamma_0(\mathbf{f}_{S,0,k}^*)$. In addition, it is seen that problems (6) and (7) amount to the optimization problems for the relaying systems without a direct link and the point-to-point MIMO systems, respectively. Thus, $\mathbf{f}_{S,r,k}^*$ and $\mathbf{f}_{S,0,k}^*$ in (6) and (7) can be readily obtained as $\mathbf{f}_{S,r,k}^* = \mathbf{u}_{1,k}$ and $\mathbf{f}_{S,0,k}^* = \mathbf{u}_{0,k}$, respectively. Then, we can get an upper bound of γ_k^* at a high SNR as

$$\begin{aligned} \gamma_k^* &\le \frac{\lambda_{1,k}\lambda_{2,k}P_S P_R}{1 + \lambda_{1,k}P_S + \lambda_{2,k}P_R} + \lambda_{0,k}P_S \\ &\approx \frac{\lambda_{1,k}\lambda_{2,k}P_S P_R}{\lambda_{1,k}P_S + \lambda_{2,k}P_R} + \lambda_{0,k}P_S \\ &\le \min(\lambda_{1,k}P_S, \lambda_{2,k}P_R) + \lambda_{0,k}P_S = \frac{P_0}{2}\min(\tilde{\lambda}_k, \hat{\lambda}_k) \end{aligned} \tag{8}$$

where $\tilde{\lambda}_k \triangleq \lambda_{0,k} + \lambda_{1,k}$, $\hat{\lambda}_k \triangleq \lambda_{0,k} + \lambda_{2,k}$, and the last inequality follows from the harmonic mean inequality, i.e., $xy/(x+y) \le \min(x,y)$.

Moreover, applying the suboptimal beamforming technique in [13], we can obtain a lower bound of γ_k^* as

$$\gamma_k^* \ge \frac{P_0}{8}\min(\bar{\lambda}_k, \hat{\lambda}_k) \tag{9}$$

where $\bar{\lambda}_k$ is defined as the largest eigenvalue of $\mathbf{H}_{0,k}^H\mathbf{H}_{0,k} + \mathbf{H}_{1,k}^H\mathbf{H}_{1,k}$. Note that $\tilde{\lambda}_k$ and $\bar{\lambda}_k$ are not equal in general, and thus, the upper and lower bounds of the diversity order in (8) and (9) may not be the same. In the following theorem, we establish the diversity order of the coded beamforming scheme by showing that the bounds of the diversity order are tight.

Theorem 1: The diversity order of the coded beamforming scheme over $(N_S \times N_R \times N_D)$ frequency-selective MIMO AF relaying channels is given by

$$D = r_{0,d_f}N_S N_D + N_R \min\left(r_{1,d_f}N_S, r_{2,d_f}N_D\right) \tag{10}$$

where $r_{l,d_f} \triangleq \min(L_l, d_f)$.

Proof: See the Appendix. ∎

It is worth noting that the diversity expression in (10) includes the previous analysis in [4] and [11] as special cases. For example, when $d_f = 1$ (uncoded) or $L_0 = L_1 = L_2 = 1$ (flat fading), we have the diversity order $D = N_S N_D + N_R \min(N_S, N_D)$ as obtained in [4]. Moreover, when the direct link is negligible ($r_{0,d_f} = 0$), the diversity order in (10) becomes $D = N_R \min(r_{1,d_f}N_S, r_{2,d_f}N_D)$, which is equal to the result in [11]. Our analysis for the lower bound of the diversity order in Appendix also illustrates that all beamforming schemes whose end-to-end SNR of the kth subcarrier γ_k satisfies $\gamma_k \ge (P_0/8)\min(\bar{\lambda}_k, \hat{\lambda}_k)$ achieve the diversity order D in (10).

In addition, the diversity order in (10) is maximized when d_f is sufficiently large, i.e., $d_f = \max(L_0, L_1, L_2)$ as

$$D_{\max} = L_0 N_S N_D + N_R \min(L_1 N_S, L_2 N_D). \tag{11}$$

However, as d_f increases, the encoding/decoding complexity becomes higher. Therefore, it may be important to find the minimum required

TABLE I
SMALLEST d_f TO ACHIEVE $D_{\max}$

	$N_S > N_D$	$N_S < N_D$	$N_S = N_D$
$L_1N_S > L_2N_D$	$\max(L_0, L_2)$	$\max\left(L_0, \left\lceil \frac{L_2N_D}{N_S} \right\rceil\right)$	$\max(L_0, L_2)$
$L_1N_S < L_2N_D$	$\max\left(L_0, \left\lceil \frac{L_1N_S}{N_D} \right\rceil\right)$	$\max(L_0, L_1)$	$\max(L_0, L_1)$
$L_1N_S = L_2N_D$	$\max(L_0, L_2)$	$\max(L_0, L_1)$	$\max(L_0, L_1) = \max(L_0, L_2)$

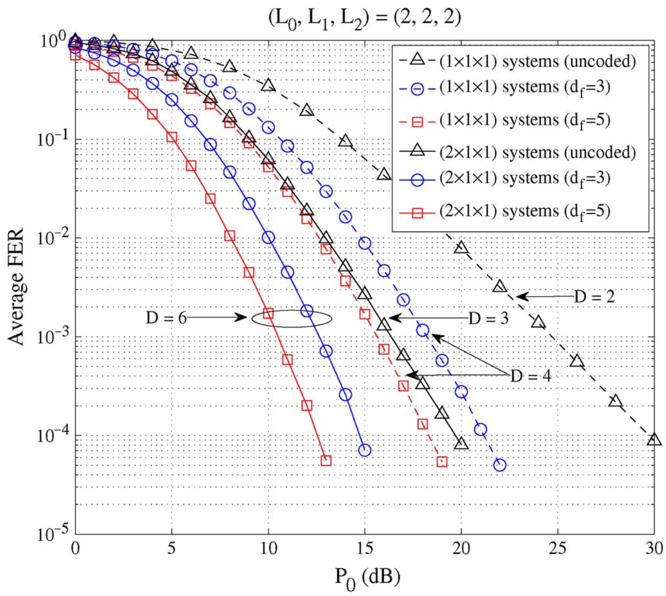

Fig. 2. Average FER performance as a function of P_0 with $(L_0, L_1, L_2) = (2, 2, 2)$.

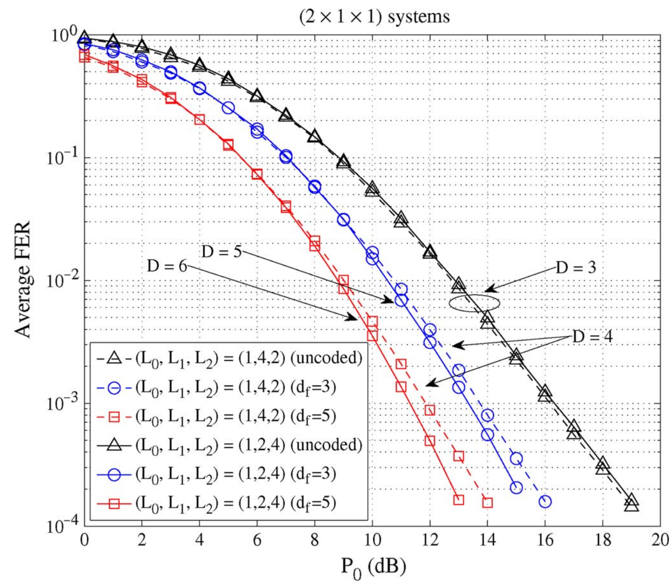

Fig. 3. Average FER performance as a function of P_0 for the $(2 \times 1 \times 1)$ systems.

free distance $d_{f,\min}$ to get the maximum diversity $D_{\max}$. In Table I, $d_{f,\min}$ is summarized for various scenarios. Interestingly, it is shown that while the first term in (11) is obtained only when $d_f \geq L_0$, the second term in $D_{\max}$ is achievable even when d_f is smaller than $\max(L_1, L_2)$. For example, when the two conditions $L_1N_S \geq L_2N_D$ and $N_S \leq N_D$ are satisfied, $d_f \geq \lceil L_2N_D/N_S \rceil$ achieves the second term in $D_{\max}$. Therefore, in this case, we have $d_{f,\min} = \max(L_0, \lceil L_2N_D/N_S \rceil)$, and the resulting diversity order is given as $D_{\max} = N_D(L_0N_S + L_2N_R)$. This result reveals that when $L_0 < \lceil L_2N_D/N_S \rceil$, increasing the number of source antennas is beneficial in terms of both the diversity gain and the coding complexity. In contrast, when $L_0 \geq \lceil L_2N_D/N_S \rceil$, the number of antennas at each node has nothing to do with $d_{f,\min}$. Similar results can be made for the case of $L_1N_S < L_2N_D$ and $N_S > N_D$.

IV. SIMULATION RESULTS

Here, we illustrate numerical results to validate our analysis. We employ a convolutional encoder with a random bit interleaver and consider OFDM modulation with $K = 64$ subcarriers and quaternary phase-shift keying (QPSK). It is assumed that the cyclic prefix length is longer than $\max(L_0, L_1, L_2) - 1$, and d_f is adjusted by varying the code memory length with a fixed code rate of 1/2. We also present uncoded systems with binary phase-shift keying to make a fair comparison with coded QPSK systems. To evaluate the system performance, we adopt the beamforming technique in [5], which shows local optimal performance.

In Fig. 2, we exhibit the average frame error rate (FER) performance for the systems with $(L_0, L_1, L_2) = (2, 2, 2)$. It is observed that coded systems show additional diversity gains compared with uncoded systems. From the result in Theorem 1, the diversity order for $(1 \times 1 \times 1)$ and $(2 \times 1 \times 1)$ systems is $D = r_{0,d_f} + \min(r_{1,d_f}, r_{2,d_f})$ and $2r_{0,d_f} + \min(2r_{1,d_f}, r_{2,d_f})$, respectively, and it is shown that the analytical results are well matched with the simulated curves. Moreover, since $d_{f,\min} = \max(L_0, L_2) = 2$, the systems with $d_f = 3$ obtain full diversity order, and the systems with $d_f = 5$ only provide an additional array gain.

Fig. 3 shows the average FER performance for $(2 \times 1 \times 1)$ systems. Here, $d_{f,\min}$ for the cases of $(L_0, L_1, L_2) = (1, 4, 2)$ and $(1, 2, 4)$ are equal to L_2. Thus, when $(L_0, L_1, L_2) = (1, 4, 2)$, the diversity orders for the systems with $d_f = 3$ and 5 are the same as 4. On the other hand, for the case of $(L_0, L_1, L_2) = (1, 2, 4)$, only the systems with $d_f = 5$ exhibit full diversity order $D_{\max} = 6$, whereas the systems with $d_f = 3$ experience the diversity order of 5.

In Fig. 4, we demonstrate the average FER performance for $(1 \times 1 \times 2)$ and $(1 \times 2 \times 2)$ systems with $(L_0, L_1, L_2) = (1, 4, 1)$. Since the number of relay antennas is irrelevant to $d_{f,\min}$, as shown in Table I, we can see that $d_{f,\min}$ for both systems is equal to 2. From simulation results, we confirm that our analysis matches well with the simulation results.

V. CONCLUSION

In this paper, we have investigated the diversity oder of coded beamforming in MIMO-OFDM AF relaying systems with a direct link. To overcome difficulty in finding the diversity order of the systems with the optimal beamforming solution, we have derived the diversity order by showing that an upper bound of the diversity order is equal to an achievable diversity order. Moreover, we have provided helpful insights for the minimum Hamming distance of the code to obtain full diversity order. The analytical result is verified with numerical simulations.

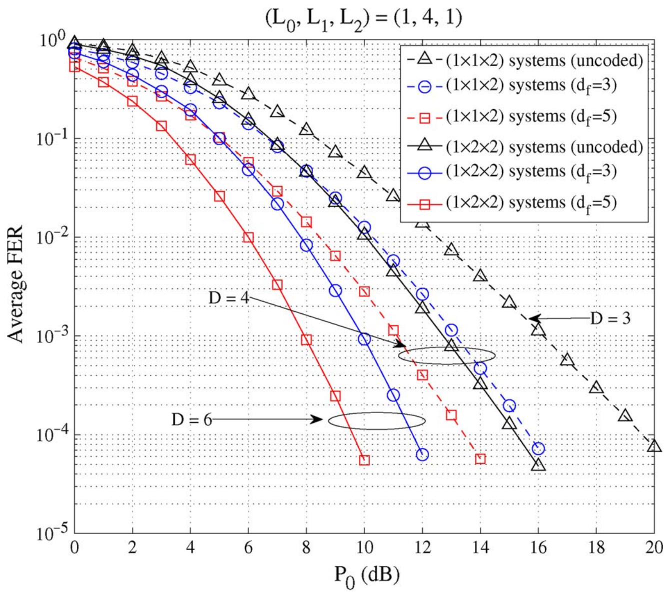

Fig. 4. Average FER performance as a function of P_0 with $(L_0, L_1, L_2) = (1, 4, 1)$.

APPENDIX

We first derive an upper bound of the diversity order. Plugging the bound in (8) into (5), the conditional PEP is lower bounded as

$$\begin{aligned} P(\mathbf{c}\to\bar{\mathbf{c}}|\mathbf{H}_{l,k},\ \forall l,k) &\dot{\geq} \exp\left(-\rho\sum_{k,d_f}^{K}\min(\tilde{\lambda}_k,\hat{\lambda}_k)\right)\\ &\dot{\geq} \exp\left(-\rho\sum_{k=1}^{d_f}\min\left(\tilde{\lambda}_{[k]},\hat{\lambda}_{[k]}\right)\right)\\ &\dot{\geq} \exp\left(-\rho\min\left(\sum_{k=1}^{d_f}\tilde{\lambda}_{[k]},\sum_{k=1}^{d_f}\hat{\lambda}_{[k]}\right)\right) \end{aligned} \tag{12}$$

where $\rho \triangleq P_0 d_{\min}^2/8$, $x_{[i]}$ is the ith smallest component of $\{x_1,\ldots,x_K\}$, the second inequality follows from [11, Lemma 1], i.e., $\sum_{k,d_f}^{K}\min(x_k,y_k)\leq\sum_{k=1}^{d_f}\min(x_{[k]},y_{[k]})$, and the last inequality is due to the fact that a sum of the minimum is always less than the minimum of the sum.

Let us define μ as $\mu\triangleq\min(\sum_{k=1}^{d_f}\tilde{\lambda}_{[k]},\sum_{k=1}^{d_f}\hat{\lambda}_{[k]})$. We denote a_k and b_k as the indexes that satisfy $\tilde{\lambda}_{[k]}=\lambda_{0,a_k}+\lambda_{1,a_k}$ and $\hat{\lambda}_{[k]}=\lambda_{0,b_k}+\lambda_{2,b_k}$, respectively. Here, $\sum_{k=1}^{d_f}\lambda_{0,a_k}\leq\sum_{k=1}^{d_f}\mathrm{Tr}(\mathbf{H}_{0,a_k}^H\mathbf{H}_{0,a_k})=\sum_{i=1}^{N_S}\sum_{j=1}^{N_D}\mathbf{h}_{0,i,j}^H\tilde{\mathbf{W}}_{0,d_f}\mathbf{h}_{0,i,j}$, where $\tilde{\mathbf{W}}_{l,d_f}\triangleq\sum_{k=1}^{d_f}\mathbf{w}_{l,a_k}\times\mathbf{w}_{l,a_k}^H$ whose rank is $r_{l,d_f}=\min(L_l,d_f)$. Let us introduce the EVD of $\tilde{\mathbf{W}}_{l,d_f}$ as $\tilde{\mathbf{W}}_{l,d_f}=\tilde{\mathbf{U}}_{l,d_f}\tilde{\mathbf{\Phi}}_{l,d_f}\tilde{\mathbf{U}}_{l,d_f}^H$, where $\tilde{\mathbf{U}}_{l,d_f}$ and $\tilde{\mathbf{\Phi}}_{l,d_f}$ represent a unitary matrix and a diagonal matrix with nonzero entries $\tilde{\phi}_{l,d_f}^1,\ldots,\tilde{\phi}_{l,d_f}^{r_{l,d_f}}$, respectively. Then, an upper bound of $\sum_{k=1}^{d_f}\lambda_{0,a_k}$ becomes $\sum_{k=1}^{d_f}\lambda_{0,a_k}\leq\sum_{i=1}^{N_S}\sum_{j=1}^{N_D}\sum_{m=1}^{r_{0,d_f}}|\tilde{h}_{0,m}^{(i,j)}|^2\tilde{\phi}_{0,d_f}^m$, where $\tilde{\mathbf{U}}_{l,d_f}^H\mathbf{h}_{l,i,j}=[\tilde{h}_{l,1}^{(i,j)},\ldots,\tilde{h}_{l,L_l}^{(i,j)}]^T$. In a similar fashion, we can obtain an upper bound of $\sum_{k=1}^{d_f}\lambda_{1,a_k}$. Then, it follows that

$$\sum_{k=1}^{d_f}\tilde{\lambda}_{[k]}\leq\tilde{U}_{d_f} \tag{13}$$

where $\tilde{U}_{d_f}\triangleq\sum_{i=1}^{N_S}(\sum_{j=1}^{N_D}\sum_{m=1}^{r_{0,d_f}}|\tilde{h}_{0,m}^{(i,j)}|^2\tilde{\phi}_{0,d_f}^m+\sum_{j=1}^{N_R}\sum_{m=1}^{r_{1,d_f}}|\tilde{h}_{1,m}^{(i,j)}|^2\tilde{\phi}_{1,d_f}^m)$.

Let us denote $\hat{\mathbf{W}}_{l,d_f}\triangleq\sum_{k=1}^{d_f}\mathbf{w}_{l,b_k}\mathbf{w}_{l,b_k}^H$ whose rank is r_{l,d_f} and its EVD as $\hat{\mathbf{W}}_{l,d_f}=\hat{\mathbf{U}}_{l,d_f}\hat{\mathbf{\Phi}}_{l,d_f}\hat{\mathbf{U}}_{l,d_f}^H$, where $\hat{\mathbf{U}}_{l,d_f}$ is a unitary matrix, and $\hat{\mathbf{\Phi}}_{l,d_f}$ is a diagonal matrix with nonzero entries $\hat{\phi}_{l,d_f}^1,\ldots,\hat{\phi}_{l,d_f}^{r_{l,d_f}}$. Then, by defining $\hat{\mathbf{U}}_{l,d_f}^H\mathbf{h}_{l,i,j}=[\hat{h}_{l,1}^{(i,j)},\ldots,\hat{h}_{l,L_l}^{(i,j)}]^T$, an upper bound of $\sum_{k=1}^{d_f}\hat{\lambda}_{[k]}$ can be derived as

$$\sum_{k=1}^{d_f}\hat{\lambda}_{[k]}\leq\hat{U}_{d_f} \tag{14}$$

where $\hat{U}_{d_f}\triangleq\sum_{i=1}^{N_D}(\sum_{i=1}^{N_S}\sum_{m=1}^{r_{0,d_f}}|\hat{h}_{0,m}^{(i,j)}|^2\hat{\phi}_{0,d_f}^m+\sum_{i=1}^{N_R}\sum_{m=1}^{r_{2,d_f}}|\hat{h}_{2,m}^{(i,j)}|^2\hat{\phi}_{2,d_f}^m)$. Finally, from (13) and (14), we obtain an upper bound of μ as $\mu\leq\mu_{\mathrm{ub}}\triangleq\min(\tilde{U}_{d_f},\hat{U}_{d_f})$.

We define Gamma distributed variables as $X\sim\mathcal{G}(\alpha,\beta)$ with parameters α and β. Then, since the unitary matrices $\tilde{\mathbf{U}}_{l,d_f}$ and $\hat{\mathbf{U}}_{l,d_f}$ do not change the statistical property of $h_{l,n}^{(i,j)}$, we have $\tilde{\phi}_{l,d_f}^m|\tilde{h}_{l,m}^{(i,j)}|^2\sim\mathcal{G}(1,1/\tilde{\phi}_{l,d_f}^m)$ and $\hat{\phi}_{l,d_f}^m|\hat{h}_{l,m}^{(i,j)}|^2\sim\mathcal{G}(1,1/\hat{\phi}_{l,d_f}^m)$. Thus, $\tilde{U}_{d_f}$ and $\hat{U}_{d_f}$ are equivalent to the sum of $N_S(N_D r_{0,d_f}+N_R r_{1,d_f})$ and $N_D(N_S r_{0,d_f}+N_R r_{2,d_f})$ independent Gamma random variables, respectively. Moreover, it is well known that the moment-generating function of a sum of K independent Gamma random variables $U=\sum_{i=1}^{K}X_i$ with $X_i\sim\mathcal{G}(\alpha_i,\beta_i)$ is $\mathcal{L}_U(s)=\int_0^\infty e^{-sx}f_U(x)dx=\prod_{i=1}^{K}(1+s/\beta_i)^{-\alpha_i}$.

Note that for a small δ (i.e., $\delta\to0^+$), the probability distribution function (pdf) of $W\triangleq\min(X_1,X_2)$ is equal to $f_W(\delta)=f_{X_1}(\delta)+f_{X_2}(\delta)$, where $f_{X_k}(\cdot)$ indicates the pdf of X_k [11]. Therefore, substituting μ_{ub} into the conditional PEP in (12), we get a lower bound of the average PEP as

$$\begin{aligned} &P(\mathbf{c}\to\bar{\mathbf{c}})\\ &\dot{\geq}\int_0^\infty e^{-\rho x}f_{\mu_{\mathrm{ub}}}(x)dx\\ &=\int_0^\infty e^{-\rho x}f_{\tilde{U}_{d_f}}(x)dx+\int_0^\infty e^{-\rho x}f_{\hat{U}_{d_f}}(x)dx\\ &\dot{\geq}\prod_{m=1}^{r_{0,d_f}}\left(1+\rho\tilde{\phi}_{0,d_f}^m\right)^{-N_SN_D}\prod_{m=1}^{r_{1,d_f}}\left(1+\rho\tilde{\phi}_{1,d_f}^m\right)^{-N_SN_R}\\ &\quad+\prod_{m=1}^{r_{0,d_f}}\left(1+\rho\hat{\phi}_{0,d_f}^m\right)^{-N_SN_D}\prod_{m=1}^{r_{2,d_f}}\left(1+\rho\hat{\phi}_{2,d_f}^m\right)^{-N_RN_D}\\ &\dot{\geq}(1+\rho\phi_{\max})^{-N_S\left(N_Dr_{0,d_f}+N_Rr_{1,d_f}\right)}\\ &\quad+(1+\rho\phi_{\max})^{-N_D\left(N_Sr_{0,d_f}+N_Rr_{2,d_f}\right)}\\ &\simeq(\rho\phi_{\max})^{-D_{\mathrm{ub}}}+o(\rho^{-D_{\mathrm{ub}}}) \end{aligned}$$

where $\phi_{\max}$ is the maximum of all nonzero eigenvalues, and $D_{\mathrm{ub}}\triangleq r_{0,d_f}N_SN_D+N_R\min(r_{1,d_f}N_S,r_{2,d_f}N_D)$. Thus, an upper bound of diversity order is established.

We now derive a lower bound of the diversity order. Substituting the bound in (9) into (5), an upper bound of the conditional PEP is

$$\begin{aligned} &P(\mathbf{c}\to\bar{\mathbf{c}}|\mathbf{H}_{l,k},\ \forall l,k)\\ &\quad\dot{\leq}\exp\left(-\tau\sum_{k,d_f}^{K}\min(\bar{\lambda}_k,\hat{\lambda}_k)\right)\\ &\quad\dot{\leq}\exp\left(-\tau\sum_{k=1}^{d_f}\min\left(\bar{\lambda}_{[d_f-k+1]},\hat{\lambda}_{[k]}\right)\right) \end{aligned} \tag{15}$$

where $\tau \triangleq P_0 d_{\min}^2/32$, and the last inequality follows from [11, Lemma 1], i.e., $\sum_{k,d_f}^{K} \min(x_k, y_k) \geq \sum_{k=1}^{d_f} \min(x_{[d_f-k+1]}, y_{[k]})$. Note that all possible values for $\sum_{k=1}^{d_f} \min(\bar{\lambda}_{[d_f-k+1]}, \hat{\lambda}_{[k]})$ are $\sum_{k=1}^{d_f-i} \bar{\lambda}_{[k]} + \sum_{k=1}^{i} \hat{\lambda}_{[k]}$ for $i = 0, 1, \ldots, d_f$. Thus, $\sum_{k=1}^{d_f} \times \min(\bar{\lambda}_{[d_f-k+1]}, \hat{\lambda}_{[k]}) \geq \nu$, where $\nu \triangleq \min_{i=0,\ldots,d_f}(\sum_{k=1}^{d_f-i} \bar{\lambda}_{[k]} + \sum_{k=1}^{i} \hat{\lambda}_{[k]})$.

Now, we focus on a lower bound of $\sum_{k=1}^{d_f-i} \bar{\lambda}_{[k]}$. Let us define $N \triangleq \max(N_S, N_R, N_D)$. Then, we have $\sum_{k=1}^{d_f-i} \bar{\lambda}_{[k]} \geq (1/N) \sum_{k=1}^{d_f-i} \mathrm{Tr}(\mathbf{H}_{0,[k]}^H \mathbf{H}_{0,[k]} + \mathbf{H}_{1,[k]}^H \mathbf{H}_{1,[k]}) = (1/N) \sum_{i=1}^{N_S}(\sum_{j=1}^{N_D} \mathbf{h}_{0,i,j}^H \bar{\mathbf{W}}_{0,d_f-i} \mathbf{h}_{0,i,j} + \sum_{j=1}^{N_R} \mathbf{h}_{1,i,j}^H \bar{\mathbf{W}}_{1,d_f-i} \mathbf{h}_{1,i,j})$, where $\bar{\mathbf{W}}_{l,n} \triangleq \sum_{k=1}^{n} \mathbf{w}_{l,[k]} \mathbf{w}_{l,[k]}^H$ whose rank is $r_{l,n} = \min(L_l, n)$. By introducing the EVD of $\bar{\mathbf{W}}_{l,n}$ as $\bar{\mathbf{W}}_{l,n} = \bar{\mathbf{U}}_{l,n} \bar{\mathbf{\Phi}}_{l,n} \bar{\mathbf{U}}_{l,n}^H$, where $\bar{\mathbf{U}}_{l,n}$ and $\bar{\mathbf{\Phi}}_{l,n}$ indicate a unitary matrix and a diagonal matrix with nonzero entries $\bar{\phi}_{l,n}^1, \ldots, \bar{\phi}_{l,n}^{r_{l,n}}$, respectively, a lower bound of $\sum_{k=1}^{d_f-i} \bar{\lambda}_{[k]}$ is given as

$$\sum_{k=1}^{d_f-i} \bar{\lambda}_{[k]} \geq \frac{1}{N} \bar{U}_{d_f-i}$$

where $\bar{U}_{d_f-i} \triangleq \sum_{i=1}^{N_S}(\sum_{j=1}^{N_D} \sum_{m=1}^{r_{0,d_f-i}} |\bar{h}_{0,m}^{(i,j)}|^2 \bar{\phi}_{0,d_f-i}^m + \sum_{j=1}^{N_R} \sum_{m=1}^{r_{1,d_f-i}} |\bar{h}_{1,m}^{(i,j)}|^2 \bar{\phi}_{1,d_f-i}^m)$, and $\bar{\mathbf{U}}_{l,n}^H \mathbf{h}_{l,i,j} = [\bar{h}_{l,1}^{(i,j)}, \ldots, \bar{h}_{l,L_l}^{(i,j)}]^T$. In a similar way as in the derivation of (14), we can get a lower bound of $\sum_{k=1}^{i} \hat{\lambda}_{[k]}$ as $\sum_{k=1}^{i} \hat{\lambda}_{[k]} \geq (1/N)\hat{U}_i$. Therefore, ν is lower bounded as $\nu \geq \nu_{\mathrm{lb}} \triangleq \min_{i=0,\ldots,d_f} \nu_i$, where $\nu_i \triangleq (1/N)(\bar{U}_{d_f-i} + \hat{U}_i)$.

Plugging ν_{lb} into the conditional PEP in (15), an upper bound of the average PEP becomes

$$P(\mathbf{c} \to \bar{\mathbf{c}}) \dot{\leq} \int_0^\infty e^{-\tau x} f_{\nu_{\mathrm{lb}}}(x) dx = \sum_{i=0}^{d_f} \int_0^\infty e^{-\tau x} f_{\nu_i}(x) dx$$
$$\simeq \left(\frac{\tau \phi_{\min}}{N}\right)^{-D_{\mathrm{lb}}} + o(\tau^{-D_{\mathrm{lb}}})$$

where $\phi_{\min}$ is the minimum of all nonzero eigenvalues, and $D_{\mathrm{lb}} \triangleq \min_{i=0,\ldots,d_f} D_{lb,i}$ with $D_{lb,i} \triangleq r_{0,d_f-i} N_S N_D + r_{1,d_f-i} N_S N_R + r_{0,i} N_S N_D + r_{2,i} N_R N_D$. Here, it can be readily shown that $D_{lb,i} \geq \min(D_{lb,0}, D_{lb,d_f})$ for $i = 1, \ldots, d_f - 1$. Thus, it follows that $D_{\mathrm{lb}} = \min(D_{lb,0}, D_{lb,d_f}) = r_{0,d_f} N_S N_D + N_R \min(r_{1,d_f} N_S, r_{2,d_f} N_D)$, which is equal to D_{ub}, and the proof is completed. ■

References

[1] T. M. Cover and A. Gamal, "Capacity theorems for the relay channels," *IEEE Trans. Inf. Theory*, vol. IT-25, no. 5, pp. 572–584, Sep. 1979.

[2] K.-J. Lee, H. Sung, E. Park, and I. Lee, "Joint optimization for one and two-way MIMO AF multiple-relay systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3671–3681, Dec. 2010.

[3] R. Cao, T. Lv, H. Gao, S. Yang, and J. M. Cioffi, "Achieving full diversity in multi-antenna two-way relay networks via symbol-based physical-layer network coding," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3445–3457, Jul. 2013.

[4] S. W. Peters and R. W. Heath, "Nonregenerative MIMO relaying with optimal transmit antenna selection," *IEEE Signal Process. Lett.*, vol. 15, pp. 421–424, Apr. 2008.

[5] B. Khoshnevis, W. Yu, and R. Adve, "Grassmannian beamforming for MIMO amplify-and-forward relaying," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1397–1407, Oct. 2008.

[6] C. Song, K.-J. Lee, and I. Lee, "MMSE-based MIMO cooperative relaying systems: Closed-form designs and outage behavior," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 8, pp. 1390–1401, Sep. 2012.

[7] I. Hammerstrom and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2798–2802, Aug. 2007.

[8] Y. Li, W. Wang, J. Kong, and M. Peng, "Subcarrier pairing for amplify-and-forward and decode-and-forward OFDM relay links," *IEEE Commun. Lett.*, vol. 13, no. 4, pp. 209–211, Apr. 2009.

[9] C. Xing, S. Ma, Y.-C. Wu, and T.-S. Ng, "Transceiver design for dual-hop nonregenerative MIMO-OFDM relay systems under channel uncertainties," *IEEE Trans. Signal Process.*, vol. 58, no. 12, pp. 6325–6339, Dec. 2010.

[10] I. Lee, A. Chan, and C.-E. W. Sundberg, "Space–time bit-interleaved coded modulation for OFDM systems," *IEEE Trans. Signal Process.*, vol. 52, no. 3, pp. 820–825, Mar. 2004.

[11] C. Song and I. Lee, "Diversity analysis of coded beamforming in MIMO-OFDM amplify-and-forward relaying systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2445–2450, Aug. 2011.

[12] J. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[13] H.-B. Kong, C. Song, H. Park, and I. Lee, "A new beamforming design for MIMO AF relaying systems with direct link," *IEEE Trans. Commun.*, vol. 62, no. 7, pp. 2286–2295, Jul. 2014.

Polarized Channel Model for Body Area Networks Using Reflection Coefficients

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***Abstract*—A geometry-based propagation model is proposed for wideband polarized body area network (BAN) channels. Reflection coefficient functions (Γ-functions) are derived as a function of geometry-based channel modeling parameters for more precise modeling of polarized BAN channels than existing polarized channel models, which exclude the reflection coefficients or use constant-valued reflection coefficients. Conservation-of-polarization (CoP) propagation planes along with geometrical Γ-functions, which we call the *CoP-Γ plane methodology*, are used to derive the channel polarization functions. Statistical characteristics of polarized BAN channels such as the cross-polarization discrimination (XPD) and time–frequency correlation function (TF-CF) are provided based on this new CoP-Γ plane methodology. The model is validated with measurement data taken at 13 GHz and shown to be in excellent agreement.**

***Index Terms*—Body area networks (BANs), channel modeling, reflection coefficient.**

I. Introduction

Empirical studies for body-area-network (BAN) channels have been reported in the literature, including [1]–[4]. A path-loss model and other properties of the body-diffracted component have been studied in [5]–[9]. Few studies have considered depolarization in BAN channels [8], [10].

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