Limited Feedback Designs for Two-Way Relaying Systems with Physical Network Coding

Young-Tae Kim, Kwangwon Lee, Youngil Jeon, and Inkyu Lee

Abstract: **This paper considers a limited feedback system for two-way wireless relaying channels with physical network coding (PNC). For full feedback systems, the optimal structure with the PNC has already been studied where a modulo operation is employed. In this case, phase and power of two end node channels are adjusted to maximize the minimum distance. Based on this result, we design new quantization methods for the phase and the power in the limited feedback system. By investigating the minimum distance of the received constellation, we present a codebook design to maximize the worst minimum distance. Especially, for quantization of the power for 16-QAM, a new power quantization scheme is proposed to maximize the performance. Also, utilizing the characteristics of the minimum distance observed in our codebook design, we present a power allocation method which does not require any feedback information. Simulation results confirm that our proposed scheme outperforms conventional systems with reduced complexity.**

Index Terms: **Limited feedback, physical network coding, two way relaying.**

I. INTRODUCTION

As the demand for higher data rates in wireless systems is constantly on the rise, relay based wireless networks have been intensively studied with a lot of interest to extend the coverage or increase the system capacity [1]–[4]. Most relay systems are assumed to be operated in the half-duplex mode where a relay node (RN) cannot receive and transmit signal simultaneously. The capacity scaling law of the half-duplex mode relay is analyzed in [5], which shows that the pre-log factor 1/2 causes a substantial loss in spectral efficiency.

A two-way relaying protocol has been proposed to overcome such a spectral efficiency loss in the half-duplex mode system [6]–[9]. In the multiple access (MA) stage, two end nodes (EN) send their messages to a RN at the same time. Subsequently in the broadcast (BC) stage, the RN retransmits the information obtained in the MA stage to both terminals. By exploiting the knowledge of their own transmitted message, each user cancels self-interference and decodes the intended message. In this way, compared to one-way relay systems, the two-way protocol is able to achieve a spectral efficiency gain.

In order to process the received signals at the relay during the MA stage, various relaying strategies such as amplify-and-forward (AF) and decode-and-forward (DF) are introduced in [6], [10]–[12]. In the context of network coding for the two-way relay channel, analog network coding (ANC) [8], [13] and physical network coding (PNC) [14]–[16] have been proposed, where both schemes allow simultaneous transmission of two users by performing joint detection. While the ANC scheme shows advantages of simple implementation compared to the PNC scheme, the PNC enables us to reduce the effect of relay noise and thus enhance the performance. Such two-way denoise-and-forward (DNF) with PNC was well investigated in [16]–[20]. In this protocol, after the ENs transmit their data simultaneously to a RN, the RN performs detection without decoding, and the PNC encodes two data considering interference at the MA stage. A performance gain of the DNF protocol over other methods were derived in [16].

The exclusive-OR (XOR) operation is a popular choice for PNC [21], since the scheme with XOR requires a simple procedure and needs no feedback information. However, it suffers from a severe performance degradation due to the MA interference as shown in [17]. To improve the performance, a modulo operation can be considered for PNC instead of the XOR. Assuming perfect synchronization, the modulo operation can remove the MA interference perfectly, and achieves almost the same bit error rate performance as one-way systems with twice the spectral efficiency [22]. Our previous paper [23] proved that employing the modulo operation and utilizing precoding to make both ENs' channels equal allow us to obtain the optimal performance in terms of the minimum distance in the presence of full feedback information.

Recently, the paper [17] proposed adaptive network coding with a limited number of feedforward bits. This scheme determines network coding functions and relay mappers offline in advance, which maximize the minimum distance for all channel conditions. Then, the RN chooses the optimal network coding function depending on instantaneous channels, and sends the information of the selected network code to both ENs via a rate-limited feedforward link. This scheme exhibits good performance compared to the XOR system. However, every nodes need to know not only a lot of network coding functions and relay mappers, but also very sophisticated selection criteria sub-

This paper was handled by the EICs.

The material in this paper was presented in part at the IEEE ICC, Ottawa, Canada, June 2012. This work was supported by the National Research Foundation of Korea (NRF) funded by the Korea Government (MSIP) under Grant 2014R1A2A1A10049769.

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Digital object identifier 10.1109/JCN.2015.000084

ject to the channel condition. For example, for 16-QAM, information of at least 400 network coding functions and 15 relay mappers is required at each node. Thus, practical implementation may become difficult.

To address the problem, in this paper, we design a limited feedback scheme for two-way relaying channels with the modulo operation for PNC. We propose quantization methods for the optimal precoder which controls phase and power of channels for two ENs. Then the quantization of the phase and power is performed separately which results in a simple solution. We first investigate the minimum distance with respect to the phase or the power, and then illustrate the quantization method based on observations on the minimum distance. Especially, for 16-QAM, we propose a new quantization method for power control to maximize the worst minimum distance. From simulation results, we confirm that our proposed system outperforms conventional systems in with reduced complexity.

In addition, utilizing characteristics of the minimum distance observed in our codebook design, a power allocation scheme is proposed, which does not require any feedback information. Power allocation is determined according to the employed modulation level. In this scheme, we decouple the received constellation points for the power control. Simulation results show that our approach results in a 4 dB gain for Rician fading channels.

This paper is organized as follows: Section II describes the system model of two-way relay channels. In Section III, we propose a limited feedback scheme for the two-way relay systems with PNC based on the modulo operation. Also, in Section IV a power allocation method is presented which operates in an open loop environment. In Section V, the simulation results are presented comparing the proposed methods with the conventional relay schemes. Finally, the paper concludes with Section VI.

II. SYSTEM DESCRIPTION

In this section, we consider a single antenna two-way relay system with finite rate feedback. It is assumed that there is no direct link between the ENs A and B, and two ENs communicate with each other through a RN. Each EN exchanges the message $s_i \in \{0, 1, \cdots, M-1\}$ $(i = A, B)$ by employing M-ary quadrature amplitude modulation (M-QAM) where M is the number of symbols in the constellation. Denoting an M-QAM symbol mapper as $\mathcal{M}(\cdot)$, EN A and B transmit the symbols $x_A = \mathcal{M}(\int_{\mathcal{A}})$ and $x_B = \mathcal{M}(\int_{\mathcal{B}})$ with $E\{|x_i|^2\} = 1$ by multiplying v_A and v_B, respectively. The weight $v_i = \sqrt{P_i}e^{j\tilde{\theta}_i}$ consists of the power allocation factor $\sqrt{P_i}$ and the phase adjustment parameter. Fig. 1 depicts the schematic diagram for our system.

The received signal of the RN during the MA stage is expressed as

$$\begin{aligned} y_R &= h_A v_A x_A + h_B v_B x_B + z_R \\ &= \sqrt{P_A}e^{j\tilde{\theta}_A} h_A x_A + \sqrt{P_B}e^{j\tilde{\theta}_B} h_B x_B + z_R \quad (1) \end{aligned}$$

where h_i stands for the channel coefficient of the link between EN i and the RN and, z_R is the complex Gaussian noise with zero mean and variance σ^2. Here, h_A and h_B are represented as $h_A = |h_A|e^{j\theta_A}$ and $h_B = |h_B|e^{j\theta_B}$, respectively. It is assumed that $\sqrt{P_i}$ satisfies EN i's power constraint $P_{i,\max}$ as $P_i \leq P_{i,\max}$. Without loss of generality, we assume that only EN B adjusts the phase difference of both channels, i.e., $\tilde{\theta}_A = 0$ and $\tilde{\theta}_B = \theta_C$ where θ_C is defined as the phase adjustment by EN B.

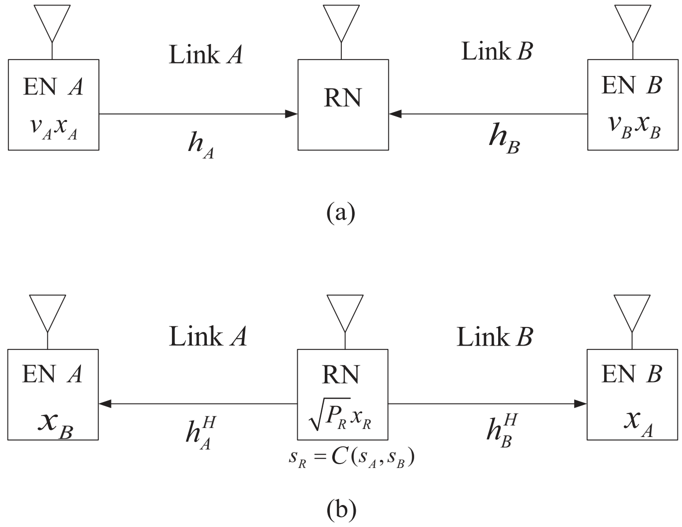

Fig. 1. Schematic diagram of a precoding scheme with PNC: (a) Multiple access stage and (b) broadcast stage.

Then, the received signal in (1) can be rewritten as

$$y_R = \sqrt{P_A}h_A x_A + \sqrt{P_B}e^{j\theta_C} h_B x_B + z_R. \quad (2)$$

After the RN receives the signal from ENs, s_A and s_B are estimated by maximum-likelihood (ML) detection as

$$\begin{aligned} (\hat{s}_A, \hat{s}_B) = \arg\min_{s_A, s_B} & |y_R - \sqrt{P_A}h_A\mathcal{M}(\int_{\mathcal{A}}) \\ & -\sqrt{P_B}e^{j\theta_C}h_B\mathcal{M}(\int_{\mathcal{B}})|^{\in}. \quad (3) \end{aligned}$$

Then, PNC using the denoising mapper $\mathcal{C}$ generates the network coded information $s_R = \mathcal{C}(\hat{\int}_{\mathcal{A}}, \hat{\int}_{\mathcal{B}})$.

At the broadcast (BC) stage, the RN transmits $x_R = \mathcal{M}_{\mathcal{R}}(\int_{\mathcal{R}})$ to both ENs, where $\mathcal{M}_{\mathcal{R}}(\cdot)$ indicates the symbol mapper at the relay. Then, the received signal at ENs can be written as

$$y_i = \sqrt{P_R}h_i x_R + z_i, \qquad \text{for } i = A \text{ and } B$$

where P_R denotes the transmitted power of the RN with relay power constraint $P_{R,\max}$ as $P_R \leq P_{R,\max}$ and $z_i \sim \mathcal{N}(\prime, \sigma^{\in})$ is the complex Gaussian noise. For simplicity, we assume that the channels of the MA and BC stages are the same due to reciprocity and the channel state information (CSI) is perfectly known at the receivers. Finally, EN i can extract the symbol s_R from the received signal, and then the other EN's symbol is obtained by using its own symbol s_i and the denoising mapper $\mathcal{C}$.

For the optimal BC stage, the RN transmits its symbol with full power ($P_R = P_{R_C}$) regardless of channel conditions. However, for the MA stage, the power P_A, P_B and the phase θ_C should be adjusted according to channel realizations. To address this, the paper [23] considered the minimum distance between the constellation points of (s_A, s_B), which determines the error probability at the MA stage. We can express the squared minimum distance as

$$d_{\min}^2 \triangleq \min_{\substack{\mathcal{C}(\int_A, \int_B) \\ \neq \mathcal{C}(\int_A', \int_B')}} \frac{\left|\sqrt{P_A} h_A \Delta x_A + \sqrt{P_B} e^{j\theta_C} h_B \Delta x_B\right|^2}{\sigma^2} \quad (4)$$

where $\Delta x_i \triangleq |\mathcal{M}(\int_\rangle) - \mathcal{M}(\int_\rangle')|$. Since the minimum distance of the MA stage cannot exceed that of link A or B due to MA interference, we have the following relation [18], [23]

$$d_{\min}^2 \leq \min\left(\frac{P_A|h_A|^2\Delta_{\min}^2}{\sigma^2}, \frac{P_B|h_B|^2\Delta_{\min}^2}{\sigma^2}\right) \triangleq d_{\text{min,opt}}^2 \quad (5)$$

where $\Delta_{\min} \triangleq \min \Delta x_A = \min \Delta x_B$ and $d_{\text{min,opt}}$ is defined as the maximum of $d_{\min}$.

We have already proven in [23] that for full feedback systems, choosing a modulo operation as the denoising mapper $\mathcal{C}(\cdot,\cdot)$ and adjusting the EN's power and the phase achieve the optimal performance, i.e., the minimum distance achieves the upper bound of (5). Defining the inphase and the quadrature of s_i as s_{iI} and s_{iQ}, respectively, s_{RI} and s_{RQ} are generated by the modulo operation as [22]

$$\begin{aligned} s_{Rk} &= \mathcal{C}_{\|}(\int_{A\|}, \int_{B\|}) \\ &= (s_{Ak} + s_{Bk}) \bmod \sqrt{M}, \qquad \text{for } k = I \text{ and } Q \end{aligned} \quad (6)$$

where $\mathcal{C}_{\|}(\cdot,\cdot)$ represents the denoising mapper for the inphase or the quadrature and $(\cdot) \bmod \sqrt{M}$ indicates the modulo operation of size $\sqrt{M}$. In (6), the modulo operation is separately applied to the inphase and quadrature components.

The optimal values for P_A, P_B and θ_C in terms of the minimum distance are given as [23]

$$P_A = P_{A_C},\ P_B = \frac{|h_A|^2}{|h_B|^2} P_{A_C} \ \text{ if } \ P_{A_C}|h_A|^2 \leq P_{B_C}|h_B|^2, \quad (7)$$

$$P_A = \frac{|h_B|^2}{|h_A|^2} P_{B_C},\ P_B = P_{B_C} \ \text{ if } \ P_{A_C}|h_A|^2 > P_{B_C}|h_B|^2,$$

$$\theta_C = \theta_A - \theta_B. \quad (8)$$

With the solutions of (7) and (8), $d_{\text{min,opt}}^2$ is achieved as $(\Delta_{\min}^2/\sigma^2)\min(P_{A,\max}|h_A|^2, P_{B,\max}|h_B|^2)$ in (5), and the two link channels become the same [23]. In fact, to obtain $d_{\text{min,opt}}$, unquantized feedback information is required for adjusting P_A, P_B and θ_C. However, in practice, limited feedback is normally adopted to reduce the overhead. In the following, we propose an efficient solution for limited feedback systems.

III. PROPOSED LIMITED FEEDBACK SCHEMES

In this section, we propose a limited feedback scheme for two-way relaying channels with PNC for QPSK and 16-QAM. Our goal is to present quantization methods for P_A, P_B, and θ_C based on the optimal values in (7) and (8) assuming that the modulo operation in (6) is adopted for the denoising mapper. Since it is difficult to design a quantization method which jointly optimizes both power control P_A and P_B and the phase adjustment θ_C, we address the problem by solving the power control and the phase adjustment separately.

A. QPSK

For QPSK, we first consider quantization for the phase by assuming that the optimal power control (7) is applied. In this case, the minimum distance between (s_A, s_B) and (s_A', s_B') becomes

$$d_{\min}^2 = \min_{\substack{\mathcal{C}(\int_A, \int_B) \\ \neq \mathcal{C}(\int_A', \int_B')}} \frac{g^2}{\sigma^2}\left|\Delta x_A + e^{j(\theta_C + \theta_B - \theta_A)}\Delta x_B\right|^2 \quad (9)$$

where $g = \sqrt{P_A}|h_A| = \sqrt{P_B}|h_B| \in \mathbb{R}$ denotes the channel gain of both ENs. Note that the optimal power control (7) results in the same channel gain.

During the MA stage, QPSK symbols from both ENs are superposed at the relay and this results in 16 possible points of (s_A, s_B). Thus, the number of candidates of $d_{\min}$ in (9) equals $(16 \times 15)/2 = 120$. Due to symmetry, however, it can be smaller since there are several signal pairs which generate the same distance. It was derived in [19] that this number can be reduced to 12 by choosing XOR as the denoising mapper. Since the XOR operation is equivalent to the modulo operation for QPSK, we can decrease the number using the method in [19]. In addition, it can be easily shown that we can further reduce the candidate number to 7 by utilizing the fact that both channel gains are equal. As a result, a set of $(\Delta x_A, \Delta x_B)$ for these candidates is given as

$$\begin{bmatrix} \frac{\Delta x_A}{\sqrt{2}} \\ \frac{\Delta x_B}{\sqrt{2}} \end{bmatrix} \in \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 1 \\ 1+j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}, \begin{bmatrix} j \\ 1+j \end{bmatrix}, \begin{bmatrix} 1 \\ -1+j \end{bmatrix}, \begin{bmatrix} j \\ 1-j \end{bmatrix} \right\}. \quad (10)$$

The minimum distance of these 7 candidates varies with $\theta_C + \theta_B - \theta_A$ in (9). Note that $\theta_C = \theta_A - \theta_B$ maximizes the minimum distance as plotted in Fig. 2(a). If θ_C is set incorrectly as $\theta_C = \theta_A - \theta_B + \pi/4$, the minimum distance becomes smaller as shown in Fig. 2(b). For more descriptions on the effect of θ_C, we plot the minimum distance ratio $R \triangleq d_{\min}/d_{\text{min,opt}}$ with respect to $\Theta(\theta_C) \triangleq \theta_C + \theta_B - \theta_A$ in Fig. 3 by using (10). When $R = 1$, we have the optimal value. It is interesting to check in this figure that not only $\Theta(\theta_C) = 0$ in (8) but also $\Theta(\theta_C) = \pi$ maximize the minimum distance. Also, the minimum distance ratio R has a period π, which allows us to save one bit in feedback, since the range $[0,\ \pi]$ can be quantized instead of the range $[0,\ 2\pi]$. Then, we uniformly quantize this range for simplicity.

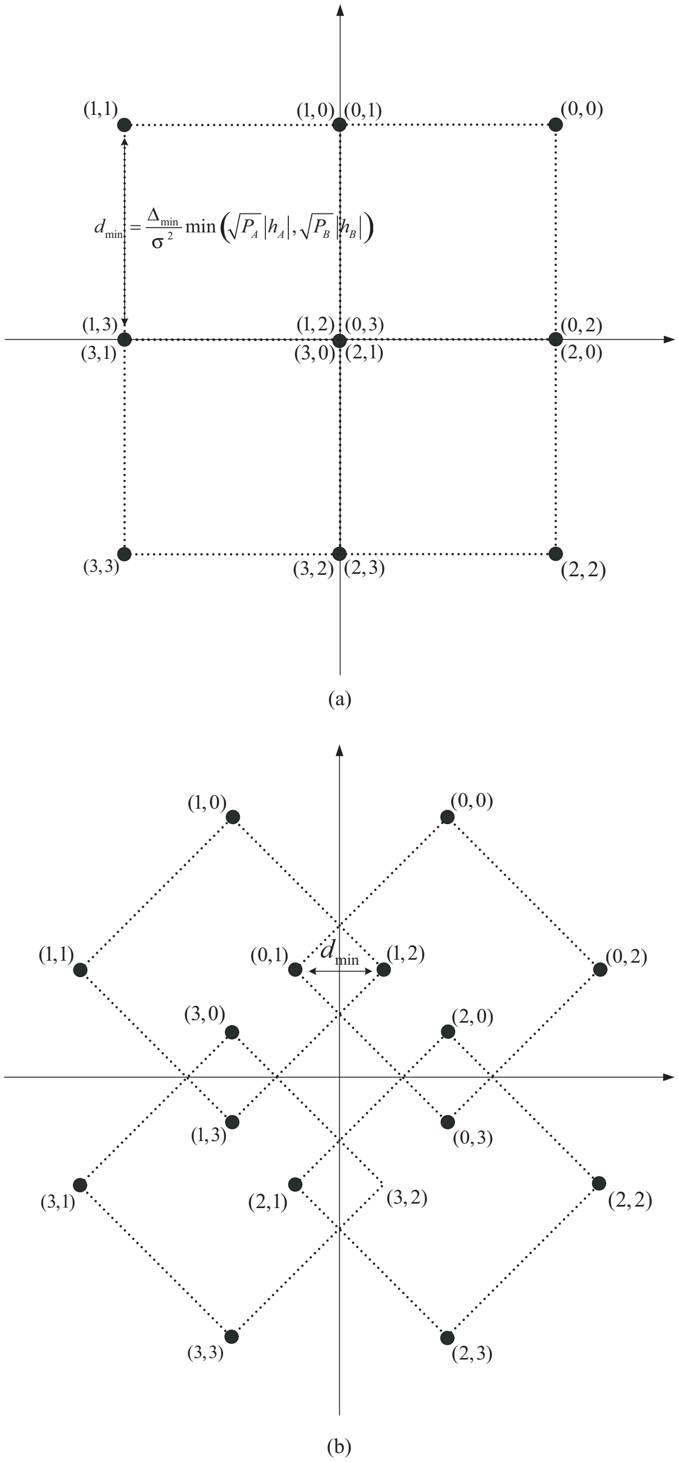

Fig. 2. Received constellation points with different phase assuming perfect power control: (a) $\theta_C + \theta_B - \theta_A = 0$ and (b) $\theta_C + \theta_B - \theta_A = \pi/4$.

As a result, the codebook for θ_C is determined on $[0,\ \pi]$ as

$$\mathrm{CB}_\theta = \left\{0, \frac{\pi}{2^{\mathrm{FB}_\theta}}, \cdots, (2^{\mathrm{FB}_\theta} - 1)\frac{\pi}{2^{\mathrm{FB}_\theta}}\right\} \tag{11}$$

where FB_θ stands for the number of feedback bits for θ_C. Then,

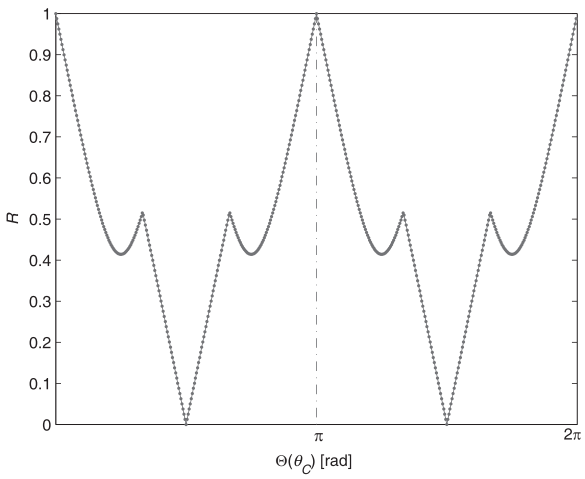

Fig. 3. The minimum distance ratio with respect to $\Theta(\theta_C)$ for QPSK.

θ_C in limited feedback systems is selected from the codebook CB_θ. Note that the minimum distance is not a monotonous function of $\Theta(\theta_C)$. Thus, the selection of θ_C should be performed not to approach $\theta_A - \theta_B$ but to minimize directly the minimum distance as

$$\hat{\theta}_C = \arg\max_{\Theta(c_i)} R, \qquad \text{for } c_i \in \mathrm{CB}_\theta \tag{12}$$

by using the precalculated ratio in Fig. 3. As the number of FB_θ increases, the selection of θ_C to be as close as possible to $\theta_A - \theta_B$ also leads to the same result as the remaining phase error is close to 0.

Next, we consider the power control for limited feedback systems by assuming that the optimal phase compensation (8) is applied. In this case, we have already proven in [23] that beside the solution (7), full power transmission ($P_A = P_{A,\max}$ and $P_B = P_{B,\max}$) also maximizes the minimum distance for QPSK. Even though this is not power efficient and a further power reduction is possible with feedback, we only focus on a simple feedback system in this paper, and thus we assume that ENs transmit their symbols with full power without any feedback.

B. 16-QAM

Similar to QPSK, for 16-QAM, we separately design a quantization method for the power control and the phase adjustment. First, we consider the phase adjustment assuming that the optimal power control in (7) is adopted. Then, among 256 possible points of (s_A, s_B), the number of candidates of the minimum distance becomes 195 by applying the method in [19]. Using these candidates, the minimum distance ratio R is plotted in Fig. 4. Note in this figure that the function R is symmetric around π, but is not periodic. Thus, unlike the case of QPSK, the system with 16-QAM has only one solution $\theta_C = \theta_A - \theta_B$ which maximizes the minimum distance. Thus,

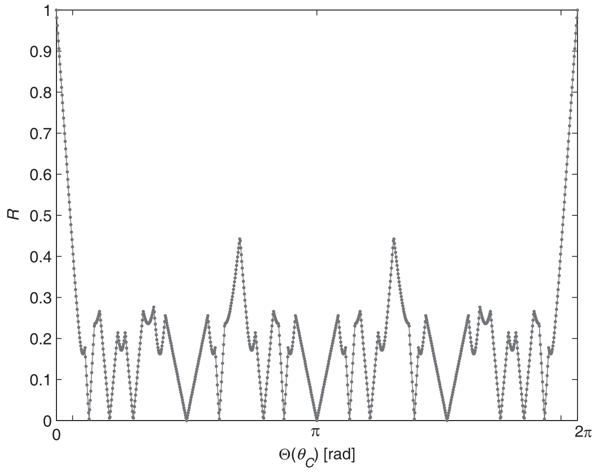

Fig. 4. The minimum distance ratio R with respect to $\Theta(\theta_C)$ for 16-QAM.

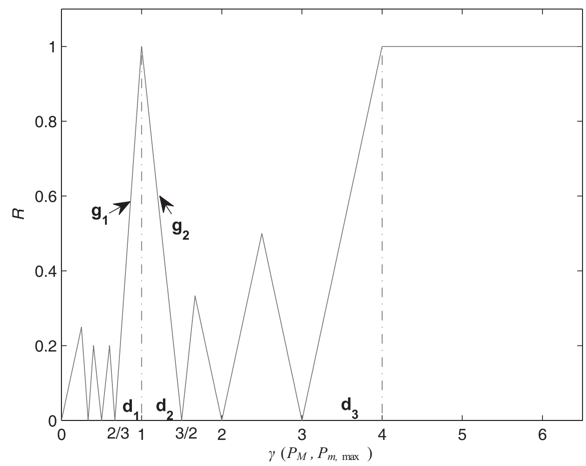

Fig. 5. The minimum distance ratio R with respect to γ for 16-QAM.

the codebook is uniformly quantized from 0 to 2π as

$$\mathrm{CB}_\theta = \left\{0, \frac{2\pi}{2^{\mathrm{FB}_\theta}}, \cdots, (2^{\mathrm{FB}_\theta} - 1)\frac{2\pi}{2^{\mathrm{FB}_\theta}}\right\}.$$

Then, θ_C is selected from CB_θ by (12).

Next, we quantize the optimal power value of ENs in (7). For simple notations, the node which has a larger channel gain is denoted as $M \triangleq \arg\max_{i\in\{A,B\}} \sqrt{P_{i,\max}}|h_i|$, and the node which has a smaller channel gain is represented as $m \triangleq \arg\min_{i\in\{A,B\}} \sqrt{P_{i,\max}}|h_i|$. In addition, we define the ratio of two channel gains as $\gamma(P_M, P_m) \triangleq \sqrt{P_M}|h_M|/\sqrt{P_m}|h_m|$. Assuming that phase adjustment is perfectly done ($\theta_C = \theta_A - \theta_B$), the minimum distance between constellation points in (4) can be expressed as

$$d_{\min}^2 = \min_{\substack{\mathcal{C}(\int_\mathcal{M}, \int_\updownarrow) \\ \neq \mathcal{C}(\int'_\mathcal{M}, \int'_\updownarrow)}} \frac{P_m|h_m|^2}{\sigma^2}\left|\Delta x_m + \gamma(P_M, P_m)\Delta x_M\right|^2. \quad (13)$$

Unlike QPSK, since the minimum distance with full power transmission at both ENs is no longer optimal, a proper quantization method for power control is required.

The optimal power control (7) indicates that in order to maximize (13), the maximum power should be adopted for the node with a smaller channel gain ($P_m^* = P_{m,\max}$), and the power of the node with a larger channel gain should be reduced to match the smaller channel gain as

$$P_M^* = \frac{P_{M,\max}}{\gamma^2(P_{M,\max}, P_{m,\max})}. \quad (14)$$

With this solution, both channel gains become equal, and thus this optimal power set leads to the optimal value $\gamma(P_M^*, P_{m,\max}) = 1$ in (13).

First of all, for limited feedbacks, we need one bit determining which node has a smaller channel gain. The power for the node with a smaller channel gain is set to full power $P_{m,\max}$. After that, $\gamma(P_{M,\max}, P_{m,\max})$ needs to be quantized to obtain P_M in (14). From the codebook, P_M should be chosen to be close to the optimal value $\gamma(P_M^*, P_{m,\max}) = 1$. Before designing a selection rule for P_M, we investigate the effect of $\gamma(P_M, P_{m,\max}) = 1$ on the minimum distance. If we assume that perfect phase adjustment in (8) is employed, we can reduce the number of candidates of $d_{\min}$ to 8 employing the method in [19] as

$$\begin{bmatrix} \frac{\Delta x_m}{\sqrt{10}} \\ \frac{\Delta x_M}{\sqrt{10}} \end{bmatrix} \in \left\{ \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \end{bmatrix} \right\}. \quad (15)$$

Using these candidates, we can calculate the minimum distance ratio R according to $\gamma(P_M, P_{m,\max})$.

In Fig. 5, R is plotted with respect to $\gamma(P_M, P_{m,\max})$ by controlling P_M when P_m is set to the full power $P_{m,\max}$. In this figure, we can confirm that if we have $1 \leq \gamma(P_{M,\max}, P_{m,\max}) \leq 4$, P_M should be reduced so that $\gamma(P_M, P_{m,\max}) = 1$ by using (14). Also, it is interesting to note in this figure that when $\gamma(P_{M,\max}, P_{m,\max}) \geq 4$, the maximum value of the minimum distance is always achieved. Thus, in our codebook design, we consider both of $\gamma(P_M^*, P_{m,\max}) = 1$ and $\gamma(P_M, P_{m,\max}) \geq 4$ maximize the minimum distance. It is important to note that P_M satisfying $\gamma(P_M, P_{m,\max}) \geq 4$ can also be a solution, since only the region of $1 \leq \gamma(P_{M,\max}, P_{m,\max}) \leq 4$ is enough to design codebooks based on the observation.

Now, we design the selection criteria for P_M. Based on (14), P_M is determined by $\gamma(P_{M,\max}, P_{m,\max})$. In (14), it would be natural to quantize $1/\gamma^2(P_{M,\max}, P_{m,\max})$ for limited feedback systems. However, for a simple approach, we quantize $\gamma(P_{M,\max}, P_{m,\max})$ in this paper. Then, our quantization model for $\gamma(P_{M,\max}, P_{m,\max})$ can be expressed as

$$\hat{\gamma}(P_{M,\max}, P_{m,\max}) = \bar{\gamma}_i \text{ if } a_{i-1} \leq \gamma(P_{M,\max}, P_{m,\max}) < a_i \quad (16)$$

for $i = 1, 2, \cdots, N$, where $N \triangleq 2^{\mathrm{FB}_P - 1}$ is the size of the codebook and FB_P indicates the number of feedback bits for the power control. Here, one bit has already been assigned for identifying the node M or m, and thus the number of quantized levels is 2^{FB_P-1}. In (16), a_0 is equal to 1 since $\gamma(P_{M,\max}, P_{m,\max}) \geq 1$.

Next, we decide the quantization levels $\bar{\gamma}_i$ and the range of a_i. First, we set $\bar{\gamma}_1 = 1$ for $a_0 \leq \gamma(P_{M,\max}, P_{m,\max}) \leq a_1$ and $\gamma(P_{M,\max}, P_{m,\max}) \geq a_{2^{\mathrm{FB}_P-1}}$ ($a_{2^{\mathrm{FB}_P-1}} \leq 4$), which value means just full power transmission. This is because for both cases of $\gamma(P_M^*, P_{m,\max}) = 1$ and $\gamma(P_M, P_{m,\max}) \geq 4$ needs full power to maximize the minimum distance, as mentioned above. In the following, we obtain $\bar{\gamma}_i$ and a_i to maximize the worst minimum distance ratio after the selected power is applied.

In order to identify $\bar{\gamma}_i$ and a_i, we adopt the following three conditions:

$$\begin{aligned}
&1.\quad \bar{\gamma}_i - a_{i-1} : a_i - \bar{\gamma}_i = 2:3 \quad \text{for } i = 2, 3, \cdots, N \\
&2.\quad \tfrac{5(a_1-a_0)}{3\bar{\gamma}_1} = \tfrac{a_2-a_1}{\bar{\gamma}_2} = \tfrac{a_3-a_2}{\bar{\gamma}_3} = \cdots = \tfrac{a_N-a_{N-1}}{\bar{\gamma}_N} \\
&3.\quad a_1 - a_0 : 4 - a_N = 1:2
\end{aligned} \tag{17}$$

We first prove the condition 1 which determines the optimal $\bar{\gamma}_i$ given a_{i-1} and a_i. After the selected power is employed, the minimum distance between the range $a_{i-1} \leq \gamma(P_{M,\max}, P_{m,\max}) < a_i$ in (16) is changed to one between $a_{i-1}/\bar{\gamma}_i \leq \gamma\left(P_{M,\max}/\bar{\gamma}_i^2, P_{m,\max}\right) < a_i/\bar{\gamma}_i$.

In order for the optimal value $\gamma\left(P_{M,\max}/\bar{\gamma}_i^2, P_{m,\max}\right) = 1$ to be located between $a_{i-1}/\bar{\gamma}_i$ and $a_i/\bar{\gamma}_i$ in Fig. 5, $\bar{\gamma}_i$ is selected between a_{i-1} and a_i as $a_{i-1} \leq \bar{\gamma}_i \leq a_i$. If the boundary points of $\gamma\left(P_{M,\max}/\bar{\gamma}_i^2, P_{m,\max}\right)$ are assumed to be $a_{i-1}/\bar{\gamma}_i \in [2/3,\ 1]$ and $a_i/\bar{\gamma}_i \in [1,\ 3/2]$, the minimum distance ratios $R_0\left(a_{i-1}/\bar{\gamma}_i\right)$ and $R_0\left(a_i/\bar{\gamma}_i\right)$ vary with different slopes g_1 and g_2 in Fig. 5, respectively, where $R_0(\alpha)$ is denoted as the minimum distance ratio with $\gamma(P_M, P_{m,\max}) = \alpha$. Here, the slopes $g_1 = |-2 + 3\gamma(P_M, P_{m,\max})|$ and $g_2 = |3 - 2\gamma(P_M, P_{m,\max})|$ are obtained by plugging $\left[\Delta x_m/\sqrt{10}, \Delta x_M/\sqrt{10}\right]^T = [-4\ \ 6]^T$ and $[6\ \ -4]^T$ in (15) into (13), respectively. Then, it can readily be checked that the smaller value of these two boundary points $R_0\left(a_{i-1}/\bar{\gamma}_i\right)$ and $R_0\left(a_i/\bar{\gamma}_i\right)$ would be a candidate of the worst minimum distance ratio.

Now, we choose $\bar{\gamma}_i$ between a_{i-1} and a_i to maximize this minimum value as

$$\bar{\gamma}_i = \arg \max_{a_{i-1} \leq \bar{\gamma}_i \leq a_i} \min\left\{R_0\left(a_{i-1}/\bar{\gamma}_i\right), R_0\left(a_i/\bar{\gamma}_i\right)\right\}. \tag{18}$$

We can see that $R_0\left(a_{i-1}/\bar{\gamma}_i\right)$ and $R_0\left(a_i/\bar{\gamma}_i\right)$ are a monotonically decreasing and a monotonically increasing function of $\bar{\gamma}_i$, respectively. Therefore, for maximizing the worst minimum distance ratio, $\bar{\gamma}_i$ should satisfy $R_0\left(a_{i-1}/\bar{\gamma}_i\right) = R_0\left(a_i/\bar{\gamma}_i\right)$. In order for these values to be equal, we have $\bar{\gamma}_i - a_{i-1} : a_i - \bar{\gamma}_i = d_1 : d_2 = 2:3$ from the slope g_1 and g_2 as in the condition 1. We can check that 3 bits ($N = 4$) are sufficient to satisfy the above assumptions $2/3 \leq a_{i-1}/\bar{\gamma}_i < 1$ and $1 \leq a_i/\bar{\gamma}_i < 3/2$.

Next, we check the condition 2. This condition means that after the selected power $\bar{\gamma}_i$ is employed, the length of the range of $a_{i-1}/\bar{\gamma}_i \leq \gamma\left(P_{M,\max}/\bar{\gamma}_i^2, P_{m,\max}\right) < a_i/\bar{\gamma}_i$ for $d_{\min}$ becomes the same for all i. If each range has different length, the longest range would have the worst minimum distance as can be checked in Fig. 5. Note that since $a_0 = \bar{\gamma}_1 = 1$, $5/3$ is multiplied for the length of the first range $(a_1 - a_0)/\bar{\gamma}_1$ to become identical with that of other ranges. Finally, similar to the condition 1, the condition 3 can be proved by observing $d_2 : d_3 = 1:2$ in Fig. 5, where $d_2 = a_1 - a_0$ and $d_3 = 4 - a_N$.

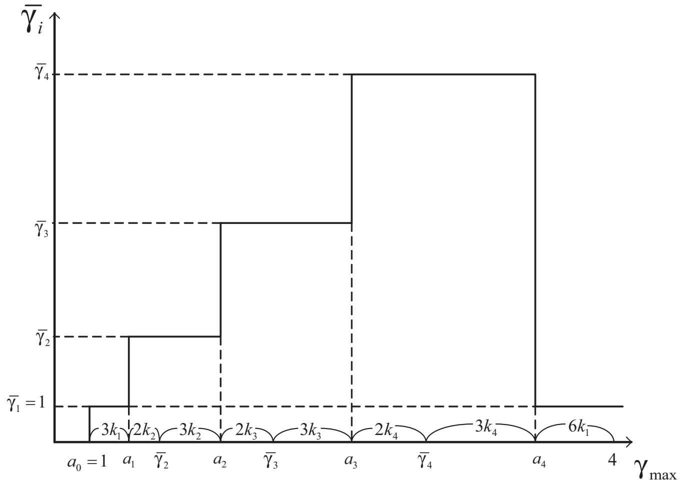

Fig. 6. Quantization levels of $\gamma_{\max}$ for power control with $N = 4$.

The above three conditions can generally be formulated as the following N equations

$$\begin{aligned}
&k_1(3k_1 + 2k_2 + 1) - k_2 = 0 \\
&k_1(3k_1 + 5k_2 + 2k_3 + 1) - k_3 = 0 \\
&\qquad\vdots \\
&k_1(3k_1 + 5\sum_{i=2}^{N-1} k_i + 2k_N + 1) - k_N = 0 \\
&9k_1 + \sum_{i=2}^{N} k_i - 3 = 0
\end{aligned} \tag{19}$$

where k_i is defined as $k_i \triangleq (a_i - a_{i-1})/5$ for $i = 2, 3, \cdots, N$ with $k_1 \triangleq (a_1 - 1)/3$. Since there are N variables and N equations, we can find k_i by a Newton method, and $\bar{\gamma}_i$ and a_i can be computed from k_i. Fig. 6 illustrates an example with $\mathrm{FB}_P = 3$.

IV. POWER ALLOCATION WITHOUT FEEDBACK

In Section III, limited feedback systems for two-way relaying channels with PNC to maximize the worst minimum distance have been studied. From the results of 16-QAM, we can make some observations on the minimum distance. As shown in Fig. 5, the minimum distance of 16-QAM in (13) can be maximized for $\gamma(P_M, P_{m,\max}) \geq 4$, i.e., $\sqrt{P_M}|h_M| \geq 4\sqrt{P_{m,\max}}|h_m|$, assuming that the difference between the phases of both channels is compensated ($\theta_A = \theta_B + \theta_C$). These results from the fact that the constellation points are not overlapped for $\sqrt{P_M}|h_M| \geq 4\sqrt{P_{m,\max}}|h_m|$ as in Fig. 7(a). Here, we denote $d_{\min,m}$ and

$d_{\min,M}$ as the minimum distance between constellation points of EN m and M, respectively. In this figure, the minimum distance $d_{\min,m}$ between the constellation points within a circle is determined as $d_{\min,m} = \sqrt{P_{m,\max}}|h_m|\Delta_{\min}$, and the minimum distance $d_{\min,M}$ between the circles are obtained by $d_{\min,M} = \sqrt{P_M}|h_M|\Delta_{\min}$. Thus, for the condition $\sqrt{P_M}|h_M| \geq 4\sqrt{P_{m,\max}}|h_m|$, the points within a circle are decoupled with those of other circles, and then the maximum of the minimum distance $d_{\min,m}$ can be achieved.

In this section, by using this decoupling characteristic of the constellation points, we examine a power allocation method which does not require any feedback information. This method focuses on decoupling of constellation points. To simplify our derivation, we tentatively assume that h_i is a Rician fading channel with a high K-factor. Then, the received signal of the RN during the MA stage in (2) can be rewritten as

$$y_R = \sqrt{P_A}e^{j\theta_A}x_A + \sqrt{P_B}e^{j\theta_B}x_B + z_R. \tag{20}$$

In (20), since we consider a system without any CSI feedback, P_A and P_B cannot be adjusted according to instant channel conditions. Instead, we set P_A and P_B in advance with no feedback. In this case, the minimum distance between the constellation points can be written as

$$d_{\min}^2 = \min_{\mathcal{C}(\mathcal{S}_A,\mathcal{S}_B)\neq\mathcal{C}(\mathcal{S}'_A,\mathcal{S}'_B)} \frac{P_A}{\sigma^2}\left|e^{j\theta_A}\Delta x_A + \frac{P_B}{P_A}e^{j\theta_B}\Delta x_B\right|^2. \tag{21}$$

Now, we first choose the node M and m according to EN i's power constraint $P_{i,\max}$. Without loss of generality, we select EN A to be node M if $P_{A,\max}$ is greater than or equal to $P_{B,\max}$. Then, P_M is set to full power $P_{M,\max}$, and P_m is reduced to decouple the constellation points. By the reduced P_m, the achieved minimum distance would also decrease, and thus it can degrade the system performance. Nevertheless, we make the circles non-overlapping, since it is difficult to adjust the overlapped constellation without any feedback.

Note that if the phase difference is zero, i.e., $\theta_A = \theta_B$, it is sufficient to set $\sqrt{P_{M,\max}} \geq 4\sqrt{P_m}$ as in Fig. 7(a) so that the circles are not overlapped. In (21), however, since the phase difference cannot be compensated in open loop systems, the above power condition is not sufficient for decoupling the circles. In this case, the points within a circle are rotated according to the phase difference. Then, when $\sqrt{P_{M,\max}} = 4\sqrt{P_m}$, some points within a circle can coincide with the points within other circles depending on $\theta_A - \theta_B$. Hence, $\sqrt{P_{M,\max}}$ must be larger than $3\sqrt{2P_m}$ as shown in Fig. 7(b). Also, to make the space between the circles larger than the minimum distance $d_{\min,m}$, the distance between the circles should be larger than $\left(3\sqrt{2}+1\right)\sqrt{P_m}\Delta_{\min}$. As a result, we have a new condition $\sqrt{P_{M,\max}} \geq \left(3\sqrt{2}+1\right)\sqrt{P_m}$ for 16-QAM systems.

Extending to general M-QAM modulation cases ($M \geq 4$), $P_{M,\max}$ and P_m can be related as

$$P_m \leq \frac{P_{M,\max}}{\left(\sqrt{2}(\sqrt{M}-1)+1\right)^2} \tag{22}$$

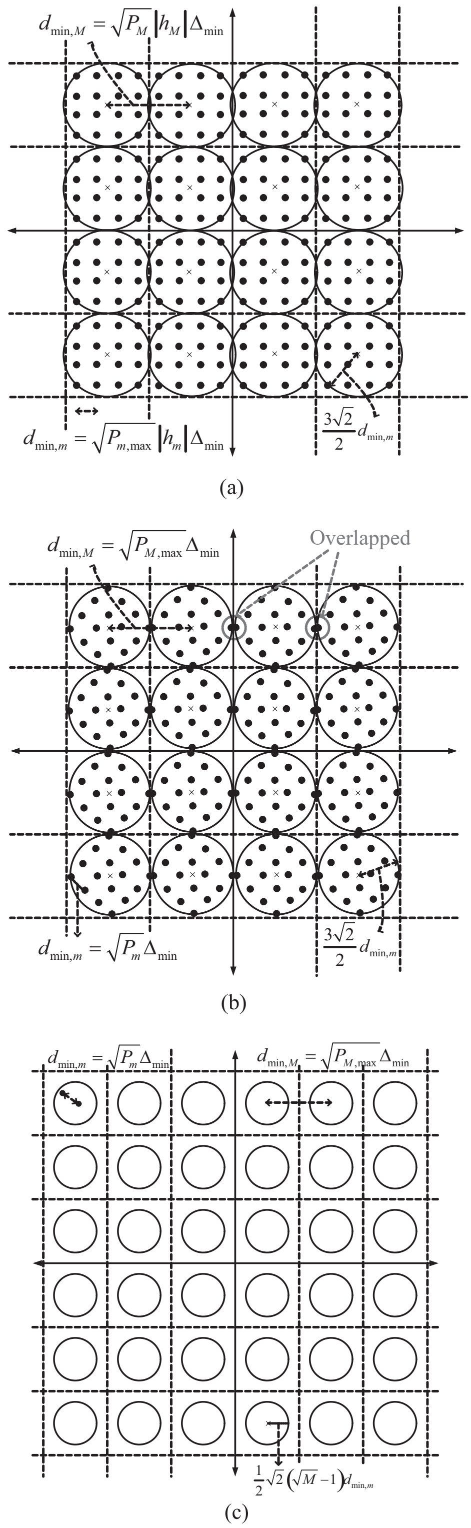

Fig. 7. The decoupled constellation points on the effective channel: (a) 16-QAM with $\sqrt{P_M}|h_M| = 4\sqrt{P_{m,\max}}|h_m|, \theta_C + \theta_B - \theta_A = 0$, (b) 16-QAM with $\sqrt{P_{m,\max}} = 3\sqrt{2P_M}$, and (c) M-QAM with $\sqrt{P_{m,\max}} \geq \sqrt{2}\left((\sqrt{M}-1)+1\right)^2\sqrt{P_M}$.

as in Fig. 7(c). This comes from the fact that the radius of a circle for M-QAM is $(1/2)\sqrt{2}(\sqrt{M}-1)\sqrt{P_m}\Delta_{\min}$ and thus the distance between the circles should be larger than $(\sqrt{2}(\sqrt{M}-1)+1)\sqrt{P_m}\Delta_{\min}$. Then, to maximize $d_{\min,m}$, the power allocation method is determined as

$$P_M = P_{M,\max} \text{ and } P_m = \frac{P_{M,\max}}{\left(\sqrt{2}(\sqrt{M}-1)+1\right)^2}. \quad (23)$$

To make a fair comparison to the equal power case, we present the sum power constraint case as $P_T = P_{A,\max} + P_{B,\max}$. By using the relation (23), the power allocation for the sum power constraint case becomes

$$P_A = \frac{P_T}{\left(\sqrt{2}(\sqrt{M}-1)+1\right)^2+1}$$

and

$$P_B = \frac{\left(\sqrt{2}(\sqrt{M}-1)+1\right)^2 P_T}{\left(\sqrt{2}(\sqrt{M}-1)+1\right)^2+1}. \quad (24)$$

Here, P_A and P_B are exchangeable. Note that the results (23) and (24) are derived assuming that the Rician factor K is high. We will show in the following section that our proposed scheme works well with finite K.

V. SIMULATION RESULTS

In this section, we evaluate the proposed limited feedback system in terms of the end-to-end throughput. We assume that h_A and h_B are frequency-flat Nakagami-Rice fading channels and the data packet frame has 256 uncoded symbols, which are the same setting as in [17]. Then, the throughput R_t is calculated as

$$R_t = \log_2 M(1-P_e)$$

where P_e denotes the packet error rate. For simplicity, all the maximum power is assumed to be the same ($P_{A,\max} = P_{B,\max} = P_{R,\max} = P$), and then the average SNR is defined as P/σ^2.

In Fig. 8, we plot the throughput of the proposed schemes and adaptive network coding (Ad-NC) with feedforward information introduced in [17] for QPSK. A Rician factor K is set to 10 dB as in [17]. The Ad-NC optimizes the denoising mapper $\mathcal{C}$ with 3 feedforward bits. In our scheme, the feedback bits are used only for phase compensation as discussed in Section III, since no power control is necessary for QPSK. First, it is remarkable to check that the performance of the proposed scheme with 3 bits is almost the same as unquantized optimal systems. Also, we can see that the proposed scheme with 3 bits achieves a 2 dB gain at 1.8 bps/Hz over Ad-NC. Moreover, the proposed scheme with $\text{FB}_\theta = 2$ is still superior to Ad-NC with a smaller overhead.

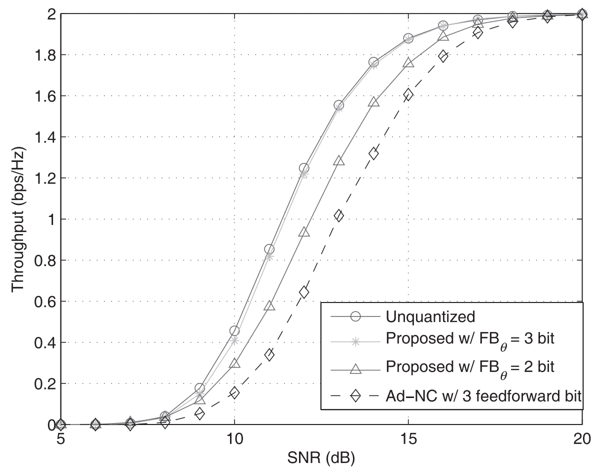

Fig. 8. Throughput of PNC systems with QPSK in Nakagami-Rice fading channels.

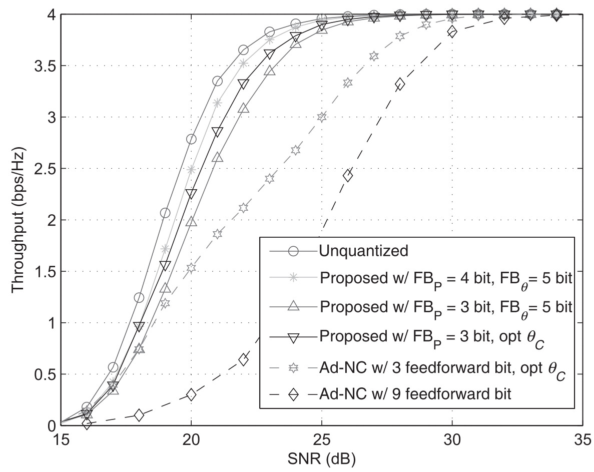

Fig. 9. Throughput of PNC systems with 16-QAM in Nakagami-Rice fading channels.

Fig. 9 depicts the simulation results of the proposed schemes with 16-QAM. Since Ad-NC in [17] adopts 9 bits for 16-QAM, the proposed scheme is set to employ the maximum of 9 feedback bits in total for fair comparison. For systems with 16-QAM, gains become greater compared to the QPSK case. First, it can be checked that the proposed scheme with total 9 bits has near optimal performance. Also, we can see in the plot that the proposed schemes exhibit huge gains over Ad-NC. Compared to Ad-NC which adaptively selects 400 network codes and 15 different QAM constellations at the RN, our scheme adopts just a modulo operation and 16-QAM, and thus requires much lower complexity at the BC stage. In addition, assuming perfect phase adjustment (opt θ_C), our scheme shows better performance than Ad-NC. Also, unlike our scheme, Ad-NC with 9 feedforward bits has quite inferior performance compared to Ad-NC with 3 feedforward bits and optimal θ_C. From

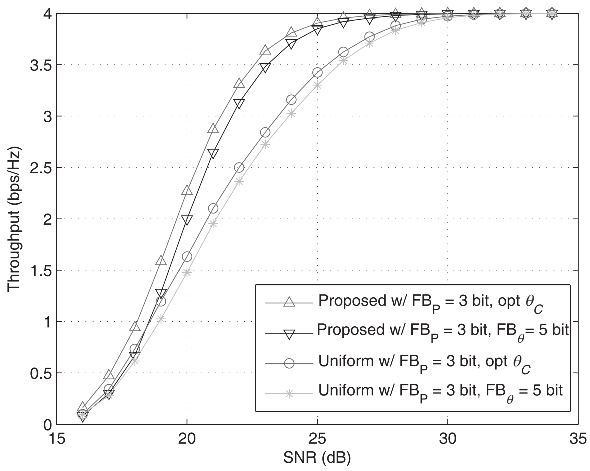

Fig. 10. Comparison between the proposed feedback quantization and the uniform quantization.

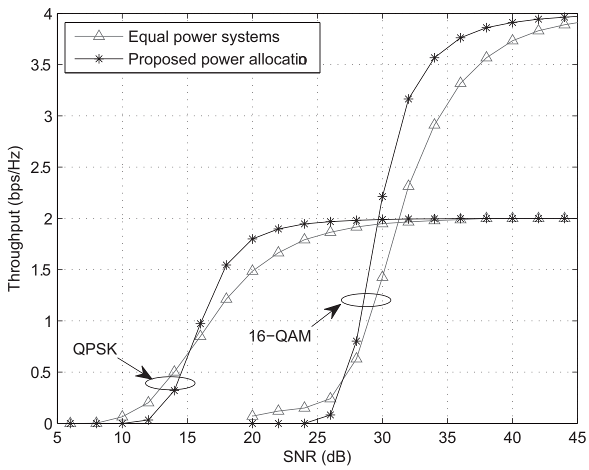

Fig. 11. Throughput of open loop systems with sum power constraints in Nakagami-Rice fading channels.

this result, we can see that the compensation of the phase difference in Ad-NC is not as efficient as in our proposed scheme.

Both Ad-NC and our proposed scheme has the same objective of maximizing the minimum distance. However, Ad-NC contains some restrictions on clustering constellation points to maximize the minimum distance. Due to the restriction that network codes made by clustering need to be decoded at each node, some points in Ad-NC must not be clustered, although the clustering may result in the larger minimum distance. This leads to the increased feedback overhead compared to the proposed one.

In order to demonstrate the efficiency of our quantization method, Fig. 10 compares the throughput performance with a uniform quantization method, where $\bar{\gamma}_i$ is uniformly distributed for $1 \leq \gamma(P_{M,\max}, P_{m,\max}) < 4$ in (16). It can be shown from Fig. 10 that our proposed method provides about 3 dB gains compared to the uniform quantization at 3.6 bps/Hz. Thus, we confirm that it is important to optimize the quantization based on the minimum distance for the power control in two-way PNC systems.

Fig. 11 exhibits the performance of our proposed scheme for open loop systems. For fair comparison, we set $P_T = 2P$. The allocated power in (24) is adopted depending on the employed modulation level. Even though no feedback information is assumed in our proposed allocation method, the proposed power allocation method provides 4 dB gains at 90% of the maximum throughput over the equal power systems for QPSK and 16-QAM. We can notice that decoupling the constellation points improves the performance in terms of the throughput. However, at low SNR, the performance of the equal power systems bcomes better than that of our scheme, because power degradation of P_A caused by the power control affects the performance more than the decoupled constellation points. In contrast, decoupling of the constellation points gets more important at high SNR. Note that the proposed power allocation method works well with finite K, although the results have been derived assuming a high K. From the simulation results, we confirm the efficacy of the proposed systems in two-way relaying channels with PNC.

VI. CONCLUSION

In this paper, we have designed a limited feedback scheme for two-way relay systems with PNC based on the optimal precoding in [23] which utilizes a modulo operation for PNC. In our scheme, new quantization methods are proposed for the phase and power of the channels. By investigating the minimum distance, we have derived quantization of the phase and power. For 16-QAM, we design a power quantization method to maximize the worst minimum distance ratio. In addition, a power allocation method for decoupling of the constellation points is proposed, which does not require any feedback information. The simulation results confirm that our proposed schemes exhibit large gains compared with conventional schemes. The limited feedback scheme for general modulation cases such as 64-QAM can be designed by using the properties described in this paper. A limited feedback scheme for multiple streams with multiple antenna relay systems remains as an interesting future work. Also, study for the relation between the proposed algorithm and the estimation error (or delayed feedback, imperfect channel reciprocity) can be researched for future work.

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