Robust Transceiver Designs in Multiuser MISO Broadcasting with Simultaneous Wireless Information and Power Transmission

Zhengyu Zhu, Zhongyong Wang, Kyoung-Jae Lee, Zheng Chu, and Inkyu Lee

Abstract: **In this paper, we address a new robust optimization problem in a multiuser multiple-input single-output broadcasting system with simultaneous wireless information and power transmission, where a multi-antenna base station (BS) sends energy and information simultaneously to multiple users equipped with a single antenna. Assuming that perfect channel-state information (CSI) for all channels is not available at the BS, the uncertainty of the CSI is modeled by an Euclidean ball-shaped uncertainty set. To optimally design transmit beamforming weights and receive power splitting, an average total transmit power minimization problem is investigated subject to the individual harvested power constraint and the received signal-to-interference-plus-noise ratio constraint at each user. Due to the channel uncertainty, the original problem becomes a homogeneous quadratically constrained quadratic problem, which is NP-hard. The original design problem is reformulated to a relaxed semidefinite program, and then two different approaches based on convex programming are proposed, which can be solved efficiently by the interior point algorithm. Numerical results are provided to validate the robustness of the proposed algorithms.**

Index Terms: **Energy harvesting, multiuser multiple-input single-output (MISO) broadcasting, robust transceiver design, simultaneous wireless information and power transmission.**

I. INTRODUCTION

MULTI-USER multiple-input multiple-output (MU-MIMO) systems have drawn a lot of attention recently due to their great potential to enhance system capacity in wireless communication systems [1]–[5]. Since the resources to be shared among mobile units are limited, this capacity increase is a big challenge which require the tremendous computational burden incurred by the signal processing for mobile units. On the other hand, due to the short lifetime of the batteries, huge power consumption demanded by the terminals lead to a situation in which the users run out of battery remarkably fast. In MU-MIMO wireless systems, this can be a serious issue.

The radio-frequency (RF) wave is usually used to transport information from a transmitter to a receiver in wireless networks. Recently, energy harvesting (EH) has been introduced which exploits energy from the natural environment (i.e., wind energy, solar power, etc.) to recharge the devices' batteries and extend the lifetime of the terminal [6], [7]. Based on this EH technique, there is a lot of interest in simultaneous wireless information and power transfer (SWIPT) based on the RF wave [8]–[10]. The idea of the SWIPT was first introduced in [6], where the essential tradeoff between the achievable rate and the transferred power is investigated. The SWIPT was later expanded to a frequency selective channel in [7].

In practice, due to potential limitations of practical circuits for harvesting energy, a user may not be able to harvest energy and decode information from the same receive signal at the same time. Under such constraint, two co-located receiver designs were proposed in [8] which include time switching (TS) and power splitting (PS), where the achievable rate-energy (R-E) region was characterized by comparing with an outer bound. [9] studied the SWIPT system via dynamic power splitting (DPS), in which the optimal power splitting rule was shown based on instantaneous channel-state information (CSI) to optimize the R-E performance trade off. To allow SWIPT at the PS receiver side, a joint transmit beamforming and receive power splitting method for a multiuser multiple-input single-output (MISO) broadcast system with the SWIPT was presented in [10].

However, all these literatures assume that perfect CSI is available at the transmitter, which cannot be satisfied in actual implementation due to limited feedback bandwidth, finite training field and channel estimation errors [11]. The work in [8] has been extended to the scheme for the imperfect CSI in [12]. Also, robust transmit beamforming design problems for the SWIPT have been investigated in some scenarios [12]–[16]. With the SWIPT scheme, robust secure communications methods for MISO systems were studied in [13]–[15], where channel uncertainties were modeled as the worst case model [17]. Considering an amplify-and-forward two-way relay system, the authors in [16] presented a robust relay beamforming design for SWIPT, where a sum rate maximization problem can be solved

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by successive convex approximation algorithms. Also, multicasting MISO systems with the SWIPT were examined in [18] where a transmit beamforming and PS ratio optimization algorithm is developed with semidefinite relaxation (SDR) technique for both perfect and imperfect CSI cases. However, its algorithm is suitable only for the system with signal-to-noise ratio (SNR) constraint for each user.

In this paper, we consider a multiuser MISO SWIPT downlink system assuming imperfect CSI based on an Euclidean ball. The average total transmit power is chosen as the performance metric subject to individual EH constraint and signal-to-interference-plus-noise (SINR) constraint at each user. Our goal is to find the transmit beamforming vector and the receive power splitting ratio which minimize the total transmit power. The proposed methods focus on a worst-case design such that the system performance is not affected by the fluctuations of instantaneous channels. To tackle this problem, the initial problem is reformulated and the uncertainty in the CSI is modeled in the covariance matrix.

By employing convex programming, we propose two different approaches. In the first method, the worst-case SINR and EH constraint are computed by utilizing bounds on the signal power. Based on the derived bounds, we develop a relaxed semi-definite programming (SDP) which can be solved by standard numerical optimization tools. Also, we show that the SDR is tight, which means that the rank of the covariance matrix of the beamforming is one. However, the first method only provides a suboptimal solution for robust beamforming and power splitting.

Different from the first method, our second approach directly identifies the minimum of SINR by applying the Lagrangian multiplier. Although this approach does not translate the original problem into an SDP, the obtained problem is convex and can be solved efficiently to obtain an exact robust solution. The proposed second method provides a performance gain over the first method at the expense of the increased computational complexity. Finally, the performance for our proposed methods are compared by numerical results.

The rest of this paper is organized as follows: In Section II, the system model is presented. Section III provides the original problem formulation and feasibility analysis. The optimization algorithms are proposed in Section IV. Section V illustrates the numerical results. Finally, we conclude the paper in Section VI.

Notation: Vectors and matrices are denoted by bold lowercase and uppercase letters, respectively. $(\cdot)^H$ represents Hermitian transpose. For a vector $\boldsymbol{x}$, $\|\boldsymbol{x}\|$ indicates the Euclidean norm. $|\cdot|$ defines the absolute value of a complex scalar. $\mathbb{C}^{M\times 1}$ and $\mathbb{H}^{M\times 1}$ describe the space of $M\times 1$ complex matrices and Hermitian matrices, respectively. For a matrix $\mathbf{A}$, $\mathbf{A}\succeq \mathbf{0}$ means that $\mathbf{A}$ is positive semi-definite. $\|\mathbf{A}\|_F$, $\mathrm{tr}(\mathbf{A})$, and $\mathrm{rank}(\mathbf{A})$ denote the Frobenius norm, trace and the rank, respectively. $\mathrm{vec}(\mathbf{A})$ stacks the elements of $\mathbf{A}$ in a column vector. A circularly symmetric complex Gaussian (CSCG) random vector $\boldsymbol{x}$ with mean $\bar{\boldsymbol{a}}$ and covariance matrix Σ is denoted as $\boldsymbol{x}\sim \mathcal{CN}(\bar{\boldsymbol{a}},\Sigma)$. $\nabla_{\boldsymbol{a}} f(\cdot)$ stands for the differentiation of a function f with respect to $\boldsymbol{a}$. Finally, $\mathrm{E}\{\cdot\}$ describes the mathematical expectation.

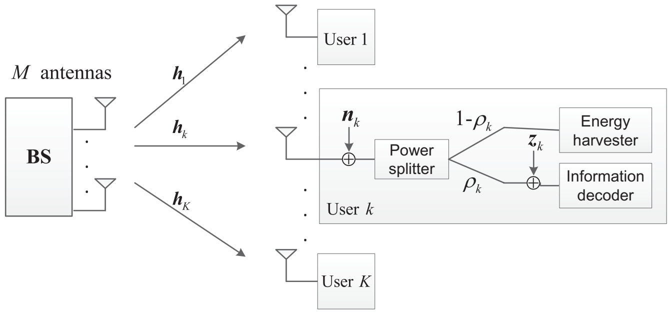


Fig. 1. A multi-user MISO SWIPT downlink system.

II. SYSTEM MODEL

We consider a multiuser MISO SWIPT downlink system, where a BS is equipped with M antennas and K users have a single antenna, as shown in Fig. 1. We assume that linear transmit beamforming is employed at the BS. Thus, the transmitted signal from the BS is defined as $\boldsymbol{x} = \sum_{k=1}^{K} \boldsymbol{v}_k s_k$ where $\boldsymbol{v}_k \in \mathbb{C}^{M\times 1}$ denotes the transmit beamforming vector for the kth user, and s_k represents the corresponding transmitted data symbol. It is assumed that s_k are independent and identically distributed (i.i.d.) CSCG random variables as $s_k \sim \mathcal{CN}(0,1)$. For the system depicted in Fig. 1, the total transmitted power is given by $E\{\|\boldsymbol{x}\|^2\} = \sum_{k=1}^{K}\|\boldsymbol{v}_k\|^2$. The received signal at the kth user is then expressed as

$$y_k = \boldsymbol{h}_k^H \sum_{j=1}^{K} \boldsymbol{v}_j s_j + n_k, \tag{1}$$

where n_k is the independent white complex Gaussian noise with zero mean and variance σ_k^2 as $n_k \sim \mathcal{CN}(0,\sigma_k^2)$.

In Fig. 1, the received signal at each user is split for information decoding (ID) and energy harvesting (EH) operations. Using a power splitter, the ρ_k $(0<\rho_k<1)$ portion of the received signal is assigned for the ID, while the remaining portion of $1-\rho_k$ is directed for the EH [8]. Accordingly, the corresponding split signal for the EH at the kth user is expressed as

$$y_k^{EH} = \sqrt{1-\rho_k}\left(\boldsymbol{h}_k^H \sum_{j=1}^{K} \boldsymbol{v}_j s_j + n_k\right). \tag{2}$$

As a result, the corresponding harvested power at the kth user, denoted by E_k, is obtained as

$$E_k = \zeta_k\left(1-\rho_k\right)\left(\sum_{j=1}^{K}\left|\boldsymbol{h}_k^H \boldsymbol{v}_j\right|^2 + \sigma_k^2\right), \tag{3}$$

where $0<\zeta_k\le 1$ represents the energy conversion efficiency at the kth user.

Thus, the split signal for the ID at the kth user is equal to

$$y_k^{ID} = \sqrt{\rho_k}\left(\boldsymbol{h}_k^H \sum_{j=1}^{K} \boldsymbol{v}_j s_j + n_k\right) + z_k, \tag{4}$$

where z_k denotes the additional noise introduced from the RF-to-signal conversion in the ID process at the kth user with $z_k \sim$

$\mathcal{CN}(0, \delta_k^2)$, which is independent of n_k. In practice, the noise power for the ID processing δ_k^2 is much smaller than the required harvested power target E_k/ζ_k, but much larger than the antenna noise power σ_k^2 [10], i.e, $E_k/\zeta_k \gg \delta_k^2 \gg \sigma_k^2$. The SINR of the ID at the kth user is then given as

$$\mathrm{SINR}_k = \frac{\rho_k |\boldsymbol{h}_k^H \boldsymbol{v}_k|^2}{\rho_k \sum_{j \neq k} |\boldsymbol{h}_k^H \boldsymbol{v}_j|^2 + \rho_k \sigma_k^2 + \delta_k^2}. \tag{5}$$

In real cellular networks, CSI is determined assuming channel reciprocity in time division duplexing (TDD) systems. In contrast, in this paper, we consider the FDD systems, where the CSI may be estimated at the receiver by using training sequences and then fed back to the transmitter via feedback channels. Estimation errors are inevitable in these cases. Due to the limited capacity of the feedback channels, the quantization errors also produce an effect on the CSI, and, imperfect CSI may degrade users' quality of service (QoS) and the system performance.

To preserve the desired QoS for all users, we investigate a robust transmit beamforming and power splitting design problem under imperfect CSI. In particular, we assume that the actual channel vector $\boldsymbol{h}_k$ lies within a ball with the radius ε_k around the estimated CSI vector $\hat{\boldsymbol{h}}_k$ from the BS to the kth user, i.e.,

$$\boldsymbol{h}_k \in \mathcal{H}_k = \left\{ \hat{\boldsymbol{h}}_k + \Delta \boldsymbol{h}_k \, | \|\Delta \boldsymbol{h}_k\| \le \varepsilon_k \right\}, \tag{6}$$

where the channel estimation error $\Delta \boldsymbol{h}_k \in \mathbb{C}^{M \times 1}$ is bounded by ε_k.

III. PROBLEM FORMULATION AND FEASIBILITY ANALYSIS

In this paper, our design objective is to minimize the average total transmit power while the SINR and the harvested energy at each user for all channel realization satisfy the given SINR target α_k and EH target e_k, respectively, i.e., $\mathrm{SINR}_k \ge \alpha_k$ and $E_k \ge e_k$, for $k = 1, \cdots, K$. Mathematically, this total transmit power minimization problem can be described as

$$\begin{aligned} (\mathrm{P0}) \min_{\{\boldsymbol{v}_k\},\ \rho_k} \quad & \sum_{k=1}^{K} \|\boldsymbol{v}_k\|^2 \\ \text{s.t.} \quad & \mathrm{SINR}_k \ge \alpha_k, \forall k, \\ & E_k \ge e_k, \forall k, \\ & \boldsymbol{h}_k \in \mathcal{H}_k, \forall k. \end{aligned} \tag{7}$$

Utilizing the result in [19], we can show that problem (P0) is feasible if and only if the SINR targets α_k satisfy the following condition:

$$\sum_{k=1}^{K} \frac{\alpha_k}{1 + \alpha_k} \le \mathrm{rank}\left(\mathbf{H}\right), \tag{8}$$

where $\mathbf{H} \triangleq [\boldsymbol{h}_1 \ \ \boldsymbol{h}_2 \ \cdots \ \boldsymbol{h}_K]$.

Key challenges in the preceding problem (P0) are the infinite number of constraints, which are caused by channel uncertainties. To deal with such a problem, one well-known method is to find the minimum values of SINR_k and E_k, respectively, relative to channel realizations that are described as the "worst ones". In that case, the original minimization problem guarantees that the smallest possible SINR and EH also satisfy all constraints.

Adopting this worst-case design methodology, we can reformulate (P0) to a simpler problem as

$$\begin{aligned} (\mathrm{P1}) \min_{\{\boldsymbol{v}_k\},\ \rho_k} \quad & \sum_{k=1}^{K} \|\boldsymbol{v}_k\|^2 \\ \text{s.t.} \quad & \min_{\boldsymbol{h}_k \in \mathcal{H}_k} \mathrm{SINR}_k \ge \alpha_k, \forall k, \\ & \min_{\boldsymbol{h}_k \in \mathcal{H}_k} E_k \ge e_k, \forall k, \\ & \|\Delta \boldsymbol{h}_k\| \le \varepsilon_k, \forall k. \end{aligned} \tag{9}$$

When computing E_k in (3) and SINR_k in (5), we need to calculate $|\boldsymbol{h}_k^H \boldsymbol{v}_j|^2$. Defining $\mathbf{W}_k = \boldsymbol{v}_k \boldsymbol{v}_k^H$ and employing the fact that $\boldsymbol{x}^H \mathbf{A} \boldsymbol{x} = \mathrm{tr}\left(\mathbf{A} \boldsymbol{x} \boldsymbol{x}^H\right)$, we can write this as

$$\begin{aligned} |\boldsymbol{h}_k^H \boldsymbol{v}_j|^2 &= |(\hat{\boldsymbol{h}}_k + \Delta \mathbf{h}_k)^H \boldsymbol{v}_j|^2 \\ &= \boldsymbol{v}_j^H (\hat{\boldsymbol{h}}_k + \Delta \mathbf{h}_k)(\hat{\boldsymbol{h}}_k + \Delta \mathbf{h}_k)^H \boldsymbol{v}_j \\ &= \mathrm{tr}\big((\hat{\boldsymbol{h}}_k + \Delta \mathbf{h}_k)(\hat{\boldsymbol{h}}_k + \Delta \mathbf{h}_k)^H \boldsymbol{v}_j \boldsymbol{v}_j^H\big) \\ &= \mathrm{tr}\big((\hat{\mathbf{H}}_k + \Delta_k) \mathbf{W}_j\big) \end{aligned} \tag{10}$$

where $\hat{\mathbf{H}}_k = \hat{\boldsymbol{h}}_k \hat{\boldsymbol{h}}_k^H$ denotes the covariance matrix of the estimated channel and $\Delta_k = \hat{\boldsymbol{h}}_k \Delta \boldsymbol{h}_k^H + \Delta \boldsymbol{h}_k \hat{\boldsymbol{h}}_k^H + \Delta \boldsymbol{h}_k \Delta \boldsymbol{h}_k^H$ represents the uncertainty in this matrix.

We can straightforwardly find the following relation as [21]

$$\begin{aligned} \|\Delta_k\|_F &= \|\hat{\boldsymbol{h}}_k \Delta \boldsymbol{h}_k^H + \Delta \boldsymbol{h}_k \hat{\boldsymbol{h}}_k^H + \Delta \boldsymbol{h}_k \Delta \boldsymbol{h}_k^H\|_F \\ &\le \|\hat{\boldsymbol{h}}_k \Delta \boldsymbol{h}_k^H\|_F + \|\Delta \boldsymbol{h}_k \hat{\boldsymbol{h}}_k^H\|_F + \|\Delta \boldsymbol{h}_k \Delta \boldsymbol{h}_k^H\|_F \\ &\le \|\hat{\boldsymbol{h}}_k\| \|\Delta \boldsymbol{h}_k^H\| + \|\Delta \boldsymbol{h}_k\| \|\hat{\boldsymbol{h}}_k^H\| + \|\Delta \boldsymbol{h}_k\|^2 \\ &= \varepsilon_k^2 + 2\varepsilon_k \|\hat{\boldsymbol{h}}_k\|. \end{aligned} \tag{11}$$

It should be noted that Δ_k is a norm-bounded matrix as $\|\Delta_k\|_F \le \xi_k$ where $\xi_k \triangleq \varepsilon_k^2 + 2\varepsilon_k \|\hat{\boldsymbol{h}}_k\|$.

Adopting the preceding notations, we can rewrite (P1) as

$$\begin{aligned} (\mathrm{P2}) \min_{\{\mathbf{W}_k \succeq 0\},\ \rho_k} \quad & \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{W}_k\right) \\ \text{s.t.} \min_{\|\Delta_k\| \le \xi_k} \quad & \frac{\rho_k \mathrm{tr}\big((\hat{\mathbf{H}}_k + \Delta_k) \mathbf{W}_k\big)}{\rho_k \sum_{j \neq k} \mathrm{tr}\big((\hat{\mathbf{H}}_k + \Delta_k) \mathbf{W}_j\big) + \rho_k \sigma_k^2 + \delta_k^2} \ge \alpha_k, \\ \min_{\|\Delta_k\| \le \xi_k} \quad & \zeta_k \left(1 - \rho_k\right) \left(\sum_{j=1}^{K} \mathrm{tr}\big((\hat{\mathbf{H}}_k + \Delta_k) \mathbf{W}_j\big) + \sigma_k^2 \right) \ge e_k, \\ & \mathrm{rank}(\mathbf{W}_k) = 1, \forall k. \end{aligned} \tag{12}$$

Since the primary problem (P1) is ill-conditioned, the previous steps are indispensable. (P1) is NP-hard and should be converted to a more tractable form [23]. The SDP relaxation is a well-known method to relieve the ill conditioned problem. It is observed that the uncertainty regions in (P2) are the result of the direct application of a quadratic transformation of the uncertainty regions of (P1). Using triangle inequality, we derive the bounds of the uncertainty, which are tight enough.

IV. ROBUST SOLUTIONS

In this section, we will deal with problem (P2) and reveal that this problem can be rewritten as a set of simple convex optimization problems. Two methods for robust optimization which minimize the SINR and EH constraints are described in the following.

A. Robust Solution based on Loose Approximation

According to [14], we can minimize the SINR by minimizing its numerator while maximizing the denominator. Then the first constraint of (P2) can be approximately rewritten as

$$\begin{aligned}&\min_{\|\Delta_k\|\le\xi_k} \rho_k \mathrm{tr}\big((\hat{\mathbf{H}}_k+\Delta_k)\mathbf{W}_k\big)\\ &-\max_{\|\Delta_k\|\le\xi_k} \alpha_k\rho_k\sum_{j\neq k}\mathrm{tr}\big((\hat{\mathbf{H}}_k+\Delta_k)\mathbf{W}_j\big)\ge\alpha_k(\rho_k\sigma_k^2+\delta_k^2).\end{aligned} \tag{13}$$

In order to minimize the numerator $\rho_k\mathrm{tr}((\hat{\mathbf{H}}_k+\Delta_k)\mathbf{W}_k)$ of the SINR constraint, employing a loose approximation [24] yields

$$\min_{\|\Delta_k\|\le\xi_k} \rho_k \mathrm{tr}\big((\hat{\mathbf{H}}_k+\Delta_k)\mathbf{W}_k\big)\le \rho_k \mathrm{tr}\big((\hat{\mathbf{H}}_k-\xi_k\mathbf{I}_M)\mathbf{W}_k\big).$$

Utilizing a similar methodology for the denominator term, it follows

$$\begin{aligned}\max_{\|\Delta_k\|\le\xi_k} \alpha_k\rho_k&\sum_{j\neq k}\mathrm{tr}\big((\hat{\mathbf{H}}_k+\Delta_k)\mathbf{W}_j\big)\\ &\ge \alpha_k\rho_k\sum_{j\neq k}\mathrm{tr}\big((\hat{\mathbf{H}}_k+\xi_k\mathbf{I}_M)\mathbf{W}_j\big).\end{aligned}$$

Using the same approach for the second constraint of problem (P2), we have

$$\begin{aligned}\min_{\|\Delta_k\|\le\xi_k} &\zeta_k(1-\rho_k)\left(\sum_{j=1}^{K}\mathrm{tr}\big((\hat{\mathbf{H}}_k+\Delta_k)\mathbf{W}_j\big)+\sigma_k^2\right)\\ &\le \zeta_k(1-\rho_k)\left(\sum_{j=1}^{K}\mathrm{tr}\big((\hat{\mathbf{H}}_k-\xi_k\mathbf{I}_M)\mathbf{W}_j\big)+\sigma_k^2\right).\end{aligned}$$

Thus, the objective function and all constraints of (P2) for the robust downlink optimization problem in MISO SWIPT systems become

$$\begin{aligned}&\text{(P3)} \min_{\{\mathbf{W}_k\succeq 0\},\ \rho_k} \sum_{k=1}^{K}\mathrm{tr}(\mathbf{W}_k)\\ &\text{s.t.}\ \mathrm{tr}\big(\hat{\mathbf{H}}_k(\mathbf{W}_k-\alpha_k\sum_{j\neq k}\mathbf{W}_j)\big)-\xi_k\mathrm{tr}\big(\mathbf{W}_k+\alpha_k\sum_{j\neq k}\mathbf{W}_j\big)\\ &\ge\alpha_k\big(\sigma_k^2+\frac{\delta_k^2}{\rho_k}\big),\forall k,\\ &\sum_{j=1}^{K}\mathrm{tr}\big((\hat{\mathbf{H}}_k-\xi_k\mathbf{I}_M)\mathbf{W}_j\big)\ge\frac{e_k}{\zeta_k(1-\rho_k)}-\sigma_k^2,\forall k.\end{aligned} \tag{14}$$

Note that in order to generate a convex problem, the nonconvex constraint $\mathrm{rank}(\mathbf{W}_k)=1$ is removed [12], [17], [21]. AS a result, (P3) is a convex optimization problem [20] since both $1/\rho_k$ and $1/(1-\rho_k)$ are convex functions over ρ_k with $0<\rho_k<1$, which is referred to as SDP and can be solved by an efficient numerical method [22]. As shown in [24], the preceding SDP relaxation is guaranteed to yield at least one optimal solution with rank one.

Let $\{\mathbf{W}_k^*\}$ and $\{\rho_k^*\}$ denote the optimal solutions of (P3). If $\{\mathbf{W}_k^*\}$ satisfies $\mathrm{rank}(\mathbf{W}_k^*)=1$, then the optimal transmit beamforming solution $\boldsymbol{v}_k{}^*$ of (P1) can be obtained from eigenvalue decomposition (EVD) of $\mathbf{W}_k^*$, and the optimal PS ratios of (P3) are also determined by the associated ρ_k^*'s. However, due to the relaxation, $\mathbf{W}_k^*$ computed from the SDP in (P3) may not have rank-one in general. If there exists any k such that $\mathrm{rank}(\mathbf{W}_k^*)>1$, the solutions $\{\mathbf{W}_k^*\}$ and $\{\rho_k^*\}$ are not optimal for (P1). In the following proposition, we prove that a robust transmit beamforming solution of (P3) satisfies $\mathrm{rank}(\mathbf{W}_k^*)=1$, i.e., the SDR is tight.

Proposition 1: For given α_k and e_k in (P3), we have the following properties.

1. $\{\mathbf{W}_k^*\}$ and $\{\rho_k^*\}$ satisfy the SINR and EH constraints of (P3) with equality.
2. $\{\mathbf{W}_k^*\}$ has $\mathrm{rank}(\mathbf{W}_k^*)=1,\forall k$.

Proof: See Appendix A.

It is revealed from the first property of Proposition 1 that the optimal robust beamforming and PS ratio solutions can be identified when both SINR and EH constraints of (P3) hold with equality for all users. Based on the second property of Proposition 1, we can see that the SDR does not lose any optimality in comparison to (P1). Thus, we can determine the global optimal solution by solving (P3) with a standard numerical optimization tool, such as CVX [22]. The procedures for solving (P3) are summarized in Algorithm 1.

The computational complexity of Algorithm 1 mainly comes from the computation of the SDP (14). According to [26], the computational complexity for solving an SDP within a tolerance ϵ is $\mathcal{O}((m_{\mathrm{sdp}}n_{\mathrm{sdp}}^{3.5}+m_{\mathrm{sdp}}^2n_{\mathrm{sdp}}^{2.5}+m_{\mathrm{sdp}}^3n_{\mathrm{sdp}}^{0.5})\cdot\log(1/\epsilon))$ where m_{sdp} is the number of linear constraints and n_{sdp} is the dimension of the semidefinite cone. For the SDP problem (14), we have $m_{\mathrm{sdp}}=2K$ and $n_{\mathrm{sdp}}=M$. Thus, the computational complexity of the SDP of (14) equals $\mathcal{O}((2KM^{3.5}+4K^2M^{2.5}+8K^3M^{0.5})\log(1/\epsilon))$.

Algorithm 1 Robust joint design of beamforming and PS for problem (P3)

1. Solve problem (P3) by CVX to obtain $\{\mathbf{W}_k^*\}$ and $\{\rho_k^*\},\forall k$.
2. Compute $\{\boldsymbol{v}_k^*\}$ by the EVD of $\{\mathbf{W}_k^*\}$.

B. Robust Solution based on Tight Approximation

In the previous subsection, we have investigated the problem of minimizing the uncertainty of SINR and EH using a traditional method. In this subsection, we will employ the Lagrangian multiplier method to find a robust solution based on tight approximation. We start with (P2) with a simple modification. By dropping the rank-one constraint for all $\mathbf{W}_k$'s [21], problem (P2) is approximately restated as

$$\begin{aligned}
&\text{(P4)} \min_{\{\mathbf{W}_k \succeq 0\},\ \rho_k} \sum_{k=1}^{K} \operatorname{tr}(\mathbf{W}_k) \\
&\text{s.t.} \min_{\|\Delta_k\| \le \xi_k} \left(\operatorname{tr}\big((\hat{\mathbf{H}}_k + \Delta_k)\mathbf{W}_k\big) - \alpha_k \sum_{j \ne k} \operatorname{tr}\big((\hat{\mathbf{H}}_k + \Delta_k)\mathbf{W}_j\big) \right) \\
&\ge \alpha_k \left(\sigma_k^2 + \frac{\delta_k^2}{\rho_k} \right), \forall k, \\
&\min_{\|\Delta_k\| \le \xi_k} \sum_{j=1}^{K} \operatorname{tr}\big((\hat{\mathbf{H}}_k + \Delta_k)\mathbf{W}_j\big) \ge \frac{e_k}{\zeta_k(1-\rho_k)} - \sigma_k^2, \forall k.
\end{aligned} \tag{15}$$

In (P4), we try to minimize the SINR and EH directly. Suppose that (P4) is feasible, and let $\{\mathbf{W}_k\}$ and $\{\rho_k\}$ be feasible solutions for SINR constraint. It can be shown that the new solutions $\{\beta\mathbf{W}_k\}$ and $\{\rho_k\}$ are also feasible for EH constraint in (P4), for $\beta > 1$. Since there must be a large enough $\beta > 1$, we can find the new solutions $\{\beta\mathbf{W}_k\}$ and $\{\rho_k\}$ which satisfy all EH constraint of problem (P4). Thus, all we need is to consider Δ_k which minimizes SINR constraint. First, we have the following proposition.

Proposition 2: For given α_k for $k = 1, \cdots, K$, $\Delta_k^{\min}$ which minimizes the SINR constraint of (P4) is obtained as

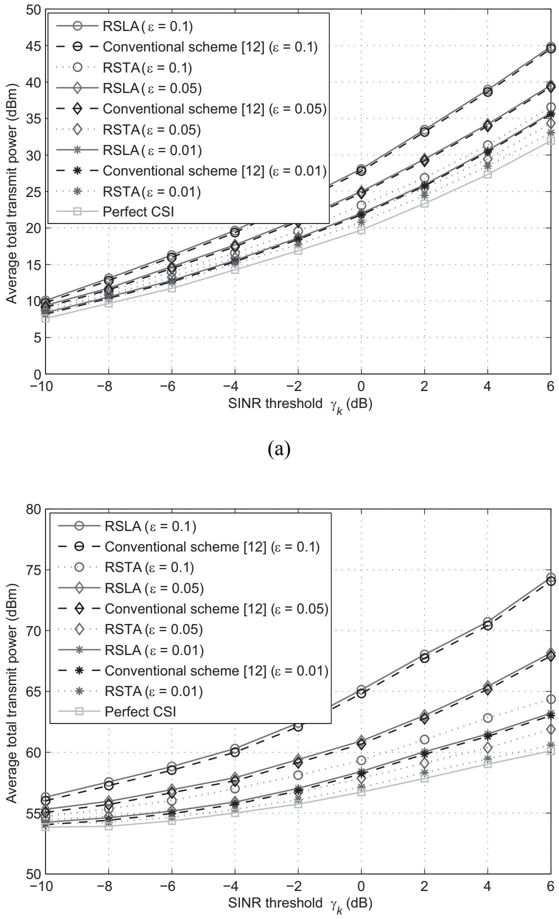
$$\Delta_k^{\min} = -\xi_k \frac{\left(\mathbf{W}_k - \alpha_k \sum\limits_{j \ne k} \mathbf{W}_j\right)^H}{\left\|\mathbf{W}_k - \alpha_k \sum\limits_{j \ne k} \mathbf{W}_j\right\|}.$$

Proof: See Appendix B. ■

By ignoring the rank-1 constraint on $\mathbf{W}_k$, the SDR of (P2) can be summarized as follows:

$$\begin{aligned}
&\text{(P5)} \min_{\{\mathbf{W}_k \succeq 0\},\ \rho_k} \sum_{k=1}^{K} \operatorname{tr}(\mathbf{W}_k) \\
&\text{s.t.} \operatorname{tr}\big(\hat{\mathbf{H}}_k(\mathbf{W}_k - \alpha_k \sum_{j \ne k} \mathbf{W}_j)\big) - \xi_k \big\|\mathbf{W}_k - \alpha_k \sum_{j \ne k} \mathbf{W}_j\big\| \\
&\ge \alpha_k \big(\sigma_k^2 + \frac{\delta_k^2}{\rho_k}\big), \forall k, \\
&\sum_{j=1}^{K} \Big(\operatorname{tr}\big(\hat{\mathbf{H}}_k \mathbf{W}_j\big) - \xi_k \|\mathbf{W}_j\| \Big) \ge \frac{e_k}{\zeta_k(1-\rho_k)} - \sigma_k^2, \forall k.
\end{aligned} \tag{16}$$

Although this problem (P5) is not SDP, it is a convex problem since the objective function and all constraint consist of matrix norms and traces that are convex [20]. Thus, (P5) can be efficiently solved by using standard numerical optimization tools [22]. Note that the beamforming vectors are also the principal eigenvectors of the solution of (P5). The proof that the optimal beamforming $\mathbf{W}_k^*$ for (P5) satisfies $\operatorname{rank}(\mathbf{W}_k) = 1$ is similar to that of Proposition 1, and thus is omitted here. According to [26], it is known that the complexity of the interior-point algorithm for solving problem (P5) is $\mathcal{O}(\sqrt{KM}(K^3M^2 + K^2M^3)\log(1/\epsilon))$. The proposed tight approximation-based robust method provides a performance gain over the scheme in Section IV. A at the expense of the increased computational complexity.



(a)

(b)

Fig. 2. The average average transmit power versus α (dB) with $K = 4$, $M = 4$: (a) $e = 1$ dBm and (b) $e = 50$ dBm.

V. NUMERICAL RESULTS

In this section, we provide numerical examples to evaluate the performance of the proposed algorithms. We set $M = 4$, $K = 4$, and $\zeta = 0.7$. It is assumed that all users have uniform system parameters, i.e., $\zeta_k = \zeta, \delta_k = \delta, \sigma_k = \sigma, \alpha_k = \alpha, \varepsilon_k = \varepsilon$, and $e_k = e$. In addition, the estimated channel $\hat{\boldsymbol{h}}_k \in \mathbb{C}^{\mathbb{M}\times\mathbb{K}}$ is randomly generated from the i.i.d. Rayleigh fading component with zero mean and unit covariance. In the plots, RSLA and RSTA indicate "robust solution based on loose approximation" and "robust solution based on tight approximation", respectively, and the "Perfect CSI" represents the case where there is no uncertainty in the CSI, i.e. $\varepsilon = 0$ [10].

Fig. 2 compares the average total transmit power versus different SINR thresholds α with various EH thresholds with $e = 1$ dBm and $e = 50$ dBm, where each point is averaged over 1000 randomly generated channel realizations. For simulations, three channel uncertainties $\varepsilon = 0.01$, 0.05, and 0.1 are considered, and the noise powers are assumed to be $\sigma^2 = -20$ dBm and $\delta^2 = -5$ dBm. As expected, the RSTA can save the transmit

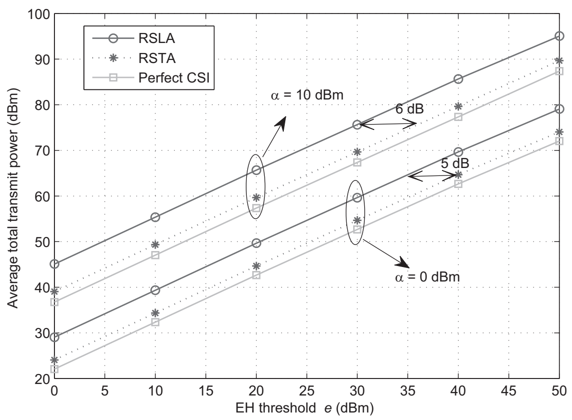


Fig. 3. The average average transmit power (dBm) versus e (dBm) with $\alpha = 0$ dB, $\alpha = 10$ dB, $K = 4$, and $M = 4$.

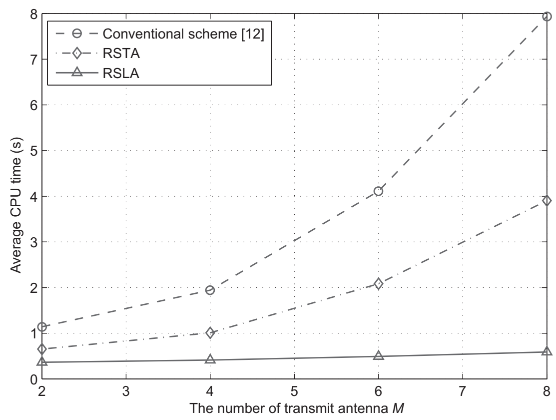


Fig. 5. The average CPU time versus M with $K = 4$, $\alpha = 10$ dB, and $e = 10$ dBm.

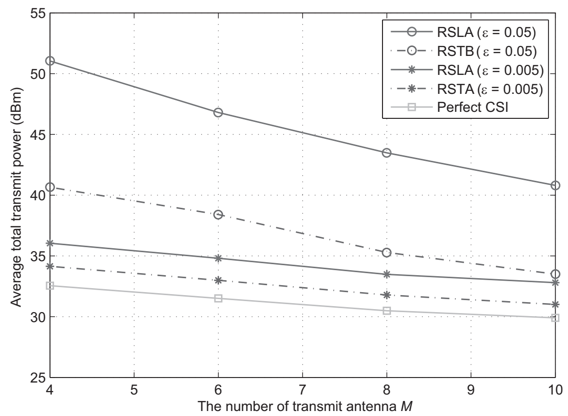


Fig. 4. The average transmit power versus M with $K = 4$, $\alpha = 10$ dB, and $e = 10$ dBm.

power compared to the RSLA at the expense of increased complexity, whereas the performance of the RSLA is almost identical with the method in [12]. In Fig. 2, the RSTA performs quite close to the Perfect CSI scheme. Also, we can check that when the SINR threshold decreases, a loss over the perfect CSI reduces for both proposed methods. When ε is increased, the required total transmit power grows for proposed methods.

Fig. 3 depicts the average total transmit power achieved by RSLA and RSTA with fixed $\gamma = 0$ dB and $\gamma = 10$ dB with respect to EH thresholds e. In this plot, we set the system for $\sigma^2 = 8$ dBm, $\delta^2 = 8$ dBm, and $\varepsilon = 0.01$. Similarly as in Fig. 2, we can observe that the RSTA achieves the lesser transmit power than the RSLA for all values of e. Furthermore, the RSTA performs very closely to the perfect CSI case. Notice that there are 6 dB and 5 dB gaps between RSLA and RSTA curves for $\gamma = 10$ dB and $\gamma = 0$ dB, respectively.

In Fig. 4, we study the impact of the number of transmit antennas on the total transmit power with $\alpha = 10$ dB and $e = 10$ dBm. In this case, we assume $\sigma^2 = 10$ dBm, $\delta^2 = 7$ dBm, $K = 4$, and $\varepsilon = 0.005$ and 0.05. Similar to Fig. 2, we can see that the RSTA achieves lower transmit power compared to the RSLA. Also, when M increases, the average total transmit power is decreased for all cases.

In Fig. 5, in terms of the average CPU running time, the computational complexities of the proposed methods are compared for a channel realization under various M on a computer.[1] In this simulation, we set $K = 4, e = 10$ dBm, $\alpha = 10$ dB, and $\varepsilon = 0.01$. It is observed that the conventional method in [12] has much higher complexity than our proposed algorithms, and RSLA has much lower complexity than the RSTA.

VI. CONCLUSION

In this paper, we have considered a transmit beamforming and receive power splitting problem for a multiuser MISO SWIPT broadcast channel under imperfect CSI. Our work has focused on the average total transmit power at a BS subject to EH and SINR constraints for each user. First, we have proposed a lower approximation robust approach based on SDP. Utilizing the SDR technique, we have optimally solved the non-convex problem, and shown that the relaxation is tight. Also, the second approach has been proposed by applying a Lagrangian multiplier method. Numerical results have confirmed the robustness and effectiveness of the proposed methods.

APPENDICES

I. PROOF OF PROPOSITION 1

Since (P3) is a SDP problem, it is convex. It is easy to verify that (P3) satisfies the Slater's condition [20] and its duality gap

[1] The central processing unit (CPU) is Intel Core i7-4700HQ 2.4GHz, and the size of random access memory (RAM) is 4GB.

is zero. Therefore, the Lagrangian of (P3) can be written as

$$\begin{aligned}\mathrm{L}\left(\mathbf{W}_k,\rho_k,\lambda_k,\mu_k\right) &\triangleq \sum_{k=1}^{K}\mathrm{tr}(\mathbf{W}_k)+\sum_{k=1}^{K}\lambda_k\left(\alpha_k\sigma_k^2+\frac{\alpha_k\delta_k^2}{\rho_k}\right)\\ &-\sum_{k=1}^{K}\lambda_k\big(\mathrm{tr}\big(\hat{\mathbf{H}}_k(\mathbf{W}_k-\alpha_k\sum_{j\neq k}\mathbf{W}_j)\big)-\xi_k\mathrm{tr}\big(\mathbf{W}_k+\alpha_k\sum_{j\neq k}\mathbf{W}_j)\big)\\ &-\sum_{k=1}^{K}\mu_k\left(\sum_{j=1}^{K}\mathrm{tr}\big((\hat{\mathbf{H}}_k-\xi_k\mathbf{I}_M)\mathbf{W}_j\big)-\frac{e_k}{\zeta_k\big(1-\rho_k\big)}+\sigma_k^2\right)\end{aligned} \tag{17}$$

where $\{\lambda_k\}$ and $\{\mu_k\}$ are the dual variables associated with the kth SINR constraint and EH constraint of (P3), respectively. As a result, the Lagrange dual function of (P3) is expressed as [20, Section 5.7.3]

$$g\left(\lambda_k,\mu_k\right)=\min_{\mathbf{W}_k\succeq\mathbf{0},\ \rho_k}\ \mathrm{L}\left(\mathbf{W}_k,\rho_k,\lambda_k,\mu_k\right).$$

After applying some algebraic manipulations, the above minimum function can be explicitly given as

$$\begin{aligned}g\left(\lambda_k,\mu_k\right)=\min_{\mathbf{W}_k\succeq\mathbf{0},\rho_k}&\left\{\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{W}_k\right)+\sum_{k=1}^{K}\left(\lambda_k\alpha_k-\mu_k\right)\sigma_k^2\right.\\ &\left.+\sum_{k=1}^{K}\left(\frac{\lambda_k\alpha_k\delta_k^2}{\rho_k}+\frac{\mu_k e_k}{\zeta_k\left(1-\rho_k\right)}\right)\right\},\end{aligned} \tag{18}$$

where $\mathbf{B}_k$ is denoted as

$$\begin{aligned}\mathbf{B}_k&=\left(1+\left(1-\alpha_k\right)\lambda_k\xi_k\right)\mathbf{I}_M-\left(1+\alpha_k\right)\lambda_k\hat{\mathbf{H}}_k\\ &+\sum_{j=1}^{K}\Big(\left(\lambda_j\alpha_j+\mu_j\right)\xi_j\mathbf{I}_M+\left(\alpha_j\lambda_j-\mu_j\right)\hat{\mathbf{H}}_j\Big).\end{aligned} \tag{19}$$

Suppose that the optimal dual solution of (P3) is $\{\lambda_k^*,\mu_k^*\}$. Then, we define $\mathbf{B}_k^*$ as $\mathbf{B}_k$ in (19) with $\lambda=\lambda^*$ and $\mu=\mu^*$.

In the following, we first consider the the Lagrangian minimization over $\mathbf{W}_k$ with fixed $\lambda_k\geq 0$ and $\mu_k\geq 0,\forall k$. By ignoring the constant terms associated with λ_k and μ_k in (19), it can be observed from (19) that $\mathbf{W}_k^*$ must be the optimal solution of the problem

$$\min_{\mathbf{W}_k\succeq\mathbf{0}}\ \sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k^*\mathbf{W}_k\right). \tag{20}$$

Furthermore, in order to satisfy the SINR constraint, it must hold $\mathbf{W}_k^*\neq\mathbf{0},\forall k$. Besides, to guarantee a bounded dual optimal value, it must follow $\mathbf{B}_k^*\succeq\mathbf{0}$. As a result, the optimal value of problem (20) should satisfy the KKT conditions as $\mathrm{tr}\left(\mathbf{B}_k^*\mathbf{W}_k^*\right)=0$. Then, by combining with $\mathbf{B}_k^*\succeq\mathbf{0}$ and $\mathbf{W}_k^*\succ\mathbf{0}$, it is equivalent to $\mathbf{B}_k^*\mathbf{W}_k^*=\mathbf{0}$. Also, it is revealed from (18) that the optimal PS solution ρ_k^* must be the optimal solution of the problem

$$\begin{aligned}\min_{\rho_k}\quad&\sum_{k=1}^{K}\left(\frac{\lambda_k^*\alpha_k\delta_k^2}{\rho_k}+\frac{\mu_k^* e_k}{\zeta_k\left(1-\rho_k\right)}\right)\\ \text{s.t.}\quad&0<\rho_k<1.\end{aligned} \tag{21}$$

Next, we prove this proposition by discussing the following three cases on λ_k^* and μ_k^*.

1) $\lambda_k^*>0$ and $\mu_k^*=0$, it can be shown that ρ_k^* approaches 1. Thus, the harvested power will become so small, which is not possible to satisfy the EH constraint.
2) If $\lambda_k^*=0$ and $\mu_k^*>0$, it can be shown that ρ_k^* approaches 0. Thus, the ρ_k^* portion of the received power for the ID approaches 0, which might not happen.
3) If $\lambda_k^*=0$ and $\mu_k^*=0$, we will prove that this case cannot happen for any user by contradiction.

Suppose that there exist some k's such that $\lambda_k^*=\mu_k^*=0$. We define a non-empty set $\Psi\triangleq\{k|\lambda_k^*=0,\mu_k^*=0,1\leq k\leq K\}$. Introducing an auxiliary matrix

$$\begin{aligned}\mathbf{C}^*&=\left(1+\left(1-\alpha_k\right)\lambda_k^*\xi_k\right)\mathbf{I}_M\\ &+\sum_{j\neq\Psi}\Big(\big(\lambda_j^*\alpha_j+\mu_j^*\big)\xi_j\mathbf{I}_M+\big(\alpha_j\lambda_j^*-\mu_j^*\big)\hat{\mathbf{H}}_j\Big),\end{aligned}$$

$\mathbf{B}_k^*$ can be rewritten as

$$\mathbf{B}_k^*=\begin{cases}\mathbf{C}^*, & \text{if } k\in\Psi\\ \mathbf{C}^*-\left(1+\alpha_k\right)\lambda_k^*\hat{\mathbf{H}}_k, & \text{if } k\notin\Psi.\end{cases} \tag{22}$$

Due to $\mathbf{B}_k^*\succeq\mathbf{0}$ and $-\left(1+\alpha_k\right)\lambda_k^*\hat{\mathbf{H}}_k\preceq\mathbf{0}$, it satisfies that $\mathbf{C}^*\succeq\mathbf{0}$. In the following, we derive $\mathbf{C}^*\succ\mathbf{0}$ by contradiction. Provided that the minimum eigenvalue of $\mathbf{C}^*$ is zero. There exists at least one nonzero $\boldsymbol{x}$ satisfying $\boldsymbol{x}^H\mathbf{C}^*\boldsymbol{x}=0$. According to (22), it follows

$$\boldsymbol{x}^H\mathbf{B}_k^*\boldsymbol{x}=-\left(1+\alpha_k\right)\lambda_k^*\boldsymbol{x}^H\hat{\mathbf{H}}_k\boldsymbol{x}\geq 0\quad\text{for } k\notin\Psi. \tag{23}$$

Note that if $k\notin\Psi$, we have $\lambda_k^*>0$. Thus, we can obtain $\boldsymbol{x}^H\hat{\mathbf{H}}_k\boldsymbol{x}=|\hat{\boldsymbol{h}}_k^H\boldsymbol{x}|^2\leq 0$ for $k\notin\Psi$ from (23). Thus, it follows $\hat{\boldsymbol{h}}_k^H\boldsymbol{x}=0$ for $k\notin\Psi$.

As a result, we can get the following formula

$$\begin{aligned}\boldsymbol{x}^H\mathbf{C}^*\boldsymbol{x}&=\boldsymbol{x}^H\Bigg(\sum_{j=1}^{K}\Big(\big(\lambda_j^*\alpha_j+\mu_j^*\big)\xi_j\mathbf{I}_M+\big(\alpha_j\lambda_j^*-\mu_j^*\big)\hat{\mathbf{H}}_j\Big)\\ &\quad+\left(1+\left(1-\alpha_k\right)\lambda_k^*\xi_k\right)\mathbf{I}_M\Bigg)\boldsymbol{x}\\ &=\boldsymbol{x}^H\boldsymbol{x}>0.\end{aligned} \tag{24}$$

It can be observed that (24) contradicts to $\boldsymbol{x}^H\mathbf{C}^*\boldsymbol{x}=0$. Therefore, it follows $\mathbf{C}^*\succ\mathbf{0}$, i.e., $\mathrm{rank}\left(\mathbf{C}^*\right)=M$. Thus from (22), we have $\mathrm{rank}\left(\mathbf{B}_k^*\right)=M$ for $k\in\Psi$. According to $\mathbf{B}_k^*\mathbf{W}_k^*=\mathbf{0}$, it follows $\mathbf{W}_k^*=\mathbf{0}$ for $k\in\Psi$ which contradicts to the assumption $\mathbf{W}_k^*\neq\mathbf{0}$. Therefore, we conclude that Ψ is empty, i.e., $\lambda_k=0$ and $\mu_k=0$ cannot happen for any k. Since for given $\alpha_k>0$ and $e_k>0$, $0<\rho_k^*<1$ must hold for all users in (P3), the case 1 and case 2 cannot happen. Therefore, it follows $\lambda_k^*>0,\mu_k^*>0,\forall k$. According to the complementary slackness conditions [20], the first part of Proposition 1 is thus proved.

Next, we will prove the second property of Proposition 1. Since Ψ is empty, we have

$$\begin{aligned}\mathbf{C}^*&=\left(1+\left(1-\alpha_k\right)\lambda_k^*\xi_k\right)\mathbf{I}_M\\ &+\sum_{j=1}^{K}\Big(\big(\lambda_j^*\alpha_j+\mu_j^*\big)\xi_j\mathbf{I}_M+\big(\alpha_j\lambda_j^*-\mu_j^*\big)\hat{\mathbf{H}}_j\Big),\end{aligned}$$

and then (22) reduces to $\mathbf{B}_k^* = \mathbf{C}^* - (1+\alpha_k)\,\lambda_k^* \hat{\mathbf{H}}_k, \forall k$. Since $\mathrm{rank}(\mathbf{C}^*) = M$, it follows $\mathrm{rank}(\mathbf{B}_k^*) \geq M-1$. It is noted that if $\mathbf{B}_k^*$ is full rank, $\mathbf{W}_k^*$ becomes $\mathbf{0}$ which contradicts to $\mathbf{W}_k^* \neq \mathbf{0}$. As a result, the rank of $\mathbf{B}_k^*$ is given as $\mathrm{rank}(\mathbf{B}_k^*) = M-1$. From $\mathbf{B}_k^*\mathbf{W}_k^* = \mathbf{0}$, we have $\mathrm{rank}(\mathbf{W}_k) = 1$. Thus, we prove the second part of Proposition 1.

II. PROOF OF PROPOSITION 2

By introducing an arbitrary positive multiplier $\theta \geq 0$, the Lagrangian function is given by

$$\begin{aligned}\mathrm{L}\left(\Delta_k, \theta\right) = \mathrm{tr}\big((\hat{\mathbf{H}}_k + \Delta_k)\mathbf{W}_k\big) - \alpha_k \sum_{j\neq k} \mathrm{tr}\big((\hat{\mathbf{H}}_k + \Delta_k)\mathbf{W}_j\big) \\ + \theta\left(\mathrm{tr}\left(\Delta_k \Delta_k^H\right) - \xi_k^2\right).\end{aligned} \tag{25}$$

We differentiate the Lagrangian function with respect to Δ_k and equate it to zero [25] as

$$\nabla_{\Delta_k} \mathrm{L}\left(\Delta_k, \theta\right) = \mathbf{W}_k^H - \alpha_k \sum_{j\neq k} \mathbf{W}_j^H + \theta \Delta_k = \mathbf{0}. \tag{26}$$

We can find the optimal solution Δ_k as

$$\Delta_k^{\mathrm{opt}} = -\frac{1}{\theta}(\mathbf{W}_k - \alpha_k \textstyle\sum_{j\neq k} \mathbf{W}_j)^H.$$

In order to remove the parameter θ, the Lagrangian function is differentiated with respect to θ and set to zero as

$$\nabla_\theta \mathrm{L}\left(\Delta_k, \theta\right) = \|\Delta_k\| - \xi_k = 0.$$

Let us denote the optimal solution for θ as $\theta^{\mathrm{opt}} = \frac{1}{\xi_k}\left\|\mathbf{W}_k - \alpha_k \sum_{j\neq k} \mathbf{W}_j\right\|$. By combining the above results, we finally get

$$\Delta_k^{\mathrm{opt}} = -\xi_k \frac{\left(\mathbf{W}_k - \alpha_k \sum_{j\neq k} \mathbf{W}_j\right)^H}{\left\|\mathbf{W}_k - \alpha_k \sum_{j\neq k} \mathbf{W}_j\right\|}.$$

To test its optimality, it is observed that the second derivative at the optimal solution point Δ_k^{opt} is positive semidefinite, i.e.,

$$\nabla^2_{\Delta_k^*} \mathrm{L}\left(\Delta_k^{\mathrm{opt}}, \theta^{\mathrm{opt}}\right) = \theta^{\mathrm{opt}}(\mathrm{vec}\left\{\mathbf{I}_M\right\} \mathrm{vec}\left\{\mathbf{I}_M\right\})^T \succeq \mathbf{0}.$$

REFERENCES

[1] S. R. Lee, S. H. Park, S. H. Moon, and I. Lee, "New beamforming schemes with optimum receive combining for multiuser MIMO systems," in *Proc. IEEE ICC*, Beijing, China, May 2008, pp. 4118–4122.

[2] H. Sung, S. R. Lee, and I. Lee, "Generalized channel inversion methods for multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3489–3499, Nov. 2009.

[3] W. Lee, H. Park, H. Kong, J. S. Kwak, and I. Lee, "A new beamforming design for multicast systems," *IEEE Trans. Veh. Technol.*, vol. 62, no. 8, pp. 4093–4097, Oct. 2013.

[4] S. R. Lee, J. S. Kim, S. H. Moon, H. B. Kong, and I. Lee, "Zero-forcing beamforming in multiuser MISO downlink dystems under per-antenna power constraint and equal-rate metric," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 228–236, Jan. 2013.

[5] S. R. Lee, H. B. Kong, H. Park, and I. Lee, "Beamforming designs based on an asymptotic approach in MISO interference channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6430–6438, Dec. 2013.

[6] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. IEEE ISIT*, Toronto, Canada, July 2008, pp. 1612–1616.

[7] P. Grover and A. Sahai, "Shannon meets Tesla: wireless information and power transfer," in *Proc. IEEE ISIT*, Austin, USA, June 2010, pp. 2363–2367.

[8] R. Zhang and C. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 1989–2001, May 2013.

[9] L. Liu, R. Zhang, and K. C. Chua, "Wireless information and power transfer: a dynamic power splitting approach," *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3990–4001, Sept. 2013.

[10] Q. Shi, L. Liu, W. Xu, and R. Zhang, "Joint transmit beamforming and receive power splitting for MISO SWIPT systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 6, pp. 3269–3280, June 2014.

[11] H. T. Kim, S. H. Lim, I. Lee, S. Kim, and S. Y. Chung, "Code design for MIMO downlink with imperfect CSIT," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 89–94, Jan. 2010.

[12] Z. Zhu, K. J. Lee, Z. Wang, and I. Lee. "Robust beamforming and power splitting design in distributed antenna system with SWIPT under bounded channel uncertainty," in *Proc. IEEE VTC*, Glasgow, Scotland, May 2015.

[13] J. Rubio and A. Pascual-Iserte, "Energy-aware broadcast multiuser-MIMO precoder design with imperfect channel and battery knowledge," *IEEE Trans. Wireless Commun.*, vol. 13, no. 6, pp. 3137–3152, June 2014.

[14] R. Feng, Q. Li, Q. Zhang, and J. Qin, "Robust secure transmission in MISO simultaneous wireless information and power transfer system," *IEEE Trans. Veh. Technol.*, accepted for publication 2014.

[15] D. W. K. Ng, E. S. Lo, and R. Schober, "Robust beamforming for secure communication in systems with wireless information and power transfer," *IEEE Trans. Wireless Commun.,* vol. 13, no. 8, pp. 4599–4615, Aug. 2014.

[16] D. Li, C. Shen, and Z. Qiu, "Sum rate maximization and energy harvesting for two-way af relay systems with imperfect CSI," in *Proc. IEEE ICASSP*, Vancouver, Canada, May 2013, pp. 4958–4962.

[17] S. Mohammadkhani, S. M. Razavizadeh, and I. Lee, "Robust filter and forward relay beamforming with spherical channel state information uncertainties," in *Proc. IEEE ICC*, Sydney, Australia, June 2014, pp. 5023–5028.

[18] M. Khandaker and K. Wong, "SWIPT in MISO multicasting systems," *IEEE Wireless Commun. Lett.*, vol. 3, no. 3, pp. 277–280, June 2014.

[19] R. Hunger and M. Joham, "A complete description of the QoS feasibility region in the vector broadcast channel," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3870–3878, July 2010.

[20] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge U.K.: Cambridge Univ. Press, 2004.

[21] Z. Q. Luo, W. K. Ma, A. M. C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems: From its practical deployments and scope of applicability to key theoretical results," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

[22] M. Grant and S. Boyd. (2012, Sept.). CVX: matlab software for disciplined convex programming, version 2.0 beta, [Online]. Available: http://cvxr.com/cvx.

[23] Z. Q. Luo and S. Zhang, "A semidefinite relaxation scheme for multivariate quartic polynomial optimization with quadratic constraints," in *Proc. Conf. Math. Oper. Res.*, pp. 13–15, Jan. 2009.

[24] M. Bengtsson and B. Ottersten, "Optimum and suboptimum transmit beamforming," in *Handbook of Antennas in Wireless Communications,* L. C. Godara, Ed., CRC Press, Aug. 2001.

[25] A. Hjrungnes and D. Gesbert, "Complex-valued matrix differentiation: techniques and key results," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2740–2746, June 2007.

[26] I. Polik and T. Terlaky, "Interior point methods for nonlinear optimization," in *Nonlinear Optimization*, 1st ed., Eds. Springer, 2010.

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