Sum-Rate Maximization for Multiuser MIMO Wireless Powered Communication Networks

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***Abstract*—This paper investigates multiuser multiple-input–multiple-output (MIMO) wireless powered communication networks where a multiantenna hybrid access point (H-AP) transfers wireless energy to multiantenna users in a downlink phase, and the users utilize the harvested energy for their information transmission to the H-AP in an uplink phase. By employing space-division multiple-access techniques, we propose an optimal algorithm that jointly computes the downlink energy precoding matrices, the uplink information precoding matrices, and time allocation between the downlink and the uplink phases for maximizing the uplink sum-rate performance. To this end, we first obtain the optimal energy and information transmit covariance matrices with given time allocation. Then, the optimal time allocation can be efficiently identified by a simple line search method. Simulation results verify that the proposed joint optimal algorithm significantly improves the average sum-rate performance, compared with a conventional scheme that determines time allocation and precoding matrices separately.**

***Index Terms*—Multiple-input multiple-output (MIMO) systems, wireless energy transfer (WET), wireless powered communication networks (WPCNs).**

I. INTRODUCTION

Far-field wireless energy transfer (WET) utilizing radio frequency (RF) signals has received much attention, owing to its capability to supply power to energy-demanding devices without replacing their batteries [1]. Recently, the WET techniques have been studied in traditional wireless communication systems where the role of the RF signals is limited to wireless information transmission (WIT). In particular, two different protocols, namely, simultaneous wireless information and power transfer (SWIPT) [2], [3] and wireless powered communication networks (WPCNs) [4]–[8], have been investigated in various system configurations. Unlike SWIPT, where the systems are confined to downlink transmission, the WPCN considers both the downlink and uplink networks.

In the WPCN systems, there exist two sequential transmission phases. First, in a downlink phase, users collect the energy of the RF signals broadcasted from a hybrid access point (H-AP). Then, in an uplink phase, the users exploit the harvested energy for their WIT to the H-AP. For single-antenna WPCN systems, Ju and Zhang in [4] and Lee *et al.* in [5] introduced dynamic time-division multiple access (TDMA) protocols where the time durations for each time slot are optimized to maximize the uplink sum-rate performance.

These works were extended to multiple-input single-output (MISO) WPCNs in [6]–[8], where the H-AP has multiple antennas. By employing the dynamic TDMA approach, Sun *et al.* in [6] proposed the optimal beamforming and time allocation algorithm for the multiuser (MU) MISO WPCN systems. Moreover, the space-division multiple-access techniques [9], [10] were applied in [7] and [8] to the MU-MISO WPCN, and the optimal WET and WIT beamforming vectors that maximize the minimum throughput among all users were derived in [7]. Yang *et al.* in [8] presented an asymptotic analysis in a large-scale MU-MISO scenario by considering the channel estimation procedures for the WPCN and identified an asymptotically optimal solution for the minimum-rate maximization problem.

In this paper, we study MU multiple-input multiple-output (MIMO) WPCN systems where both the H-AP and users are equipped with multiple antennas. In this configuration, we maximize the uplink sum-rate performance by jointly optimizing the downlink WET precoding matrices, the uplink WIT precoding matrices, and the time allocation between the downlink and uplink phases. To this end, we first present the optimal energy and information transmit covariance matrices with given time allocation and then determine the optimal time allocation based on the obtained covariance matrices.

We show that the optimal downlink energy covariance matrices for each user are identical, and this matrix is rank-one. In other words, the optimal downlink WET strategy is *multicasting* and *beamforming*, and thus, the H-AP transmits the common energy RF signal for all users with the rank-one beamforming vector aligned to the maximum eigenmode of the weighted sum of downlink channel matrices. For computing the optimal uplink information covariance matrices with given time allocation, we provide an iterative procedure based on a waterfilling scheme [11].

Finally, the optimal time allocation can be efficiently calculated by applying a simple line search method, and the corresponding joint optimal covariance matrices are identified. Through simulation results, we confirm that the proposed joint optimal algorithm offers a significant performance gain compared with a conventional scheme that separately designs the time allocation, the downlink WET precoding, and the uplink WIT precoding.

II. SYSTEM MODEL

Here, we describe a system model for MU-MIMO WPCNs where an H-AP with M antennas communicates with K users, each equipped with N antennas as shown in Fig. 1. It is assumed that the H-AP has fixed power supply, whereas the users have no embedded energy sources. Thus, in the WPCN, the users first collect the energy of the RF signals radiated from the H-AP in the downlink phase and then transmit their information signals to the H-AP in the uplink phase by utilizing the harvested energy.[1] Without loss of generality, we assume that the total operation time for the system is 1, and the time durations for the downlink and uplink phases are given by τ and $1 - \tau$, respectively $(0 \leq \tau \leq 1)$. Moreover, it is assumed that channel state information is perfectly known at the H-AP.

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[1]In this paper, we focus on a theoretical design of a WPCN. See [1] for the implementation issues on RF energy harvesting hardware.

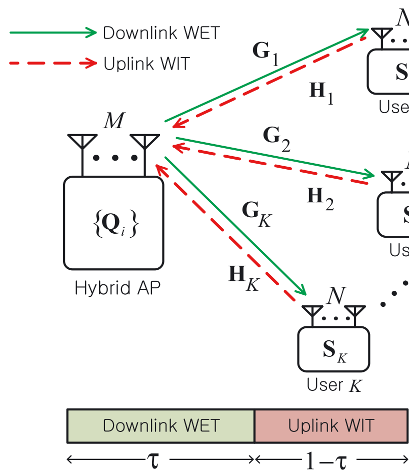

Fig. 1. Schematic diagram for an MU-MIMO WPCN.

In the downlink phase, the H-AP transfers the independent energy signals $\mathbf{x}_i \in \mathbb{C}^{M\times 1}$ of user i for $i = 1,\ldots,K$ with the energy transmit covariance matrix $\mathbf{Q}_i = \mathbb{E}[\mathbf{x}_i\mathbf{x}_i^H] \in \mathbb{C}^{M\times M}$. The transmit power constraint at the H-AP is defined as P, i.e., $\sum_{i=1}^K \mathrm{tr}(\mathbf{Q}_i) \le P$. Denoting $\mathbf{G}_i \in \mathbb{C}^{N\times M}$ as the downlink channel matrix between the H-AP and user i, the received signal $\mathbf{r}_i \in \mathbb{C}^{N\times 1}$ at user i is written as

$$\mathbf{r}_i = \mathbf{G}_i\mathbf{x}_E + \mathbf{n}_i$$

where $\mathbf{x}_E = \sum_{j=1}^K \mathbf{x}_j$ stands for the transmitted energy signal with $\mathbb{E}[\mathbf{x}_E\mathbf{x}_E^H] = \mathbf{Q}_E \triangleq \sum_{j=1}^K \mathbf{Q}_j$, $\mathbf{n}_i \sim \mathcal{CN}(0, \mathbf{I}_N)$ represents the complex Gaussian noise at user i, and $\mathbf{I}_m$ accounts for an identity matrix of size $m\times m$. Then, the harvested energy E_i at user i becomes [2], [3]

$$E_i = \eta_i\tau\sum_{j=1}^K \mathrm{tr}\left(\mathbf{G}_i\mathbf{Q}_j\mathbf{G}_i^H\right) \tag{1}$$

where $0 < \eta_i \le 1$ indicates the energy efficiency at user i.

In the uplink phase, the users transmit their uplink information signals $\mathbf{s}_i \in \mathbb{C}^{N\times 1}$ to the H-AP by exploiting the harvested energy E_i. The received signal $\mathbf{y} \in \mathbb{C}^{M\times 1}$ at the H-AP can be expressed as

$$\mathbf{y} = \sum_{i=1}^K \mathbf{H}_i\mathbf{s}_i + \mathbf{z}$$

where $\mathbf{H}_i \in \mathbb{C}^{M\times N}$ denotes the uplink channel matrix between the H-AP and user i, and $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I}_M)$ indicates the complex Gaussian noise at the H-AP. At the H-AP, we adopt a successive interference cancelation technique for information decoding as in [11]. Then, the achievable uplink sum rate is given by

$$R = (1-\tau)\log_2\left|\mathbf{I}_M + \sum_{i=1}^K \mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right|$$

where $\mathbf{S}_i = \mathbb{E}[\mathbf{s}_i\mathbf{s}_i^H] \in \mathbb{C}^{N\times N}$ is equal to the information transmit covariance matrix of user i.

In this paper, we aim to maximize the uplink sum-rate performance for the MU-MIMO WPCN by jointly optimizing the time allocation τ, the energy transmit covariance matrices $\{\mathbf{Q}_i\}$, and the information transmit covariance matrices $\{\mathbf{S}_i\}$. Then, we can formulate the uplink sum-rate maximization problem as

$$\max_{\tau,\{\mathbf{S}_i\succeq 0\},\{\mathbf{Q}_i\succeq 0\}} (1-\tau)\log_2\left|\mathbf{I}_M + \sum_{i=1}^K \mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right| \tag{2}$$

$$\text{s.t.}\ \mathrm{tr}(\mathbf{S}_i) \le \frac{\eta_i\tau}{1-\tau}\sum_{j=1}^K \mathrm{tr}\left(\mathbf{G}_i\mathbf{Q}_j\mathbf{G}_i^H\right),\quad i = 1,\ldots,K \tag{3}$$

$$\sum_{i=1}^K \mathrm{tr}(\mathbf{Q}_i) \le P \tag{4}$$

$$0 \le \tau \le 1$$

where (3) represents the uplink transmit power constraint at user i, i.e., user i cannot use more energy than the harvested energy E_i in (1), and (4) stands for the downlink transmit power constraint at the H-AP. It is worth noting that problem (2) is convex for a fixed τ; however, in general, it is jointly nonconvex due to the coupled variables in constraint (3), and thus, it is not straightforward to find an optimal solution. In the following section, we provide optimal methods to solve problem (2).

III. Optimal Solution

Here, we present the optimal solution for the nonconvex problem in (2) by reformulating (2) into the equivalent convex problem. Defining new variables $\mathbf{V}_i \in \mathbb{C}^{N\times N}$ for $i = 1,\ldots,K$ and $\mathbf{W} \in \mathbb{C}^{M\times M}$ as $\mathbf{V}_i \triangleq (1-\tau)\mathbf{S}_i$ and $\mathbf{W} \triangleq \tau\mathbf{Q}_E = \tau\sum_{i=1}^K \mathbf{Q}_i$, respectively, problem (2) can be rewritten as

$$\begin{aligned}\max_{\tau,\{\mathbf{V}_i\succeq 0\},\mathbf{W}\succeq 0}\ & (1-\tau)\log_2\left|\mathbf{I}_M + \frac{1}{1-\tau}\sum_{i=1}^K \mathbf{H}_i\mathbf{V}_i\mathbf{H}_i^H\right| \\ \text{s.t.}\ & \mathrm{tr}(\mathbf{V}_i) \le \eta_i\mathrm{tr}\left(\mathbf{G}_i\mathbf{W}\mathbf{G}_i^H\right),\quad i = 1,\ldots,K \\ & \mathrm{tr}(\mathbf{W}) \le \tau P \\ & 0 \le \tau \le 1.\end{aligned} \tag{5}$$

Note that the objective function of problem (5) is jointly concave on τ and $\{\mathbf{V}_i\}$, since it is a perspective function of a concave function $\log_2|\mathbf{I}_M + \sum_{i=1}^K \mathbf{H}_i\mathbf{V}_i\mathbf{H}_i^H|$ [12]. In addition, all the constraints of problem (5) are affine. Therefore, problem (5) is jointly convex with respect to τ, $\{\mathbf{V}_i\}$, and $\mathbf{W}$. To solve (5) efficiently, we first obtain the optimal transmit covariance matrices with a given τ in Section III-A and B. Next, the optimal value of the time duration τ will be determined in Section III-C.

A. Optimal Energy Transmit Covariance Matrix

Here, we provide the optimal energy transmit covariance matrix $\mathbf{W}^\star$ for a given τ. Problem (5) with a fixed τ can be recast to

$$\mathcal{R}(\tau) \triangleq \max_{\{\mathbf{V}_i\succeq 0\},\mathbf{W}\succeq 0} (1-\tau)\log_2\left|\mathbf{I}_M + \frac{1}{1-\tau}\sum_{i=1}^K \mathbf{H}_i\mathbf{V}_i\mathbf{H}_i^H\right| \tag{6}$$

$$\text{s.t.}\ \mathrm{tr}(\mathbf{V}_i) \le \eta_i\mathrm{tr}\left(\mathbf{G}_i\mathbf{W}\mathbf{G}_i^H\right),\quad i = 1,\ldots,K \tag{7}$$

$$\mathrm{tr}(\mathbf{W}) \le \tau P \tag{8}$$

where $\mathcal{R}(\tau)$ represents the optimal value of problem (6).[2]

Since problem (6) is convex and satisfies Slater's condition, the duality gap is zero. Therefore, we can optimally solve (6) by applying

[2]With fixed $\mathbf{W}$, problem (6) reduces to the sum-rate maximization problem of the conventional MIMO multiple-access channels [11].

the Lagrange duality method [12]. The Lagrangian of problem (6) becomes

$$\mathcal{L}\left(\{\mathbf{V}_i\}, \mathbf{W}, \{\mu_i\}, \nu\right) = (1-\tau)\log_2\left|\mathbf{I}_M + \frac{1}{1-\tau}\sum_{i=1}^{K}\mathbf{H}_i\mathbf{V}_i\mathbf{H}_i^H\right| - \sum_{i=1}^{K}\mu_i \mathrm{tr}(\mathbf{V}_i) + \nu\tau P + \mathrm{tr}(\mathbf{FW})$$

where $\mathbf{F} \triangleq \sum_{i=1}^{K}\mu_i\eta_i\mathbf{G}_i^H\mathbf{G}_i - \nu\mathbf{I}_M$, and $\{\mu_i\}$ and ν stand for the nonnegative dual variables corresponding to constraints (7) and (8), respectively.

The dual function of (6) can be written as

$$\mathcal{G}(\{\mu_i\}, \nu) = \max_{\{\mathbf{V}_i \succeq 0\}, \mathbf{W} \succeq 0} \mathcal{L}\left(\{\mathbf{V}_i\}, \mathbf{W}, \{\mu_i\}, \nu\right) \tag{9}$$

and the dual problem is given by $\min_{\{\mu_i \geq 0\}, \nu \geq 0}\mathcal{G}(\{\mu_i\}, \nu)$. Due to the zero duality gap, we can equivalently address the primal problem (6) via its dual problem [12]. Therefore, in what follows, we first obtain the dual function $\mathcal{G}(\{\mu_i\}, \nu)$ from problem (9) with given dual variables and then determine the optimal dual variables $\{\mu_i^\star\}$ and $\nu^\star$, which minimize the dual function. In the following theorem, we present the optimal energy covariance matrix $\mathbf{W}^\star$ by investigating problem (6).

Theorem 1: The optimal solution $\mathbf{W}^\star$ for problem (6) is expressed as

$$\mathbf{W}^\star = \tau P \mathbf{u}_{\mathbf{B},1}\mathbf{u}_{\mathbf{B},1}^H \tag{10}$$

where $\mathbf{u}_{\mathbf{B},1}$ is the unit-norm eigenvector of a matrix $\mathbf{B} \triangleq \sum_{i=1}^{K}\mu_i^\star\eta_i\mathbf{G}_i^H\mathbf{G}_i$ corresponding to the maximum eigenvalue $\lambda_{\mathbf{B},1}$. Moreover, the optimal dual variables must satisfy $\mu_i^\star > 0$ for $i = 1, \ldots, K$ and $\nu^\star = \lambda_{\mathbf{B},1}$.

Proof: See the Appendix. ■

Theorem 1 implies that for the optimal downlink WET, the H-AP transmits the same energy symbol for all users, i.e., *multicasting* [13] is the optimal transmission policy. In addition, the optimal energy covariance matrix (10) is rank-one, and this proves that *beamforming*, which transmits a single stream, is the optimal strategy for the downlink WET. Therefore, the optimal energy signal $\mathbf{x}_E$ can be determined as $\mathbf{x}_E = \sqrt{P}\mathbf{u}_{\mathbf{B},1}x_E$, where x_E indicates an arbitrary random scalar with unit variance. Note that the beamforming vector $\mathbf{u}_{\mathbf{B},1}$ is aligned to the maximum eigenmode of matrix $\mathbf{B}$, which is the weighted sum of the downlink channel covariance matrices $\mathbf{G}_i^H\mathbf{G}_i$.

B. *Optimal Information Transmit Covariance Matrix*

Here, we provide the optimal information transmit covariance matrix $\{\mathbf{V}_i^\star\}$ with a given τ. Denoting $\bar{\mathbf{V}}_i = (\mu_i/(1-\tau))\mathbf{V}_i$ for $i = 1, \ldots, K$ and neglecting the irrelevant terms, problem (9) for obtaining $\{\mathbf{V}_i^\star\}$ can be recast to

$$\max_{\{\bar{\mathbf{V}}_i \succeq 0\}} \log_2\left|\mathbf{I}_M + \sum_{i=1}^{K}\frac{1}{\mu_i}\mathbf{H}_i\bar{\mathbf{V}}_i\mathbf{H}_i^H\right| - \sum_{i=1}^{K}\mathrm{tr}(\bar{\mathbf{V}}_i). \tag{11}$$

Due to the term $\sum_{i=1}^{K}\mathrm{tr}(\bar{\mathbf{V}}_i)$, problem (11) is different from MIMO multiple-access channels [11].

To solve (11) optimally, we first identify a solution of $\bar{\mathbf{V}}_i$ with given other $\bar{\mathbf{V}}_j$ for $j \neq i$, and then, the optimal $\mathbf{V}_i^\star$ for $i = 1, \ldots, K$ will be determined. The first term in (11) is rewritten as

$$\begin{aligned}\log_2&\left|\mathbf{I}_M + \sum_{i=1}^{K}\frac{1}{\mu_i}\mathbf{H}_i\bar{\mathbf{V}}_i\mathbf{H}_i^H\right| \\ &= \log_2\left|\mathbf{I}_M + \mathbf{C}_i\bar{\mathbf{V}}_i\mathbf{C}_i^H\right| + \log_2\left|\mathbf{I}_M + \sum_{j\neq i}\frac{1}{\mu_j}\mathbf{H}_j\bar{\mathbf{V}}_j\mathbf{H}_j^H\right|\end{aligned} \tag{12}$$

where $\mathbf{C}_i = (\mathbf{I}_M + \sum_{j\neq i}(1/\mu_j)\mathbf{H}_j\bar{\mathbf{V}}_j\mathbf{H}_j^H)^{-1/2}\sqrt{(1/\mu_i)}\mathbf{H}_i$. By collecting the relevant terms to $\bar{\mathbf{V}}_i$ in (11) and (12), we can reformulate problem (11) as

$$\max_{\bar{\mathbf{V}}_i \succeq 0} \log_2\left|\mathbf{I}_M + \mathbf{C}_i\bar{\mathbf{V}}_i\mathbf{C}_i^H\right| - \mathrm{tr}(\bar{\mathbf{V}}_i). \tag{13}$$

Let us define the singular value decomposition of $\mathbf{C}_i$ as $\mathbf{C}_i = \mathbf{R}_i\mathbf{\Sigma}_i\mathbf{M}_i^H$, where the matrices $\mathbf{R}_i \in \mathbb{C}^{M\times r_i}$ and $\mathbf{M}_i \in \mathbb{C}^{M\times r_i}$ contain the first r_i left and right singular vectors of $\mathbf{C}_i$, respectively, with $r_i = \mathrm{rank}(\mathbf{C}_i)$, and the diagonal matrix $\mathbf{\Sigma}_i = \mathrm{diag}(\sigma_{i,1}, \ldots, \sigma_{i,r_i})$ consists of singular values $\sigma_{i,k}$ for $k = 1, \ldots, r_i$ with $\sigma_{i,1} \geq \cdots \geq \sigma_{i,r_i}$. Then, with given $\{\mathbf{V}_j\}$ for $j \neq i$, a solution for problem (13) is expressed as [2]

$$\mathbf{V}_i = \frac{1-\tau}{\mu_i}\mathbf{M}_i\mathbf{D}_i\mathbf{M}_i^H \tag{14}$$

where $\mathbf{D}_i = \mathrm{diag}(d_{i,1}, \ldots, d_{i,r_i})$ with $d_{i,k} = \max(0, \log_2 e - (1/\sigma_{i,k}^2))$ for $k = 1, \ldots, r_i$.

Now, we will show by contradiction that the optimal solution $\{\mathbf{V}_i^\star\}$ for problem (6) indeed becomes (14), i.e., $\mathbf{V}_i^\star = ((1-\tau)/\mu_i)\mathbf{M}_i\mathbf{D}_i\mathbf{M}_i^H$ for $i = 1, \ldots, K$. Suppose that the optimal $\mathbf{V}_i^\star$ does not fulfill (14). Then, by fixing other $\mathbf{V}_j$ for $j \neq i$ as $\mathbf{V}_j = \mathbf{V}_j^\star$, we can increase the objective value of (6) by setting $\mathbf{V}_i^\star$ to (14), since a solution in (14) satisfies the Karush–Kuhn–Tucker (KKT) conditions [2]. This contradicts the assumption, and thus, the optimal $\mathbf{V}_i^\star$ for $i = 1, \ldots, K$ is calculated by (14).

To compute the optimal $\mathbf{V}_i^\star$ from (14), other $\mathbf{V}_j^\star$ values for $j \neq i$ are required. Thus, we propose an iterative procedure for identifying $\{\mathbf{V}_i^\star\}$ based on an iterative waterfilling algorithm [11]. In this procedure, we first initialize $\mathbf{V}_i = \mathbf{0}$ for $i = 1, \ldots, K$. At each iteration, we sequentially update $\mathbf{V}_i$ from $i = 1$ to K by using (14). This process is repeated until $\{\mathbf{V}_i\}$ values converge. It is known that this iterative procedure converges to the optimal solution $\{\mathbf{V}_i^\star\}$ [11].

From (10) and (14), we can determine the dual function in (9). Then, the remaining part is to solve the dual problem $\min_{\{\mu_i \geq 0\}, \nu \geq 0}\mathcal{G}(\{\mu_i\}, \nu)$, which finds the optimal dual variables $\{\mu_i^\star\}$ and $\nu^\star$. These optimal dual variables can be attained by the subgradient methods, e.g., the ellipsoid method [12]. Since the optimal $\nu^\star$ is given by $\nu^\star = \lambda_{\mathbf{B},1}$ from Theorem 1, we only need to update μ_i for $i = 1, \ldots, K$ by using the subgradient $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_K]^T$. Here, the subgradient δ_i of the dual function (9) with respect to μ_i at a certain point $\tilde{\boldsymbol{\mu}} = [\tilde{\mu}_1, \ldots, \tilde{\mu}_K]^T$ is written as

$$\begin{aligned}\delta_i &= \mathrm{tr}\left(\eta_i\mathbf{G}_i\mathbf{W}^\star\mathbf{G}_i^H - \mathbf{V}_i^\star\right) \\ &\quad + \eta_i\left(\tau P - \mathrm{tr}(\mathbf{W}^\star)\right)\mathbf{u}_{\tilde{\mathbf{B}},1}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{u}_{\tilde{\mathbf{B}},1} \\ &= \mathrm{tr}\left(\eta_i\mathbf{G}_i\mathbf{W}^\star\mathbf{G}_i^H - \mathbf{V}_i^\star\right)\end{aligned} \tag{15}$$

where $\tilde{\mathbf{B}} \triangleq \sum_{i=1}^{K}\tilde{\mu}_i\eta_i\mathbf{G}_i^H\mathbf{G}_i$, and (15) comes from the fact that $\mathrm{tr}(\mathbf{W}^\star) = \tau P\mathrm{tr}(\mathbf{u}_{\mathbf{B}}\mathbf{u}_{\mathbf{B}}^H) = \tau P$. We summarize the overall algorithm for problem (6) with a given τ in Table I.

TABLE I
ALGORITHM 1: OPTIMAL ALGORITHM FOR PROBLEM (6) WITH A GIVEN τ

Initialize $\mu_i > 0$ for $i = 1, \cdots, K$.
Repeat
Compute $\mathbf{W}^\star$ from (10).
Initialize $\mathbf{V}_i = \mathbf{0}$ for $i = 1, \cdots, K$.
Repeat
For $i = 1 : K$
Update $\mathbf{V}_i = \frac{1-\tau}{\mu_i}\mathbf{M}_i\mathbf{D}_i\mathbf{M}_i^H$.
End
Until $\{\mathbf{V}_i\}$ converge
Obtain $\mathbf{V}_i^\star = \mathbf{V}_i$ for $i = 1, \cdots, K$.
Update $\{\mu_i\}$ by applying the ellipsoid method.
Until $\{\mu_i\}$ converge

TABLE II
ALGORITHM 2: OPTIMAL ALGORITHM FOR PROBLEM (2)

Initialize $a = 0$, $b = 1$, and $\varphi = (-1 + \sqrt{5})/2$.
Repeat
Update $\tau_1 = b - \varphi(b-a)$ and $\tau_2 = a + \varphi(b-a)$.
Obtain $\mathcal{R}(\tau_1)$ and $\mathcal{R}(\tau_2)$ from Algorithm 1.
If $\mathcal{R}(\tau_2) < \mathcal{R}(\tau_1)$, set $b = \tau_2$. Else, set $a = \tau_1$.
Until $\lvert b-a \rvert$ converges
Obtain $\tau^\star = \frac{(a+b)}{2}$, $\mathbf{Q}_E^\star = \frac{\mathbf{W}^\star}{\tau^\star}$, and $\mathbf{S}_i^\star = \frac{\mathbf{V}_i^\star}{1-\tau^\star}$ for $i = 1, \cdots, K$.

C. Optimal Time Allocation

So far, the optimal transmit covariance matrices $\mathbf{W}^\star$ and $\{\mathbf{V}_i^\star\}$ have been found when τ is fixed. Here, we identify the optimal time allocation $\tau^\star = \arg\max_\tau \mathcal{R}(\tau)$ by investigating the function $\mathcal{R}(\tau)$ in (6). As previously mentioned, the objective function of problem (5) is jointly concave on τ and $\{\mathbf{V}_i\}$. Note that $\mathcal{R}(\tau)$ is the maximum of the objective function in (5) with respect to $\mathbf{W}$ and $\{\mathbf{V}_i\}$ over the convex set specified by constraints (7) and (8).

It is known that maximizing a jointly concave function with respect to some variables over a convex set also yields a concave function [12], and thereby, $\mathcal{R}(\tau)$ is concave with respect to τ. As a result, the optimal $\tau^\star$ can be efficiently determined by the golden section search method.[3] In the golden section search method, the function value $\mathcal{R}(\tilde{\tau})$ is required at a certain point $\tilde{\tau}$, and this can be achieved by Algorithm 1. To summarize, we provide Algorithm 2, which solves the original problem (2) in Table II.

IV. SIMULATION RESULTS

Here, we evaluate the average sum-rate performance of MU-MIMO WPCN systems through numerical simulations. For convenience, we adopt the notation (M,N,K) to represent a K-user WPCN with M H-AP and N user antennas. In the simulations, we set the energy harvesting efficiency η_i as $\eta_i = 0.7\ \forall i$, and the noise variance as -50 dBm. The elements of the downlink and uplink channel matrices are independently generated from the zero-mean complex Gaussian distribution, i.e., the Rayleigh fading channel model is employed, with 30-dB average signal attenuation from the H-AP to all users.

To demonstrate the impact of the time allocation τ, in Fig. 2, we present the average sum-rate performance of the proposed optimal algorithm as a function of τ for (2, 2, 2) systems. It is observed that the average sum rate changes a lot as τ varies, and this tendency becomes more pronounced as the transmit power at H-AP P increases. Thus, the time allocation between the downlink and uplink phases is important particularly when P is large. Moreover, we can see that the optimal $\tau^\star$, which provides the maximum sum-rate performance, decreases as P grows. This is due to the fact that with small P, i.e., when the H-AP

[3]By employing the golden section search method, an extreme point of a unimodal function can be efficiently identified without knowledge of the derivative [14].

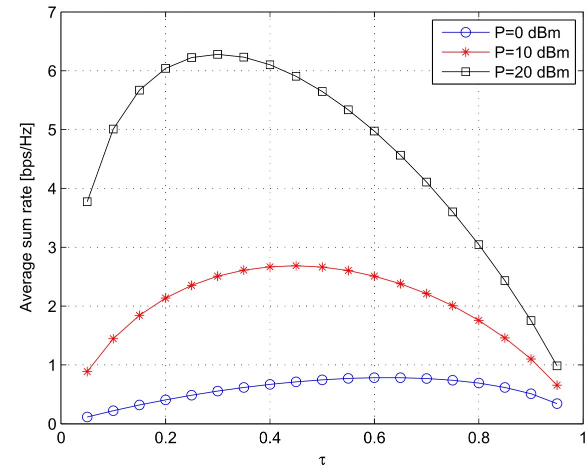

Fig. 2. Average sum-rate performance as a function of τ for (2, 2, 2) systems with different P values.

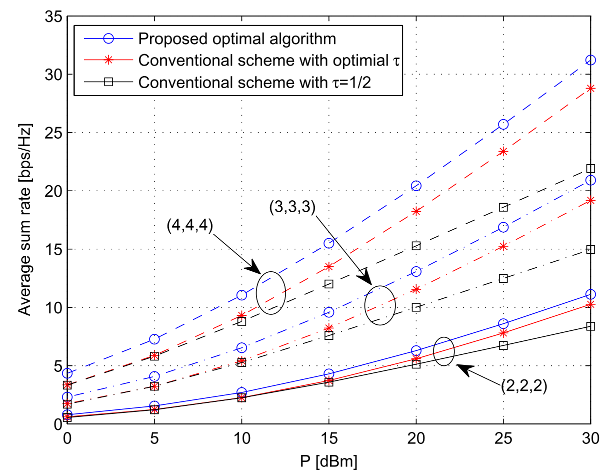

Fig. 3. Average sum-rate performance as a function of P for (2, 2, 2), (3, 3, 3), and (4, 4, 4) systems.

does not have enough transmit power, the downlink phase duration should be increased so that more energy can be harvested. On the other hand, with large transmit power, the H-AP can transfer enough energy during a short time duration. As a result, the sum-rate performance can be improved by allocating more time duration to the uplink WIT phase with large P.

Fig. 3 shows the average sum-rate performance of the proposed optimal algorithm for (2, 2, 2), (3, 3, 3), and (4, 4, 4) systems. For comparison, we also plot the performance of a conventional scheme based on [11]. In this scheme, we apply a naive downlink WET covariance matrix $\mathbf{Q}_E = (P/M)\mathbf{I}_M$, and the uplink WIT covariance matrices $\{\mathbf{S}_i\}$ are computed from the conventional iterative water-filling algorithm in [11] with user i's uplink power constraint $p_i = (\tau/(1-\tau))\mathrm{tr}(\mathbf{G}_i\mathbf{Q}_E\mathbf{G}_i^H) = (P\tau/M(1-\tau))\mathrm{tr}(\mathbf{G}_i^H\mathbf{G}_i)$. The time allocation τ in the conventional scheme is optimally calculated based on Algorithm 2 presented in Section III-C (marked with an asterisk) or is equally allocated as $\tau = 1/2$ (marked with a square). In Fig. 3, it is shown that the proposed optimal algorithm outperforms the conventional schemes, and the performance gap increases as P and (M,N,K) grow. By comparing the performance of the two conventional schemes, we can see that optimizing τ via Algorithm 2 significantly improves the average sum rate of the WPCN. Furthermore, it is observed that at $P = 20$ dBm, the proposed joint optimal algorithm offers about 23% and 34% gains over the conventional scheme with $\tau = 1/2$ in (2, 2, 2) and (4, 4, 4) systems, respectively.

V. Conclusion

In this paper, we have investigated MU-MIMO WPCN systems where both the H-AP and users are equipped with multiple antennas. In this system, we have optimally solved a joint time allocation and transmit covariance matrix optimization problem to maximize the sum rate. Simulation results have confirmed that the proposed optimal algorithm provides a substantial performance gain over a conventional scheme.

Appendix
Proof of Theorem 1

Before verifying (10), we first show $\mu_i^\star > 0, \forall i$. Suppose that $\mu_j^\star$ is equal to 0. In this case, problem (9) yields a solution $\mathbf{V}_j^\star = \alpha\mathbf{T}$ for any $\alpha \geq 0$ and $\mathbf{T} \succeq 0$. Note that this solution is infeasible since the dual function becomes unbounded with $\alpha \to \infty$, i.e., $\mathcal{G}(\{\mu_i\}, \nu) \to \infty$. Thus, it follows $\mu_i^\star > 0, \forall i$.

Next, we prove $\nu^\star = \lambda_{\mathbf{B},1}$, where $\lambda_{\mathbf{B},1}$ reflects the maximum eigenvalue of the matrix $\mathbf{B} \triangleq \sum_{i=1}^K \mu_i^\star \eta_i \mathbf{G}_i^H \mathbf{G}_i$. To avoid the unbounded dual function, the matrix $\mathbf{A} \triangleq \mathbf{B} - \nu^\star \mathbf{I}_M$ must have no positive eigenvalues, i.e., $\mathbf{A}$ should be negative semidefinite, since, otherwise, we have an infeasible solution $\mathbf{W}^\star = \alpha \mathbf{u}_\mathbf{A} \mathbf{u}_\mathbf{A}^H$ with $\alpha \to \infty$, where $\mathbf{u}_\mathbf{A}$ represents the eigenvector of $\mathbf{A}$ associated with the positive eigenvalue $\lambda_\mathbf{A}$.

Let us denote the eigenvalue decomposition of matrix $\mathbf{A}$ as $\mathbf{A} = \mathbf{U}_\mathbf{B}(\mathbf{\Lambda}_\mathbf{B} - \nu^\star \mathbf{I}_M)\mathbf{U}_\mathbf{B}^H$, where $\mathbf{U}_\mathbf{B} \in \mathbb{C}^{M\times M}$ and $\mathbf{\Lambda}_\mathbf{B} = \mathrm{diag}(\lambda_{\mathbf{B},1}, \ldots, \lambda_{\mathbf{B},M})$ with $\lambda_{\mathbf{B},1} \geq \cdots \geq \lambda_{\mathbf{B},M}$ stand for the eigenvector matrix and the eigenvalue matrix of $\mathbf{B}$, respectively. Due to the fact $\mu_i^\star > 0$, matrix $\mathbf{B}$ is always positive semidefinite, and thus, the eigenvalues $\lambda_{\mathbf{B},k}$ for $k = 1, \ldots, M$ are nonnegative. Therefore, to ensure that $\mathbf{A}$ has nonpositive eigenvalues, i.e., $\lambda_{\mathbf{B},k} - \nu^\star \leq 0$ for $k = 1, \ldots, M$, it follows $\nu^\star \geq \lambda_{\mathbf{B},1} > 0$.[4]

The KKT conditions of problem (6) with respect to $\mathbf{W}^\star$ are expressed as [3], [12]

$$\mathbf{A}\mathbf{W}^\star = \mathbf{0} \tag{16}$$

$$\nu^\star \left(\tau P - \mathrm{tr}(\mathbf{W}^\star)\right) = 0 \tag{17}$$

$$\mathbf{W}^\star \succeq \mathbf{0}. \tag{18}$$

It is clear that if $\nu^\star > \lambda_{\mathbf{B},1}$, matrix $\mathbf{A} = \mathbf{B} - \nu^\star \mathbf{I}_M$ becomes a negative-definite and full-rank matrix. Thus, we have $\mathbf{W}^\star = \mathbf{0}$ from (16), which contradicts the complementary slackness condition (17) since $\nu^\star > 0$. As a result, the optimal dual variable $\nu^\star$ is given by $\nu^\star = \lambda_{\mathbf{B},1}$.

Now, we show that the optimal $\mathbf{W}^\star$ is expressed as (10). It is worthwhile to note that since $\nu^\star = \lambda_{\mathbf{B},1}$, the null space of $\mathbf{A}$ is spanned by a vector $\mathbf{u}_{\mathbf{B},1}$, i.e., $\mathbf{A}\mathbf{u}_{\mathbf{B},1} = \mathbf{0}$, where $\mathbf{u}_{\mathbf{B},1}$ accounts for the unit-norm eigenvector of $\mathbf{B}$ corresponding to the maximum eigenvalue $\lambda_{\mathbf{B},1}$. For this reason, the optimal conditions (16) and (18) are achieved with $\mathbf{W}^\star = \beta \mathbf{u}_{\mathbf{B},1}\mathbf{u}_{\mathbf{B},1}^H$ for any nonnegative number β. To find β, we employ the complementary slackness condition (17). Due to the fact $\nu^\star > 0$, it follows $\tau P - \mathrm{tr}(\mathbf{W}^\star) = 0$ from (17). Therefore, we have $\mathrm{tr}(\mathbf{W}^\star) = \beta = \tau P$. This completes the proof.

[4]The maximum eigenvalue $\lambda_{\mathbf{B},1}$ of matrix $\mathbf{B}$ is always positive, since the rank of $\mathbf{B}$ is greater than 1 in general due to the fact $\mu_i^\star > 0 \, \forall i$.

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