

Maximization of Minimum Rate for Wireless Powered Communication Networks in Interference Channel

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Abstract—We investigate wireless powered communication networks in an interference channel. In this system, due to asymmetric time allocation of the downlink and the uplink among multiple cells, cross-link interference may occur, which significantly affects overall performance. Considering this interference issue, we study a minimum rate maximization problem to overcome a severe imbalance on a rate distribution among users. The minimum rate maximization problem is non-convex; thus we propose an algorithm that updates the time allocation and the users' transmit power based on the Lagrangian duality method and the Perron–Frobenius theorem, respectively. Simulation results verify that the proposed methods outperform conventional schemes.

Index Terms—Wireless powered communication network (WPCN), interference channel.

I. INTRODUCTION

ENERGY harvesting (EH) techniques based on radio frequency (RF) signals have attracted much attention recently due to their capability of charging electronic devices without wires [1]. One of the most popular RF EH structures is a wireless powered communication network (WPCN) [2]–[4] where a hybrid access point (H-AP) broadcasts the wireless energy transfer (WET) signals in a downlink (DL), while a user node harvests those signals to transmit the wireless information transmission (WIT) signals in an uplink (UL) [3]. The sum-rate maximization methods for the WPCN system were widely studied with different configurations [2], [3].

It has been known that the WPCN systems suffer from the “doubly near-far” phenomenon [3], in which a higher data rate is assigned to only users with good communications conditions, when the sum-rate maximization is considered. Therefore, a fairness issue becomes important for practical WPCN designs. There have been a few studies on fairness-aware performance maximization in the WPCN systems [3], [4]. However, it should be emphasized that most works on the WPCN have been dedicated to a single cell scenario, and [5] is the only one which considered a two-user interference channel (IFC) and maximized the sum-rate performance to the best of our knowledge.

In this letter, we study the WPCN in a IFC where the WET and the WIT operations in each cell are conducted in a time division duplex (TDD) mode. In this TDD based WPCN system, the time duration for the WET and the WIT in different

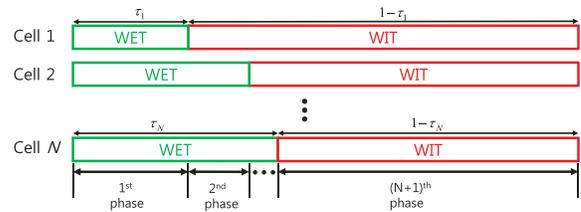


Fig. 1. Illustration of the WET and WIT operation in each cell.

cells may not be same in general, and as a result, cross-link interference occurs [5]. In other words, the WET signal from a certain cell detrimentally affects the information decoding at other cells. Therefore, the DL and the UL should be jointly taken into account when designing the WPCN in the IFC.

For this asymmetric configuration, to overcome the aforementioned unfairness issue in sum-rate maximization methods, we examine a problem which maximizes the minimum rate. Notice that the conventional sum-rate maximization technique in [5] cannot be directly applied to our general multi-cell WPCN scenario, due to the extended system model and the difference in the objective function. We thus propose an algorithm which iteratively optimizes the power and the time duration. First, by applying the Perron-Frobenius theorem [6], we present a power allocation solution with given time. Subsequently, by fixing power, we allocate the DL time duration of each cell by utilizing the Lagrangian duality method. From simulation results, we confirm that by exploiting cross-link interference efficiently, the asymmetric protocol outperforms the symmetric protocol, which is quite different from the conventional communication systems.

II. SYSTEM MODEL

We depict a WPCN in a N -user IFC [7], [8], where each H-AP in N cells serves its user in the same frequency band. It is assumed that all the H-APs and the users are equipped with a single antenna and global channel state information (CSI) is available for every cell. As shown in Fig. 1, we adopt the harvest-then-transmit protocol where each H-AP conducts the WET and WIT in a TDD manner. We assume that the total operation time for one system block is equal to one and denote the time durations of the DL and the UL for cell j ($j = 1, \dots, N$) as τ_j and $1 - \tau_j$, respectively. Without loss of generality, we consider $\tau_1 \leq \tau_2 \leq \dots \leq \tau_N$.

Then, the system can be divided into $N + 1$ phases (Fig. 1). In the first phase $0 \leq t \leq \tau_1$, all H-APs perform the WET operation. Let us define the UL channel between H-AP j and user i as $h_{i,j}$ for $i, j = 1, 2, \dots, N$. Then, assuming channel reciprocity, the DL channel becomes $h_{i,j}^*$. Denoting $r_{i,n}$ as the received signal of user i in the n th phase, the received signal of user i during the first phase is given by $r_{i,1} = \sum_{\forall k} \sqrt{p_{d,k}} h_{i,k}^* x_k + z_{u,i}$, $\forall i$, where $p_{d,j}$ and x_j with $\mathbb{E}[|x_j|^2] = 1$ stand for the transmit power and the inde-

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pendent WET signal in the DL at H-AP j , respectively, and $z_{u,i}$ indicates the complex Gaussian noise at user i with zero mean and variance σ^2 . Note the WET interference signals from the other cells act as an additional source to charge energy.

Consequently, the amount of the harvested energy $E_{i,1}$ at user i during the first phase can be written by $E_{i,1} = \eta\tau_1 \sum_{k=1}^N p_{d,k} |h_{i,k}|^2$, $\forall i$, where η represents the conversion efficiency of the EH process. Here, we ignore the harvested energy from the noise power since it is negligible compared to $p_{d,j}$ [1].

In the following n th phase ($n = 2, \dots, N+1$), H-AP j ($j = n, n+1, \dots, N$) still carries out the WET operation in the DL, whereas the remaining H-AP i ($i = 1, 2, \dots, n-1$) receives the WIT signals transmitted from user i , and as a result, cross-link interference occurs at H-AP i . The received signal $y_{i,n}$ of H-AP i is obtained as

$$y_{i,n} = \sqrt{p_{u,i}} h_{i,i} s_i + \underbrace{\sum_{k=1, k \neq i}^{n-1} \sqrt{p_{u,k}} h_{k,i} s_k}_{\text{WIT Interference}} + \underbrace{\sum_{k=n}^N \sqrt{p_{d,k}} g_{i,k} x_k}_{\text{cross-link WET Interference}} + z_{H,i}, \quad (1)$$

where $p_{u,i}$ and s_i with $\mathbb{E}[|s_i|^2] = 1$ denote the UL transmit power and the UL WIT signal of user i , respectively, $g_{j,i}$ represents the channel coefficient between H-AP j and i , and $z_{H,i}$ accounts for the complex Gaussian noise at H-AP i with zero mean and variance σ^2 . Note two different types of interference exist in (1), i.e., WIT and cross-link WET.

Since the WET signals $\{x_j\}$ do not carry any information, it can be determined in advance as an arbitrary random variable and shared among H-APs [7]. Thus, each H-AP may cancel the cross-link WET interference in (1) if such coordination is possible. Based on this, the achievable information rate $R_{i,n}$ of H-AP i ($i = 1, \dots, n-1$) at the n th phase in bits/Hz is expressed as

$$R_{i,n} = (\tau_n - \tau_{n-1}) \times \log \left(1 + \frac{p_{u,i} |h_{i,i}|^2}{\sigma^2 + \sum_{k=1, k \neq i}^{n-1} p_{u,k} |h_{k,i}|^2 + \sum_{k=n}^N \beta p_{d,k} |g_{i,k}|^2} \right) \quad (2)$$

where $\beta \in (0, 1)$ indicates the attenuation factor after the cancelation of the WET signal. Then, the total achievable rate of user i in bits/second/Hz (bps/Hz) is obtained as $R_i \triangleq \sum_{n=i+1}^{N+1} R_{i,n}$.

Meanwhile, user j ($j = n, n+1, \dots, N$) still executes the WET operation in the n th phase ($n = 2, 3, \dots, N$). Therefore, the received signal at user i in the n th phase becomes

$$r_{i,n} = \sum_{k=n}^N \sqrt{p_{d,k}} h_{i,k}^* x_k + \underbrace{\sum_{k=1, k \neq i}^{n-1} \sqrt{p_{u,k}} g_{i,k}^{\text{user}} s_k}_{\text{cross-link WIT interference}} + z_{u,i},$$

where $g_{i,k}^{\text{user}}$ equals the channel coefficient between user i and user k . Accordingly, the amount of the harvested energy at user i in the n th phase is derived as

$$E_{i,n} = \eta(\tau_n - \tau_{n-1}) \sum_{k=n}^N p_{d,k} |h_{i,k}|^2, \quad (3)$$

in which we ignore the negligible energy harvested from the WIT interference signals and noise power in (3) [9].

III. MINIMUM RATE MAXIMIZATION

In this section, we propose an iterative algorithm which finds a solution of the minimum rate maximization problem. To this end, a power allocation solution is first derived by

utilizing the Perron-Frobenius Theorem with fixed time. Then, we discuss an approach to identify a time allocation solution for given power using the Lagrangian duality method.

A. Power Allocation

We first determine a power allocation solution with a given τ by applying the Perron-Frobenius Theorem [6], which exploits a fixed point equation as will be described later. By introducing a new variable $R_{\min,E} > 0$ and the UL energy variable $\mathbf{E}_u = [E_{u,1}, \dots, E_{u,N}]^T$ where $E_{u,i} = (1 - \tau_i)p_{u,i}$, the minimum rate maximization problem with a given τ can be formulated as

$$\max_{\mathbf{E}_u, R_{\min,E}} R_{\min,E} \quad (4)$$

$$s.t. R_i(\mathbf{E}_u) \geq R_{\min,E}, \quad \forall i, \quad (5)$$

$$E_{u,i} \leq E_i, \quad \forall i. \quad (6)$$

We first present the following lemma which explains an important characteristic of the optimal solution for problem (4).

Lemma 1: For a given τ , the inequality in constraint (5) holds with equality at the optimal, i.e., $R_i(\mathbf{E}_u^*) = R_{\min,E}^*$, $\forall i$.

Proof: Since it can be easily proved, we omit the proof for the sake of brevity. ■

By inspecting Lemma 1, we confirm that the optimal $(\mathbf{E}_u^*, R_{\min,E}^*)$ for problem (4) is a solution of the fixed point equation $A(\mathbf{E}_u) = \frac{1}{R_{\min,E}} \mathbf{E}_u$, where

$$A(\mathbf{E}_u) \triangleq [A_1(\mathbf{E}_u), A_2(\mathbf{E}_u), \dots, A_N(\mathbf{E}_u)]^T \quad (7)$$

with $A_i(\mathbf{E}_u) \triangleq \frac{E_{u,i}}{R_i(\mathbf{E}_u)}$.

Next, let us denote the power constraint function $\rho(\mathbf{E}_u)$ as $\rho(\mathbf{E}_u) \triangleq \max_i \frac{E_{u,i}}{E_i}$, which scales \mathbf{E}_u into the largest feasible region in the direction of \mathbf{E}_u , i.e., $\mathbf{E}_u / \rho(\mathbf{E}_u)$. Then, it can be shown that the following properties, which will be later utilized for the Perron-Frobenius, hold for $\rho(\mathbf{E}_u)$:

1) Positive homogeneity: $\rho(\lambda \mathbf{E}_u) = \lambda \rho(\mathbf{E}_u)$.

2) Monotonicity: $\mathbf{E}_u \leq \hat{\mathbf{E}}_u$ implies $\rho(\mathbf{E}_u) \leq \rho(\hat{\mathbf{E}}_u)$.

We can check that the optimal \mathbf{E}_u^* of problem (4) is in the set $\mathcal{U} \triangleq \{\mathbf{E}_u | \rho(\mathbf{E}_u) = 1\}$, since at least one constraint in (6) should meet with equality at the optimal \mathbf{E}_u^* , i.e., $E_{u,i}^* = E_i$, for some i [10, Lemma 1]. Hence, problem (4) can be reformulated into the conditional eigenvalue problem as

$$\text{Find } R_{\min,E}, \mathbf{E}_u \quad s.t. A(\mathbf{E}_u) = \frac{1}{R_{\min,E}} \mathbf{E}_u, \quad \text{for } \mathbf{E}_u \in \mathcal{U}. \quad (8)$$

To solve problem (8) optimally, we first prove the conditions related to the function $A(\mathbf{E}_u)$ in the following lemma.

Lemma 2: The function $A(\mathbf{E}_u)$ in (7) satisfies the following:

1) There exist positive numbers b, c , and a vector $\mathbf{d} > \mathbf{0}$ such that $b\mathbf{d} \leq A(\mathbf{E}_u) \leq c\mathbf{d}$ for all $\mathbf{E}_u \in \mathcal{U}$.

2) If $\lambda \mathbf{E}_u \leq \tilde{\mathbf{E}}_u$, then $\lambda A(\mathbf{E}_u) \leq A(\tilde{\mathbf{E}}_u)$ for any $\mathbf{E}_u, \tilde{\mathbf{E}}_u \in \mathcal{U}$ and $0 \leq \lambda \leq 1$.

Proof: Refer to Appendix I. ■

Now, for a given τ , we can optimally compute the power allocation solution \mathbf{E}_u^* by applying the Perron-Frobenius Theorem [6] as stated in the following theorem.

Theorem 1: If $A(\mathbf{E}_u)$ satisfies the conditions in Lemma 2 and (\mathbf{E}_u) fulfills positive homogeneity and monotonicity, the optimal solution of (8) exhibits the following properties:

1) $A(\mathbf{E}_u) = \frac{1}{R_{\min,E}} \mathbf{E}_u$ has a unique solution $\mathbf{E}_u^* \in \mathcal{U}$ and $R_{\min,E}^* > 0$.

2) Defining $\tilde{A}^k(\mathbf{E}_u)$ as the value of $\tilde{A}(\mathbf{E}_u) = A(\mathbf{E}_u)/\rho(\mathbf{E}_u)$ at the k th iteration, it follows $\mathbf{E}_u^* = \lim_{k \rightarrow \infty} \tilde{A}^k(\mathbf{E}_u)$ for any $\mathbf{E}_u \geq 0$.

Proof: Refer to [6]. \blacksquare

Theorem 1 indicates that the fixed point iteration, which calculates $\mathbf{E}_u = A(\mathbf{E}_u)$ and $\mathbf{E}_u = \frac{\mathbf{E}_u}{\rho(\mathbf{E}_u)}$ iteratively, yields the optimal \mathbf{E}_u^* and $R_{\min,E}^*$ of problem (8).

B. Time Allocation

We next identify a time allocation solution for a fixed \mathbf{E}_u . We adopt the change of variable as $a_i = \tau_{i+1} - \tau_i \forall i$, and correspondingly the achievable rate of R_i can be rewritten by

$$R_i(\mathbf{a}) = \sum_{n=i+1}^{N+1} a_{n-1} \times \log \left(1 + \frac{E_{u,i}^n |h_{i,i}|^2 / a_{n-1}}{\sigma^2 + \sum_{k \neq i}^{n-1} \frac{E_{u,k}^n |h_{k,i}|^2}{a_{n-1}} + \sum_{k=n}^N \beta p_{d,k} |g_{i,k}|^2} \right), \quad (9)$$

where $E_{u,i}^n \triangleq E_{u,i} \frac{\tau_n - \tau_{n-1}}{1 - \tau_i}$ represents the energy transmitted by user i in the n th phase and $\mathbf{a} = [a_1, \dots, a_N]^T$. Then, by introducing an auxiliary variable $R_{\min,T}$, the minimum rate maximization problem for given \mathbf{E}_u can be formulated as

$$\max_{\mathbf{a} \geq 0, R_{\min,T}} R_{\min,T} \quad (11)$$

$$s.t. R_i(\mathbf{a}) \geq R_{\min,T}, \forall i, \quad (12)$$

$$E_i(\mathbf{a}) \geq E_{u,i}, \forall i, \quad (13)$$

$$\mathbf{1}^T \mathbf{a} \leq 1. \quad (14)$$

To obtain the globally optimal solution for problem (11), we employ a two-layer algorithm where $R_{\min,T}$ is identified by a line search method in the outer loop, while \mathbf{a} is computed in the inner loop by solving the following feasibility problem with a given $R_{\min,T}$:

$$\text{Find } \mathbf{a}, s.t. (12), (13), (14). \quad (15)$$

It is emphasized that the feasibility of problem (15) can be judged by comparing $R_i(\mathbf{a})$ and $R_{\min,T}$. If $R_i(\mathbf{a}) < R_{\min,T}$ for some i , problem (15) is infeasible. Thus, $R_{\min,T}$ should be diminished to make problem (15) feasible. In contrast, if $R_i(\mathbf{a}) \geq R_{\min,T}, \forall i$, which indicates that problem (15) is feasible, we could increase $R_{\min,T}$ in the next iteration. As a result, $R_{\min,T}^*$ can be determined in the outer loop by a simple bisection search method. In what follows, we will focus on solving the inner feasibility problem (15) with a fixed $R_{\min,T}$.

By examining the Hessian matrix of $R_i(\mathbf{a})$, it can be verified that $R_i(\mathbf{a})$ is a concave function with respect to \mathbf{a} . Also, since (13) and (14) are affine constraints, problem (15) is a convex feasibility problem. Therefore, problem (15) can be optimally solved by leveraging the Lagrange duality method with zero duality gap [3]. The Lagrangian of problem (15) is given by $\mathcal{L}_1(\mathbf{a}, \boldsymbol{\omega}) = \sum_{i=1}^N \omega_i (R_i(\mathbf{a}) - R_{\min,T})$, where $\boldsymbol{\omega} = [\omega_1, \dots, \omega_N]^T$ is the non-negative dual variable vector associated with the constraint (12).

The dual function $\mathcal{G}_1(\boldsymbol{\omega})$ is then determined as

$$\mathcal{G}_1(\boldsymbol{\omega}) = \max_{\mathbf{a}} \mathcal{L}_1(\mathbf{a}, \boldsymbol{\omega}), \quad s.t. (13), (14). \quad (16)$$

Since problem (16) now becomes the weighted sum-rate maximization problem with a fixed $\boldsymbol{\omega}$, the gradient projection method can be applied as in [5]. A detailed procedure of solving (16) is omitted for the sake of brevity.

Once the dual function is identified for a given $\boldsymbol{\omega}$, the dual problem $\min_{\boldsymbol{\omega} \geq 0} \mathcal{G}_1(\boldsymbol{\omega})$ can be addressed by the ellipsoid

method. The sub-gradient of the dual function $\mathcal{G}_1(\boldsymbol{\omega})$ is expressed by $\nu_{\omega_i} = R_i(\mathbf{a}) - R_{\min,T}, \forall i$. Note that the time allocation solution \mathbf{a}^* for problem (15) is obtained when $\boldsymbol{\omega}$ converges to the optimal dual variable $\boldsymbol{\omega}^*$. Correspondingly, the minimum achievable rate can be computed by $R_i(\mathbf{a}^*)$. In summary, with a given \mathbf{E}_u , the optimal τ^* for problem (11) can be calculated by solving the feasibility problem (15) with the converged $R_{\min,T}^*$.

The proposed algorithm is presented in Algorithm 1. Note that $R_{\min,E}$ derived in Section IV-A is set as an initial R_L , which represents a lower bound for the bisection search, in order to guarantee the monotonic increase at every alternation. Owing to the characteristic of the alternating procedure, to improve the performance, we randomly generate M feasible τ as the initial points and choose the best one as a solution in Algorithm 1.

Algorithm 1 Time and Power Allocation Algorithm

Initialize τ .

Repeat

Initialize \mathbf{E}_u .

Repeat

Update $\mathbf{E}_u = A(\mathbf{E}_u)$ and normalize the energy as $\mathbf{E}_u = \frac{\mathbf{E}_u}{\rho(\mathbf{E}_u)}$.

Until $R_{\min,E}^*$ and \mathbf{E}_u^* converge

Set $R_L = R_{\min,E}^*$ and initialize R_U as any large number.

Repeat

Set $R_{\min,T} = \frac{1}{2}(R_L + R_U)$.

Obtain τ^* by solving problem (15).

If $R_i(\tau_i) \leq R_{\min,T}$ for some i

Update $R_U = R_{\min,T}$.

Else

Update $R_L = R_{\min,T}$.

Until convergence

Until convergence

IV. SIMULATION RESULTS

In this section, we evaluate the minimum rate performance of the WPCN in the IFC. Throughout this section, as in [8], the UL and DL channel gains are generated as $|h_{i,j}|^2 = d_{i,j}^{-\alpha} \kappa_{i,j} |\tilde{h}_{i,j}|^2$, where $d_{i,j}$ and $\tilde{h}_{i,j}$ are the distance and the small-scale Rayleigh fading coefficient between H-AP i and user j , respectively, $\alpha = 3$ represents the pathloss exponent, and $\kappa_{i,j}$ indicates the strength of the interference channel. We set $d_{i,j} = 10$ m $\forall i, j$, $\kappa_{i,i} = 1 \forall i$, and $\kappa_{i,j} = \tilde{\kappa}$ for $i \neq j$ otherwise stated. Similarly, the channel gain of the cross-link $|g_{j,i}|^2$ is determined as $|g_{j,i}|^2 = d_H^{-\alpha} \tilde{\kappa} |\tilde{g}_{j,i}|^2$, where the distance equals $d_H = 15$ m. Also, we fix the conversion efficiency of the EH process as $\eta = 0.5$, the noise power as $\sigma^2 = -50$ dBm [2], the transmit power $p_{d,j}$ at H-AP j as $p_{d,j} = P_T \forall j$, the number of initial points for Algorithm 1 to $M = 20$, and the attenuation faction as $\beta = -50$ dBm.

We compare the following two baseline schemes:

- Symmetric protocol: The DL time allocation for each cell is same, i.e., $\tau_1 = \dots = \tau_N \triangleq \tau$. Then, the time allocation τ can be obtained through an one-dimensional exhaustive search over $0 \leq \tau \leq 1$ in the outer loop. For a given τ , the UL power allocation is implemented by the proposed method in Section III-A.
- Non-cooperation scheme: Each cell assigns its UL transmit power and time allocation based on the single cell WPCN solution in [3] with $\beta = 1$.

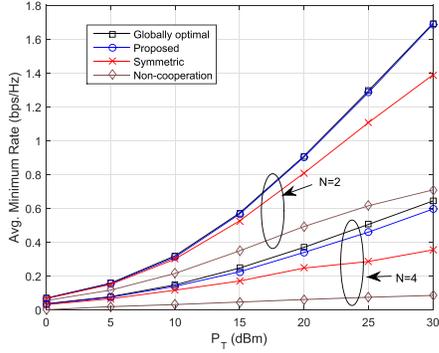
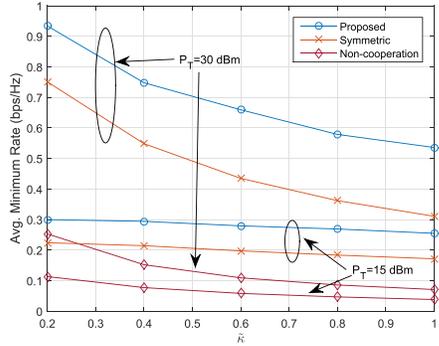
Fig. 2. Average minimum rate performance with respect to P_T .Fig. 3. Average minimum rate performance with respect to $\tilde{\kappa}$.

Fig. 2 presents the minimum rate performance of the WPCN in the IFC with $\tilde{\kappa} = 0.8$ as a function of P_T . We also plot the performance of the globally optimal scheme where time allocation solution is obtained through exhaustive search and the uplink power allocation is implemented by the proposed method in Section III.A. First, only negligible performance gap is observed between the optimal scheme and the proposed algorithm. Next, we can confirm that the proposed asymmetric protocol outperforms the symmetric protocol, and the performance gap increases as P_T and N grow. This is because through the cross-link interference link, the harvested energy of users is efficiently utilized when the H-APs and the users operate in the asymmetric manner. Moreover, the non-cooperation scheme exhibits a significant performance degradation, since each cell egoistically allocates the time and the UL power, resulting in severe WIT and cross-link WET interferences to the other cells. For quantitative comparison, the proposed minimum rate maximization algorithm exhibits 32.9% and 82.3% performance gains at $P_T = 25$ dBm and $N = 4$ compared to the symmetric protocol and the non-cooperation scheme, respectively.

Next, to see the impact of the interference channel condition on overall performance, Fig. 3 illustrates the minimum rate performance as a function of $\tilde{\kappa}$ with $N = 4$. It is observed that the performance of all three schemes constantly decreases as the power of the interference link $\tilde{\kappa}$ grows. This infers that although there exists a trade-off between the amount of the harvested energy and the WIT interference, the channel condition has more influence on the WIT interference.

APPENDIX I :PROOF OF LEMMA 2

Condition 1 in Lemma 2 is straightforward to verify, and thus we omit the proof. For condition 2, we will prove by

establishing the following inequalities

$$\lambda A(\mathbf{E}_u) \leq A(\lambda \mathbf{E}_u) \leq A(\tilde{\mathbf{E}}_u). \quad (17)$$

Since $R_i(\mathbf{E}_u)$ decreases as all components of \mathbf{E}_u are multiplied by $0 \leq \lambda \leq 1$, we have $\lambda A_i(\mathbf{E}_u) = \frac{\lambda E_{u,i}}{R_i(\mathbf{E}_u)} \leq \frac{\lambda E_{u,i}}{R_i(\lambda \mathbf{E}_u)} = A(\lambda \mathbf{E}_u)$, which is the first inequality in (17).

In order to establish the second inequality in (17), we show that the function $A(\mathbf{E}_u)$ is an increasing function with respect to \mathbf{E}_u . Taking the derivative with respect to $E_{u,k}$, it follows

$$\frac{\partial A_i(\mathbf{E}_u)}{\partial E_{u,k}} = \begin{cases} \frac{\sum_{n=i+1}^{N+1} a_{n-1} \left(\log(1 + B_{i,n}) - \frac{B_{i,n}}{1+B_{i,n}} \right)}{\left(\sum_{n=i+1}^{N+1} a_{n-1} \log(1 + B_{i,n}) \right)^2}, & k = i, \\ \frac{\sum_{n=i+1}^{N+1} \frac{B_{i,n}^2 |h_{k,i}|^2}{1+B_{i,n}}}{\left(\sum_{n=i+1}^{N+1} a_{n-1} \log(1 + B_{i,n}) \right)^2}, & k \neq i, \end{cases}$$

where

$$B_{i,n} \triangleq E_{u,i}^n |h_{i,i}|^2 / \left(a_{n-1} (\sigma^2 + \sum_{k \neq i}^{n-1} \frac{E_{u,k}^n |h_{k,i}|^2}{a_{n-1}} + \sum_{k=n}^N \beta p_{d,k} |g_{i,k}|^2) \right)$$

represents the signal to interference and noise ratio in the n th phase of user i .

When $k \neq i$, we can confirm that $\frac{\partial A_i(\mathbf{E}_u)}{\partial E_{u,k}} > 0$ for $B_{i,n} > 0$. For the case of $k = i$, in order to show $\frac{\partial A_i(\mathbf{E}_u)}{\partial E_{u,i}} > 0$, we check the numerator function $\gamma(B_{i,n}) \triangleq \log(1 + B_{i,n}) - \frac{B_{i,n}}{1+B_{i,n}}$. Since $\frac{\partial \gamma(B_{i,n})}{\partial B_{i,n}} = \frac{B_{i,n}}{(1+B_{i,n})^2}$ is always positive for $B_{i,n} > 0$ and $\gamma(0) = 0$, this indicates $\gamma(B_{i,n}) > 0$. Then, we can conclude that $\frac{\partial A_i(\mathbf{E}_u)}{\partial E_{u,k}} > 0$ for all $E_{u,k}$, and thus $A(\mathbf{E}_u)$ increases as \mathbf{E}_u grows. To conclude, due to the monotonically increasing property of $A(\mathbf{E}_u)$, the second inequality in (17) always holds, and thus we prove $\lambda A(\mathbf{E}_u) \leq A(\tilde{\mathbf{E}}_u)$.

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