Energy Efficient Online Power Allocation for Two Users With Energy Harvesting

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***Abstract*—In this letter, we propose an online power allocation (PA) method to maximize energy efficiency (EE) in energy harvesting systems with realistic battery constraints for two receivers. The optimization problem in this system configuration is challenging since it has a non-convex fractional form and the information of channel quality and the harvested energy can only be obtained causally in practice. To overcome these issues, we design time-average EE maximization through Lyapunov optimization techniques where the transmission power is computed with current battery information and channel fading. Moreover, based on random matrix theory, we present a simplified online EE algorithm. Numerical experiments verify that the proposed PA outperforms conventional online approaches with much reduced computational complexity.**

***Index Terms*—Energy harvesting, Lyapunov optimization, online power allocation.**

I. Introduction

ENERGY efficiency (EE) has been regarded as an important metric for designing green communication systems. Also, energy harvesting technique has been recognized as a promising method which utilizes renewable energy as a meaningful energy source to support devices [1], [2] and improve the EE performance of wireless networks [3]. Since, the EE is an important issue for communication systems with energy harvesting [4]–[8].

Several off-line EE power allocation (PA) designs were proposed for orthogonal frequency division multiple access systems [4] and fading channels [9], where all the amount of the harvested energy is assumed to be known beforehand. However, for practical systems, the information of both the channel quality and the harvested energy can only be acquired causally. Some works have presented online PA strategies which maximize spectral efficiency (SE) based on the causal system information [10], [11]. However, these works require the information on statistics of the harvested energy and channel fading at the transmitter, and produce solutions with high computational complexity. Therefore, an online EE PA scheme is desirable which relies only on the current harvested energy, and the fading condition without their statistical knowledge.

In the context of PA without statistical knowledge, many recent works have optimized transmission rate dependent utility functions with dual stochastic optimization techniques. A stochastic descent-based algorithm was proposed in [12] to maximize ergodic transmission rate. The work [13] presented cross-layer resource allocation to optimize linear and logarithmic functions of the transmission rates. An online EE algorithm for a single user scenario was introduced in [14]. However, its implementation requires prediction of the future channels which can increase the complexity of the online algorithm. Also, the authors in [15] studied a weighted sum rate maximization routing algorithm, where transmitters have energy harvesting capabilities, and the maximization is performed under energy causality and data queue stability constraints. A multi-input multi-output downlink system with energy trading between a base station (BS) and the main grid was investigated in [16], where the throughput is maximized under energy cost constraints.

The Lyapunov technique, which can be seen as a specific form of a dual stochastic algorithm [17], [18], can be used to solve the optimization problems that involve online allocation of communication resources. In recent years, the authors in [19] have proposed an online PA method for SE maximization based on the Lyapunov technique which does not require the information on the random process. Motivated by this scheme, we investigate the online PA for EE maximization for a single transmitter and two receiver system where the transmitter can harvest energy. The main differences between our work and [14] lie in the fact that we consider a two-user scenario and energy harvesting at the transmitter. Although a system with more than two receivers is more practical, the problem becomes intractable since a closed form solution is not feasible, and this can be an interesting topic for future work. One possible approximate solution could be to gather channel state information of all users in each time slot and pick only two users. Then, the proposed algorithm can be applied for the selected two users.

We first derive the battery constraint with the harvested energy and formulate the EE maximization problem from the derived battery constraint. Then, we relax the battery constraint as a time-average expression to deal with information causality, and represent the battery constraint as a virtual queue. Next, we reformulate the EE maximization problem as a queue stability problem and obtain an online PA for EE maximization by solving the queue stability problem. Also, in order to reduce

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the computational complexity, we propose a simplified PA algorithm based on random matrix theory (RMT).

II. System Model and Problem Formulation

We consider a wireless transmission system, where a transmitter equipped with N antennas is operated by energy harvesting devices and supports two single antenna receivers. It is assumed that the system operates in discrete time t with duration Δt. The channel vector $\mathbf{h}_i(t) \in \mathbb{C}^N$ between the transmitter and the i-th receiver $(i = 1, 2)$ remains constant during the time t, and $n_i(t) \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise at the i-th receiver. Let us denote $E_h(t)$ as the amount of the harvested energy into the battery at the transmitter at time t and $E_b(t)$ as the battery level at time t which is expressed by $E_{\min} \le E_b(t) \le E_{\max}$, where $E_{\min}$ and $E_{\max}$ represent a lower bound and an upper bound of energy levels in the battery, respectively. Let $E_{c,\max}$ and $P_{\max}$ be the maximum charging amount and the maximum transmit power which satisfies $\Delta t P_{\max} \le E_{\max} - E_{\min}$, respectively. We assume that $E_{c,\max} \le \Delta t P_{\max}$. Defining $P(t)$ as the transmit power at time t, which remains unchanged from the time t to $t+1$, we have $0 \le P(t) \le P_{\max}, \forall t$.

The dynamics of $E_b(t)$ over time is computed in [20] as

$$E_b(t+1) = E_b(t) - \Delta t\left(\zeta P(t) + P_c\right) + E_h(t), \tag{1}$$

where ζ and P_c indicate the inefficiency of a power amplifier and the constant power consumption term, respectively. Then, $P(t)$ is bounded by $\Delta t P(t) \le E_b(t) - E_{\min} - \Delta t P_c$. The harvested energy $E_h(t)$ is determined by

$$E_h(t) = \min\{E_{c,\max}, E_a(t), E_{h,\max}(t)\}, \tag{2}$$

where $E_a(t)$ represents the arrived energy and the maximum harvestable energy $E_{h,\max}(t)$ is defined as $E_{h,\max}(t) = E_{\max} - E_b(t) + \Delta t(\zeta P(t) + P_c)$.

Assuming that the i-th user is allocated with ρ_i portion of the total power $P(t)$ and μ_i portion of the total bandwidth, we can write the rate of the i-th user as $R_i(t) = \mu_i \log(1 + \frac{\rho_i \gamma_i(t) P(t)}{\mu_i})$, where $\gamma_i(t) \triangleq ||\mathbf{h}_i(t)||^2/\sigma^2$. In this letter, for the ease of analysis, the total bandwidth and the transmit power are equally shared between two users during every time slots ($\mu_i = \rho_i = \frac{1}{2}$). In this case, the instantaneous rate over the channel for the i-th user is $R_i(t) \triangleq \frac{1}{2}\log(1 + \gamma_i(t)P(t))$. Then, we propose a PA algorithm to maximize the time-average EE while satisfying the battery operational constraints. The time-average EE is defined by the ratio between the cumulative sum of the instantaneous rate and the corresponding power consumption.

The problem can be formulated as

$$\max_{\{P(t)\}} \quad \lim_{T\to\infty} \frac{\sum_{t=0}^{T-1}(R_1(t) + R_2(t))}{\sum_{t=0}^{T-1}(\zeta P(t) + P_c)} \tag{3}$$

$$\begin{aligned} \text{s.t.} \quad & 0 \le P(t) \le P_{\max}, \\ & E_b(t+1) = E_b(t) - \Delta t\left(\zeta P(t) + P_c\right) + E_h(t), \\ & \Delta t\left(\zeta P(t) + P_c\right) \le E_b(t) - E_{\min}. \end{aligned} \tag{4}$$

Due to the randomness of $E_h(t)$ and $\gamma_i(t)$, the optimization problem (3) is difficult to solve. Furthermore, the constraints depend on $E_b(t)$, which has time-coupling dynamics over time.

This results in the PA decisions $\{P(t)\}$ being correlated over time. If the random processes $\{\gamma_i(t)\}$ and $\{E_h(t)\}$ are Markov and their statistics are known, it is possible to solve problem (3) through DP. However, this approach typically faces a high dimensionality issue, and thus it is not easy to provide a solution efficiently. Moreover, in practice, the statistical knowledge of $\{\gamma_i(t)\}$ and $\{E_h(t)\}$ may not be available in advance, making such an assumption less realistic.

In this letter, we aim to suggest an online EE PA algorithm which does not resort to the statistical information of $\{\gamma_i(t)\}$ and $\{E_h(t)\}$. To this end, we employ the Lyapunov optimization framework [21] so that a time-average optimization problem is transformed into a queue stability problem. First, in order to apply the Lyapunov optimization method, we relax the time-coupled dynamics on the time slot constraints $E_b(t)$, $E_h(t)$, and $P(t)$ by adopting the time-average relation.

From (1), the relation of the battery dynamics over time T can be expressed as

$$E_b(T) - E_b(0) = \sum_{t=0}^{T-1}(E_h(t) - \Delta t(\zeta P(t) + P_c)). \tag{5}$$

Then, after some mathematical manipulations with $T \to \infty$, we can obtain the time-average equation by

$$\bar{E}_h - \Delta t(\zeta \bar{P} + P_c) = 0, \tag{6}$$

where $\bar{E}_h = \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} E_h(t)$ and $\bar{P} = \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} P(t)$ [20].

Now, the optimization problem in (3) is relaxed by replacing the dynamics in (4) with the long-term time-average constraint (6). Then, the equivalent problem of (3) is written as

$$\begin{aligned} \max_{\{P(t)\}, \eta \in \mathbb{R}^+} \quad & \eta \\ \text{s.t.} \quad & \lim_{T\to\infty} \sum_{t=0}^{T-1} \left(\sum_{i=1}^{2} R_i(t) - \eta(\zeta P(t) + P_c) \right) \ge 0, \\ & 0 \le P(t) \le P_{\max}, \bar{E}_h - \Delta t(\zeta \bar{P} + P_c) = 0. \end{aligned} \tag{7}$$

For a given parameter η, this problem turns into a feasibility problem in $\{P(t)\}$. Denoting a function $F(\eta)$ as

$$F(\eta) = \max_{\{P(t)\}} \lim_{T\to\infty} \sum_{t=0}^{T-1} \left(\sum_{i=1}^{2} R_i(t) - \eta(\zeta P(t) + P_c) \right), \tag{8}$$

the problem is solved by finding a root of the function $F(\eta)$, i.e., $F(\eta) = 0$, as shown in [22]. Consequently, the EE formulation in (3) can be transformed into a parametric subtractive form, and a two-layer optimization is required to solve the EE problem [23]. We compute the PA $\{P(t)\}$ for a fixed η in the inner layer, and find the optimal η which satisfies $F(\eta) = 0$ by exploiting an one-dimensional search method in the outer layer.

III. Online EE Power Allocation

A. *EE Power Allocation With Lyapunov Optimization*

Let us introduce the virtual queue $Q(t)$ for $E_b(t)$ as $Q(t) = E_b(t) - C$, where C is a time-independent constant. Note that stabilizing the queue $Q(t)$ is equivalent to satisfying (6). Then, the dynamics of $Q(t)$ is given by

$$Q(t+1) = Q(t) + E_h(t) - \Delta t(\zeta P(t) + P_c). \tag{9}$$

For a given η, the drift-plus-cost metric with the constant $V > 0$ is written as

$$\mathcal{D} = \Delta(Q(t)) - V \cdot G(\eta, t),$$

where $G(\eta, t) = \sum_{i=1}^{2} R_i(t) - \eta(\zeta P(t) + P_c)$.

For any value of $Q(t)$ and $V \geq 0$, the drift-plus-cost metric can be upper bounded as [19]

$$\mathcal{D} \leq \phi + Q(t)\left(E_h(t) - \Delta t(\zeta P(t) + P_c)\right) - VG(\eta, t), \quad (10)$$

where $\phi = \Delta t^2 (P_{\max} + P_c)^2/2$. Hence, we have the equivalent optimization problem as

$$\min_{0 \leq P(t) \leq P_{\max}} Q(t)\left(E_h(t) - \Delta t(\zeta P(t) + P_c)\right) - VG(\eta, t). \quad (11)$$

Since problem (11) is convex and differentiable, it can be solved with the derivative of the objective function in (11) with respect to $P(t)$. Note that if we do not assume equal power and bandwidth sharing, then (11) is not a convex optimization problem. The non-convexity of the optimization problem makes the analysis intractable. Furthermore, for $K > 2$ users, finding $P^*(t)$ in closed form is not possible. Therefore, we only consider the two-user case which provides a closed form solution.

Taking the first derivative of the objective function of (11) and equating it to zero, we get

$$\Delta t \zeta Q(t) - V\eta\zeta + \frac{1}{2}\sum_{i=1}^{2} \frac{\gamma_i(t)}{(1 + P(t)\gamma_i(t))} = 0.$$

It can be shown that a possible solution for $P^*(t)$ is given as $P_1^*(t) = \frac{1}{2}(-(\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \theta) + \sqrt{(\frac{1}{\gamma_1} - \frac{1}{\gamma_2})^2 + \theta^2})$, where $\theta = V/(\Delta t \zeta Q(t) - V\eta\zeta)$. Since, we have $0 \leq P^*(t) \leq P_{\max}$, for a given η, we can compute the optimal $P^*(t)$ in closed form as

$$P^*(t) = \begin{cases} 0 & \text{for } Q(t) < \alpha, \\ P_1^*(t) & \text{for } \alpha \leq Q(t) \leq \beta, \\ P_{\max} & \text{for } Q(t) > \beta, \end{cases} \quad (12)$$

where $\alpha = V(\eta\zeta - \frac{\gamma_1(t) + \gamma_2(t)}{2})/(\Delta t \zeta)$, $\beta = V(\eta\zeta - \kappa)/(\Delta t \zeta)$, $\kappa = (\frac{1}{2\gamma_1} + \frac{1}{2\gamma_2} + P_{\max})/(P_{\max}^2 + \frac{P_{\max}}{\gamma_1} + \frac{P_{\max}}{\gamma_2} + \frac{1}{\gamma_1\gamma_2})$ and. Thus, the transmitter at time slot t utilizes the system state $\{E_b(t), \gamma_1(t), \gamma_2(t)\}$ and identifies the transmit power $P^*(t)$ using (12). Then, $Q(t)$ is updated according to (9). Provided that $Q(t)$ is given, we note that calculating $P^*(t)$ does not depend on any statistical information of $E_h(t)$ or $\gamma_i(t)$.

As mentioned earlier, a solution of (11) may not be feasible to (3) since (7) is a relaxed problem. To make sure that the online EE power solution is feasible for the original problem, we need to check if $E_b(t)$ satisfies the battery constraint by determining the values of C and V. To provide a feasible online solution, it is assumed that the channel gains $||\mathbf{h}_i(t)||^2$ are bounded as $\gamma_i(t) \leq \gamma_{\max}$, where $\gamma_{\max}$ indicates the maximum gain. Then, in the following, we provide upper and lower bounds of the virtual queue $Q(t)$ as in [19].

With the proposed EE PA solution $P^*(t)$, the virtual queue $Q(t)$ is bounded for all t as $Q_{\min} \leq Q(t) \leq Q_{\max}$, where $Q_{\min} = -\Delta t P_{\max} + \frac{V}{\Delta t}(\eta\zeta - \gamma_{\max})$ and $Q_{\max} = E_{c,\max}$. In (12), we know that if $Q(t) < \alpha$, then $P^*(t) = 0$. In the subsequent time slot, this makes $Q(t+1)$ always increasing, i.e., $Q(t+1) \geq Q(t)$. Therefore, we can show the relation as

$$Q(t+1) \geq Q(t) - \Delta t P_{\max} \geq \frac{V}{\Delta t}(\eta\zeta - \gamma_{\max}) - \Delta t P_{\max}.$$

The above inequality holds for any t. In a similar manner, by utilizing the relation between $Q(t)$ and $Q(t+1)$, we can check that $Q(t)$ is upper-bounded by $E_{c,\max}$. Through these upper and lower bounds of $Q(t)$, the parameters V and C for the Lyapunov optimization are calculated as

$$V \in \left(0, \frac{(E_{\max} - E_{\min} - \Delta t P_{\max} - E_{c,\max})\Delta t \zeta}{\gamma_{\max} - \eta\zeta}\right],$$

$$C = E_{\min} + \Delta t P_{\max} - \frac{V}{\Delta t}(\eta\zeta - \gamma_{\max}).$$

For practical batteries, $E_{\max} - E_{\min} - \Delta t P_{\max} - E_{c,\max}$ is a positive value [19]. Also, it can be shown that any $\eta > \frac{\gamma_{\max}}{\zeta}$ violates the first constraint of (7). Hence, it follows $V > 0$. A closed form expression for PA for the two-user helps in obtaining the values of V and C which make sure that the battery constraints are met for all time slots. Without the close form expression for the PA, it is not possible to find the appropriate value of V.

B. Simplified EE Power Allocation Based on RMT

From the above results in Section III-A, we can address the online EE PA with current information on $E_h(t)$ and $\gamma_i(t)$. However, the computational complexity is still high, since the EE algorithm should be conducted iteratively by two layer optimization scheme [24]. The outer layer determines the EE variable η, while a subtractive problem $F(\eta)$ is solved in the inner layer. The inner layer algorithm is repeatedly computed whenever η is updated at the outer layer.

To circumvent the complexity issue, we further propose a simplified EE PA method by exploiting large system analysis based on RMT [25]. In order to utilize the results of the RMT, we consider that the channel vector is expressed as $\mathbf{h}_i(t) = \sqrt{N}\mathbf{R}_i^{1/2}\mathbf{z}(t)$, where $\mathbf{R}_i$ represents the transmit covariance matrix between the transmitter and the i-th receiver, and $\mathbf{z}(t)$ indicates a column vector whose element is an independent and identically distributed complex Gaussian random variable of zero mean and variance $1/N$.

Then, applying the trace lemma in [25], we can write $\gamma(t) - \gamma_i^\circ \xrightarrow[a.s.]{N\to\infty} 0$, where $\gamma_i^\circ = \mathrm{tr}(\mathbf{R}_i)$, and $\mathrm{tr}(\mathbf{X})$ is the trace of matrix $\mathbf{X}$. Through this approach, the channel gain becomes deterministic with respect to any time slot t in a large system limit. Then, the EE maximization problem becomes

$$\max_{P} \eta^\circ(P) \triangleq \frac{T\sum_{i=1}^{2} \log(1 + \gamma_i^\circ P)}{T(\zeta P + P_c)}. \quad (13)$$

Note that $\eta^\circ(P)$ in (13) is strictly quasi-concave and the value of P that maximizes $\eta^\circ(P)$ can be obtained by differentiating $\eta^\circ(P)$ with respect to P and equating to zero.

For a special case of $\gamma_1^\circ = \gamma_2^\circ = \gamma^\circ$, the optimal transmit power can be written in close form as

$$P^\circ = \frac{1}{\gamma^\circ}\left(\exp\left(1 + \mathcal{W}\left(\frac{1}{e}\left(\frac{\gamma^\circ P_c}{\zeta} - 1\right)\right)\right) - 1\right),$$

where $\mathcal{W}(\cdot)$ means the Lambert W function. The special case $\gamma_1^\circ = \gamma_2^\circ = \gamma^\circ$ can occur in practical systems when two users are located at the same distance from the transmitter. From the above large system analysis, the time-average EE performance in a large system limit can be calculated as $\eta^\circ(P^\circ)$. We note that the large system analysis is adopted only to derive the deterministic EE parameter $\eta^\circ(P^\circ)$, and the obtained results from the large system analysis can be applied to $N < \infty$ case. With the derived deterministic EE parameter $\eta^\circ(P^\circ)$, we can compute the online

EE PA without an iterative method. Thus, the simplified EE PA algorithm achieves a significant computational complexity savings compared to the proposed algorithm in Section III-A.

C. Performance Gap Analysis

Suppose the optimal energy efficiency value $\eta^\dagger < \gamma_{\max}/\zeta$. Then we have

$$\lim_{T\to\infty}\sum_{t=0}^{T-1}\left(\sum_{i=1}^{2}R_i(t)-\eta^\dagger(\zeta P(t)+P_c)\right)\le 0. \tag{14}$$

Let us assume that there is the optimal power allocation, denoted by $\{P^{opt}(t)\}$, which maximizes $\sum_{t=0}^{T-1}(\sum_{i=1}^{2}R_i(t)-\eta^\dagger(\zeta P(t)+P_c))$ under the constraints $0\le P(t)\le P_{\max}$ and $\bar{E}_h-\Delta t(\zeta\bar{P}+P_c)=0$. As the proposed power allocation minimizes an upper bound of $\Delta(Q(t))-VG(\eta,t)$ over all possible power allocation solutions, by setting $\eta=\eta^\dagger$ for all t in the proposed algorithm, we get the following inequality

$$\begin{aligned}\Delta(Q(t))-VG^{*}(\eta^\dagger,t)&\le\phi-VG^{opt}(\eta^\dagger,t)\\&\quad+Q(t)(E_h(t)-\Delta t(\zeta P^{opt}(t)+P_c)),\end{aligned} \tag{15}$$

where $G^{\mathrm{x}}(\eta^\dagger,t)$ represents the weighted difference of the sum rate and the consumed power for time slot t and is given as $G^{\mathrm{x}}(\eta^\dagger,t)=\frac{1}{2}\sum_{i=1}^{2}\log(1+\gamma_i(t)P^{\mathrm{x}}(t))-\eta^\dagger(\zeta P^{\mathrm{x}}(t)+P_c)$ with $\mathrm{x}\in\{*,\mathrm{opt}\}$.

Summing (15) over t from 0 to $T-1$ and dividing by T yield

$$\frac{\phi}{V}\ge\lim_{T\to\infty}\frac{1}{T}\left(\sum_{t=0}^{T-1}G^{\mathrm{opt}}(\eta^\dagger,t)-\sum_{t=0}^{T-1}G^{*}(\eta^\dagger,t)\right).$$

Hence, the upper bound on the performance gap between $\{P^{\mathrm{opt}}(t)\}$ and $\{P^{*}(t)\}$ can be reduced by increasing V. However, due to battery capacity constraint, it follows $V\le(E_{\max}-E_{min}-\Delta tP_{\max}-E_{c,\max})\Delta t\zeta/(\gamma_{\max}-\eta^\dagger\zeta)$. Also, since the maximum possible value of V increases with $\eta^\dagger$, the upper bound on the performance gap between $\{P^{\mathrm{opt}}(t)\}$ and $\{P^{*}(t)\}$ decreases as $\eta^\dagger$ grows.

IV. Simulation Results

In this section, we examine the EE performance of the proposed online EE PA algorithms. For performance comparison, we consider two naive schemes. In the first scheme, denoted as the average harvested energy (AHE) scheme, the transmit power is set to the average harvested energy per time slot. In the second scheme, denoted as the minimum-channel (Min-Ch) scheme, a transmission is made with the maximum transmit power $P_{\max}$ when the minimum of both channel gains is higher than a certain threshold $\gamma_{th}=\frac{\gamma_{\max}}{3}$. The amount of the energy arrival $E_a(t)$ is generated by a compound Poisson process with a uniform distribution between $[0,0.4]$, and the Poisson arrival rate is fixed to $\lambda=2.5$ unit/slot. A lower bound and an upper bound of the battery level are equal to $E_{min}=0$ and $E_{\max}=10$ J, respectively. Also, we have $E_{c,\max}=0.5$ J, $P_{\max}=0.5$ W, and $\Delta t=1$ sec.

In Fig. 1, we plot the EE performance with respect to the time slots with $E_b(0)=E_{\max}/2$. We observe that our proposed EE algorithms show a performance gain over Min- Ch and AHE schemes when the performance converges. The EE performance with η° in large system analysis exhibits performance quite close to the Lyapunov optimization scheme with much reduced computational complexity. Also, we emphasize that the proposed scheme based on the RMT generates good performance even in the finite antenna dimension. The EE for the AHE scheme is lower than the proposed schemes because it does not use channel state information (CSI) at all, and thus allocates constant transmit power regardless of the channel conditions. On the other hand, although the Min-Ch scheme utilizes the CSI, the performance is poor since the transmit power is based on the on-off decision.

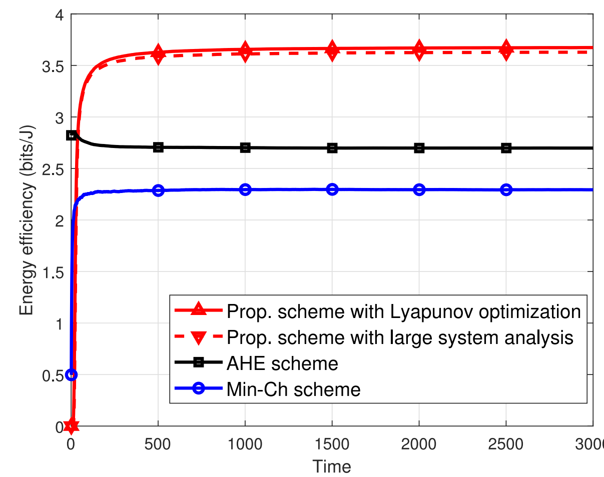

Fig. 1. Time-average EE performance with respect to time slot t when $N=8$.

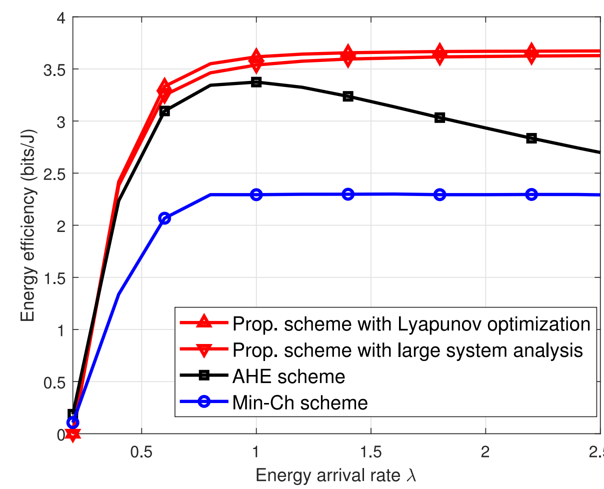

Fig. 2. Time-average EE performance with respect to λ when $N=8$.

Fig. 2 illustrates the EE performance for various energy arrival rate λ. We can see that the proposed scheme outperforms both the Min-Ch scheme and the AHE scheme. Further, we observe that as λ grows, the performance of the AHE scheme degrades after $\lambda=1$. This can be attributed to a fact that as the average harvested energy increases, the loss in EE due to the inefficient allocation of power becomes more severe.

V. Conclusion

In this letter, we have investigated an online PA to maximize long-term EE in energy harvesting systems with finite battery for two users. First, we have formulated the stochastic EE optimization problem and then developed techniques which employ Lyapunov optimization. Also, we have proposed a low-complexity algorithm based on RMT. The proposed schemes outperform the conventional schemes.

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