Asynchronous Designs for Multiuser MIMO Wireless Powered Communication Networks

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***Abstract*—This paper considers multiuser multiple-input multiple-output (MIMO) wireless powered communication networks (WPCNs). In this system, a multiantenna power beacon (PB) transfers wireless energy to several multiantenna users, and then the users transmit data to a multiantenna base station. Time durations for the energy harvesting (EH) operations at each user can be differently set based on the users' channel conditions, which results in an asynchronous protocol on the EH and the information transmission. We maximize the sum rate performance of the asynchronous WPCN by jointly optimizing the energy precoding matrices at the PB, the information precoding matrices, and the EH time durations at the users. By using convex optimization techniques, the optimal algorithm for the sum rate maximization problem is provided with analytical expressions of the optimal precoding matrices. Simulation results verify that the proposed optimal algorithm outperforms conventional schemes.**

***Index Terms*—Multiple-input multiple-output (MIMO), wireless energy transfer (WET), wireless powered communication networks (WPCNs).**

I. Introduction

In RECENT years, wireless energy transfer (WET) methods exploiting radio frequency signals have been widely studied in wireless communication areas [1], [2]. In particular, simultaneous wireless information and power transfer [3]–[6] and wireless powered communication network (WPCN) [7]–[17] have received great attention owing to their potential for supplying power to small devices in wireless sensor networks or Internet-of-Things (IoT) environments. The feasibility of the WPCN has been demonstrated in recent experimental works [18].

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A. Related Works and Motivations

In a WPCN, to support data transmission of users, a power beacon (PB) first broadcasts energy signals to charge the users in a downlink phase. By collecting this wireless energy, the users now transmit their information signals to a base station (BS) in the subsequent uplink phase. A dynamic time division multiple access (TDMA) protocol was first introduced in [7] for synchronous single-input single-output (SISO) multiuser WPCN, and the optimal time durations for the downlink WET and the uplink wireless information transmission (WIT) were provided. Various TDMA-based resource allocation algorithms for the SISO WPCN have been investigated in full-duplex scenarios [8]–[10], orthogonal frequency division multiple access systems [11], and wiretap channels [12].

Multiantenna techniques were also employed in the multiuser WPCN with the space division multiple access (SDMA) [13]–[16]. For the multiple-input single-output WPCN, Zhang *et al.* [13] and [14] examined the minimum rate maximization problems. The optimal beamforming vectors were identified in [13], and massive antenna array scenarios were presented in [14]. Recently, multiple-input multiple-output (MIMO) WPCNs have been investigated in [15] and [16], where the sum rate maximization problems were optimally solved under the SDMA setup. Boshkovska *et al.* [17] proposed robust designs for the TDMA-based MIMO WPCN so that the worst-case sum rate and minimum rate are maximized. These multiantenna WPCN works assumed that the energy harvesting (EH) time durations of multiple users are the same, and thus the WET and the WIT operations at all users are perfectly synchronized.

To further improve the system performance of a multiuser WPCN, users can set their EH time durations differently, and thus the operations of multiple users becomes asynchronous [8]–[10]. In this asynchronous setup, we should carefully adjust the EH time durations for each user as well as the energy and information transmission (IT) strategies. It is emphasized that the asynchronous protocols presented in [8]–[10] are only applicable to the SISO WPCN, where the users' uplink transmission is supported in a TDMA manner. In contrast, the performance of the asynchronous MIMO WPCN has not been fully analyzed so far.

B. Contributions and Organization

In this paper, an asynchronous protocol is considered in a multiuser MIMO WPCN when a BS performs multiuser detection

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based on successive interference cancellation (SIC) [19]. The proposed system model can be employed to communicate with energy-constrained devices in sensor networks, IoT, and radio frequency identification (RFID), where replacing batteries of devices might be costly and inefficient [2]. We aim to maximize the sum rate performance by jointly optimizing the downlink energy precoding at the PB, and the uplink information precoding as well as the EH time durations at users. Since the proposed asynchronous WPCN includes conventional synchronous schemes and the TDMA as special cases, the system capacity can be improved. However, to control the asynchronous operations of the users, we need to introduce additional optimization variables, which makes the problem more difficult compared to the synchronous case. Also, the proposed asynchronous WPCN may require carefully designed control signals, which result in a design tradeoff between the synchronization complexity and the system capacity.

We first identify the optimal precoding matrices with given time durations. By examining the Karush–Kuhn–Tucker (KKT) optimality conditions, it is revealed that the optimal energy transmission strategy is a beamforming with a proper power control policy for multiuser charging. Also, the optimal information precoding matrices can be determined by an iterative procedure. Then, with the optimal precoding matrices at hand, we can find the optimal time durations via the subgradient method. Simulation results show that the proposed asynchronous scheme for the MIMO WPCN outperforms the conventional WPCN protocols.

The rest of this paper is organized as follows. In Section II, we explain the system model for the asynchronous MIMO WPCN and formulate the sum rate maximization problem. Section III proposes the optimal algorithm that computes the optimal time durations and the transmit covariance matrices. The conventional WPCN protocols are briefly reviewed in Section IV. We demonstrate the efficacy of the proposed asynchronous MIMO WPCN over the conventional schemes in Section V via numerical results. Finally, the paper is terminated with conclusions in Section VI.

Throughout this paper, we represent an identity matrix of size m-by-m by $\mathbf{I}_m$. Also, $(x)^+$ is defined as $\max\{0, x\}$, $\mathrm{diag}(x_1, \ldots, x_m)$ stands for a diagonal matrix with diagonal elements $x_1, \ldots, x_m$, and the complement of a set $\mathcal{X}$ is denoted by $\mathcal{X}^c$.

II. System Model

We consider a multiuser WPCN shown in Fig. 1 in which a PB, a BS, and K users are equipped with N_P, N_B, and N_U antennas, respectively. It is assumed that the users do not have any external energy source. Also, the length of one transmission frame is assumed to be 1 without loss of generality. To communicate with the BS, user i $(i = 1, \ldots, K)$ first harvests energy from the received signals during τ_i seconds. Then, for the remaining duration $1 - \tau_i$, user i transmits its information signal to the BS by using the harvested energy.

In the multiuser WPCN, the EH durations $\{\tau_i\}$ would be different for each user, and the EH and the IT operations of the users are not synchronized as illustrated in Fig. 2. The

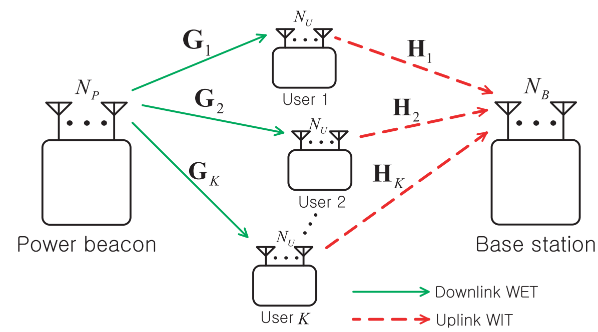

Fig. 1. Schematic diagram of a multiuser MIMO WPCN.

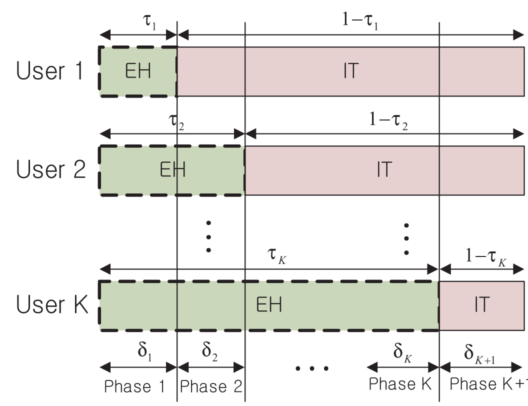

Fig. 2. Frame structure of an asynchronous WPCN.

synchronous WPCN protocol in [15] where the EH durations $\{\tau_i\}$ are the same for all users, i.e., $\tau_1 = \cdots = \tau_K$, can be regarded as a special case of this asynchronous WPCN. In contrast to the synchronous WPCN, there exist $K + 1$ different phases in the asynchronous WPCN according to the users' operations (see Fig. 2). Without loss of generality, we assume that the users are sorted such that $\tau_1 \leq \cdots \leq \tau_K$ for the rest of this paper [8]–[10]. Let us define $\delta_j \triangleq \tau_j - \tau_{j-1}$ as the time duration of phase j $(j = 1, \ldots, K)$, where $\tau_0 \triangleq 0$ and $\tau_{K+1} \triangleq 1$. Then, in phase j, users $i = 1, \ldots, j-1$ transmit their information signals to the BS, while the remaining users $k = j, j+1, \ldots, K$ collect the energy of the received signals from the PB.

First, we explain the WET procedure in phase j. Denoting $\mathbf{G}_i \in \mathbb{C}^{N_U \times N_P}$ as the channel matrix between the PB and user i, the received signal $\mathbf{y}_{ij} \in \mathbb{C}^{N_U \times 1}$ at user $i = j, \ldots, K$ in phase j can be written by

$$\mathbf{y}_{ij} = \mathbf{G}_i \mathbf{x}_j + \mathbf{n}_{ij} \tag{1}$$

where $\mathbf{x}_j \in \mathbb{C}^{N_P \times 1}$ stands for the transmitted signal at the PB in phase j with zero mean and covariance matrix $\mathbf{Q}_j = \mathbb{E}[\mathbf{x}_j \mathbf{x}_j^H] \in \mathbb{C}^{N_P \times N_P}$, and $\mathbf{n}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_U})$ indicates the complex Gaussian noise at user i in phase j. Throughout this paper, we assume that all the channel state information (CSI) is available to the network, which can be achieved by the channel estimation methods proposed for the WPCN in [14].

Let us define P_A and P_P as the average and the peak power budget at the PB, respectively. Then, the energy transmit covariance matrices $\mathbf{Q}_j$ for $j = 1, \ldots, K$ should satisfy the following power constraints:

$$\sum_{j=1}^{K} \delta_j \operatorname{tr}(\mathbf{Q}_j) \leq P_A, \tag{2}$$

$$\operatorname{tr}(\mathbf{Q}_j) \leq P_P, \text{ for } j = 1, \ldots, K. \tag{3}$$

For the EH procedure at the users, we ignore the power of the noise and the WIT signals transmitted from other users, since it is practically much smaller than the transmit power of the PB [3], [8]–[10]. Then, the harvested energy E_{ij} at user $i = j, \ldots, K$ in phase j is given by

$$E_{ij} = \eta_i \delta_j \mathbb{E}[\|\mathbf{y}_{ij}\|^2] = \eta_i \delta_j \operatorname{tr}(\mathbf{G}_i \mathbf{Q}_j \mathbf{G}_i^H) \tag{4}$$

where $\eta_i \in (0, 1]$ accounts for the EH efficiency of user i. Since user i harvests energy during phases $j = 1, \ldots, i$ as shown in Fig. 2, the total harvested energy E_i at user i can be expressed as

$$E_i = \sum_{j=1}^{i} E_{ij} = \eta_i \sum_{j=1}^{i} \delta_j \operatorname{tr}(\mathbf{G}_i \mathbf{Q}_j \mathbf{G}_i^H). \tag{5}$$

Next, we consider the WIT of the asynchronous WPCN. By utilizing the harvested energy E_i, user i transmits its information signal $\mathbf{b}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_{ij}) \in \mathbb{C}^{N_U \times 1}$ during phases $j = i+1, i+2, \ldots, K+1$, where $\mathbf{S}_{ij} \in \mathbb{C}^{N_U \times N_U}$ equals the information transmit covariance matrix of user i in phase j. Then, the energy constraint at user i can be written as

$$\sum_{j=i+1}^{K+1} \delta_j \operatorname{tr}(\mathbf{S}_{ij}) \leq \eta_i \sum_{j=1}^{i} \delta_j \operatorname{tr}(\mathbf{G}_i \mathbf{Q}_j \mathbf{G}_i^H), \quad \text{for } i = 1, \ldots, K. \tag{6}$$

It is assumed that the energy signal interference from the PB to the BS can be perfectly canceled as in [4], [9], and [10]. Denoting $\mathbf{H}_i \in \mathbb{C}^{N_B \times N_U}$ as the channel matrix between the BS and user i, the received signal $\mathbf{r}_j \in \mathbb{C}^{N_B \times 1}$ at the BS in phase j from users $i = 1, \ldots, j-1$ is obtained as

$$\mathbf{r}_j = \sum_{i=1}^{j-1} \mathbf{H}_i \mathbf{b}_{ij} + \mathbf{z}_j \tag{7}$$

where $\mathbf{z}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_B})$ indicates the complex Gaussian noise at the BS in phase j.

Accordingly, the achievable rate of user i during phases $j = i+1, i+2, \ldots, K+1$ is written by

$$R_i = \delta_{i+1} \log \left| \mathbf{I}_{N_B} + \mathbf{H}_i \mathbf{S}_{ii+1} \mathbf{H}_i^H \right| + \sum_{j=i+2}^{K+1} \delta_j \log \frac{\left| \mathbf{I}_{N_B} + \sum_{k=i}^{j-1} \mathbf{H}_k \mathbf{S}_{kj} \mathbf{H}_k^H \right|}{\left| \mathbf{I}_{N_B} + \sum_{k=i+1}^{j-1} \mathbf{H}_k \mathbf{S}_{kj} \mathbf{H}_k^H \right|} \tag{8}$$

TABLE I
SUMMARY OF NOTATIONS

Symbol	Meaning
τ_i	EH duration of user i
δ_j	Duration of phase j
$\mathbf{G}_i$	Channel matrix between the PB and user i
$\mathbf{y}_{ij}$	Received signal at user i in phase j
$\mathbf{x}_j$	Transmitted signal at the PB in phase j
$\mathbf{Q}_j$	Transmit covariance matrix at the PB i in phase j
P_A, P_P	Average power and peak power constraints at the PB
E_{ij}	Harvested energy at user i in phase j
η_i	EH efficiency of user i
$\mathbf{b}_{ij}$	Transmitted signal at user i in phase j
$\mathbf{S}_{ij}$	Transmit covariance matrix at user i in phase j
$\mathbf{H}_i$	Channel matrix between the BS and user i
$\mathbf{r}_j$	Received signal at the BS in phase j
R_i, R	Achievable rate of user i and sum rate

where the BS performs SIC with the decoding order from user 1 to $j-1$ [19]. Therefore, the sum rate of all users is given by

$$R = \sum_{i=1}^{K} R_i = \sum_{j=2}^{K+1} \delta_j \log \left| \mathbf{I}_{N_B} + \sum_{i=1}^{j-1} \mathbf{H}_i \mathbf{S}_{ij} \mathbf{H}_i^H \right|. \tag{9}$$

Notice that the decoding order does not affect the sum rate performance in (9). Table I summarizes the notations utilized throughout this paper.

In this paper, we maximize the sum rate (9) by jointly optimizing the time durations $\{\delta_j\}$, the energy transmit covariance matrices $\{\mathbf{Q}_j\}$ at the PB, and the information transmit covariance matrices $\{\mathbf{S}_{ij}\}$ at the users. The problem can be formulated as

(P1) :

$$\max_{\{\delta_j \geq 0\}, \{\mathbf{Q}_j \succeq \mathbf{0}\}, \{\mathbf{S}_{ij} \succeq \mathbf{0}\}} \sum_{j=2}^{K+1} \delta_j \log \left| \mathbf{I}_{N_B} + \sum_{i=1}^{j-1} \mathbf{H}_i \mathbf{S}_{ij} \mathbf{H}_i^H \right| \tag{10}$$

$$\text{s.t.} \sum_{j=i+1}^{K+1} \delta_j \operatorname{tr}(\mathbf{S}_{ij}) \leq \eta_i \sum_{j=1}^{i} \delta_j \operatorname{tr}(\mathbf{G}_i \mathbf{Q}_j \mathbf{G}_i^H), \quad \text{for } i = 1, \ldots, K, \tag{11}$$

$$\sum_{j=1}^{K} \delta_j \operatorname{tr}(\mathbf{Q}_j) \leq P_A, \tag{12}$$

$$\operatorname{tr}(\mathbf{Q}_j) \leq P_P, \text{ for } j = 1, \ldots, K, \tag{13}$$

$$\sum_{j=1}^{K+1} \delta_j = 1 \tag{14}$$

where the constraint in (14) is obtained since $\sum_{j=1}^{K+1} \delta_j = \sum_{j=1}^{K+1} (\tau_j - \tau_{j-1}) = \tau_{K+1} = 1$.

In general, (P1) is a nonconvex problem due to the nonconvex objective function and constraints (11) and (12). It is worth pointing out that (P1) is totally different from the conventional synchronous MIMO WPCN [15] whose sum rate maximization problem becomes a special case of (P1) by setting $\tau = \tau_i$, $\forall i$,

$\hat{\mathbf{Q}} = \mathbf{Q}_j, \forall j$, and $\hat{\mathbf{S}}_i = \mathbf{S}_{ij}, \forall j$. Also, the SISO WPCN with the TDMA uplink WIT [8]–[10] cannot be directly applied to (P1). In the following sections, we propose an algorithm to find the globally optimal solution of (P1).

III. Optimal Solution For Asynchronous WPCN

To tackle nonconvexity of (P1), we introduce new variables $\mathbf{W}_j \triangleq \delta_j \mathbf{Q}_j$ and $\mathbf{V}_{ij} \triangleq \delta_j \mathbf{S}_{ij}$ for $i = 1, \ldots, K$ and $j = 1, \ldots, K+1$. Then, (P1) is reformulated as

$$\text{(P2)}: \max_{\{\delta_j \geq 0\}, \{\mathbf{W}_j \succeq \mathbf{0}\}, \{\mathbf{V}_{ij} \succeq \mathbf{0}\}} \sum_{j=2}^{K+1} \tilde{R}_j \tag{15}$$

$$\text{s.t.} \sum_{j=i+1}^{K+1} \text{tr}(\mathbf{V}_{ij}) \leq \eta_i \sum_{j=1}^{i} \text{tr}(\mathbf{G}_i \mathbf{W}_j \mathbf{G}_i^H), \text{for } i = 1, \ldots, K, \tag{16}$$

$$\sum_{j=1}^{K} \text{tr}(\mathbf{W}_j) \leq P_A, \tag{17}$$

$$\text{tr}(\mathbf{W}_j) \leq P_P \delta_j, \text{ for } j = 1, \ldots, K, \tag{18}$$

$$\sum_{j=1}^{K+1} \delta_j = 1 \tag{19}$$

where

$$\tilde{R}_j \triangleq \delta_j \log \left| \mathbf{I}_{N_B} + \frac{1}{\delta_j} \sum_{i=1}^{j-1} \mathbf{H}_i \mathbf{V}_{ij} \mathbf{H}_i^H \right|. \tag{20}$$

Note that $\tilde{R}_j$ is a perspective function of $\log|\mathbf{I}_{N_B} + \sum_{i=1}^{j-1} \mathbf{H}_i \mathbf{V}_{ij} \mathbf{H}_i^H|$, which is concave in general. Thus, $\tilde{R}_j$ is jointly convex with respect to δ_j and $\{\mathbf{V}_{ij}\}$ [20]. Since all the constraints of (P2) are affine, we can verify that (P2) is a convex problem.

In the following, we first derive analytic expressions of the optimal transmit covariance matrices $\{\mathbf{W}_j^\star\}$ and $\{\mathbf{V}_{ij}^\star\}$ with given time durations $\{\delta_j\}$. Then, an algorithm for identifying the optimal $\{\delta_j^\star\}$ will be given. Thanks to the joint convexity of (P2), our approach does not lose the optimality.[1]

A. Optimal Energy Transmission

In this section, we derive an analytical form for the optimal $\{\mathbf{W}_j^\star\}$ for fixed $\{\delta_j\}$. With given $\{\delta_j\}$, (P2) is convex and satisfies the Slater's condition. Thus, the strong duality holds for (P2), and the optimal solutions can be attained by the Lagrange duality method. The Lagrangian is written by [20]

$$\begin{aligned}&\mathcal{L}(\{\mathbf{W}_j\}, \{\mathbf{V}_{ij}\}, \{\mu_i\}, \beta, \{\nu_j\}) \\ &= \sum_{j=2}^{K+1} \tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}\}, \{\mu_i\}) + \sum_{j=1}^{K} \text{tr}(\mathbf{A}_j \mathbf{W}_j) + P_P \sum_{j=1}^{K} \nu_j \delta_j + \beta P_A\end{aligned} \tag{21}$$

where

$$\tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}\}, \{\mu_i\}) \triangleq \tilde{R}_j - \sum_{i=1}^{j-1} \mu_i \text{tr}(\mathbf{V}_{ij}), \tag{22}$$

$$\mathbf{A}_j \triangleq \sum_{i=j}^{K} \mu_i \eta_i \mathbf{G}_i^H \mathbf{G}_i - (\nu_j + \beta) \mathbf{I}_{N_P}. \tag{23}$$

Here, $\{\mu_i\}$, β, and $\{\nu_j\}$ denote the dual variables corresponding to the constraints in (16), (17), and (18), respectively.

The dual function $\mathcal{G}(\{\mu_i\}, \beta, \{\nu_j\})$ is computed by [20]

$$\mathcal{G}(\{\mu_i\}, \beta, \{\nu_j\}) \triangleq \max_{\{\mathbf{W}_j \succeq \mathbf{0}\}, \{\mathbf{V}_{ij} \succeq \mathbf{0}\}} \mathcal{L}(\{\mathbf{W}_j\}, \{\mathbf{V}_{ij}\}, \{\mu_i\}, \beta, \{\nu_j\}). \tag{24}$$

Then, the dual problem which minimizes the dual function becomes [20]

$$\min_{\{\mu_i \geq 0\}, \beta \geq 0, \{\nu_j \geq 0\}} \mathcal{G}(\{\mu_i\}, \beta, \{\nu_j\}). \tag{25}$$

From (24), it is not difficult to verify that in order to ensure the bounded dual function $\mathcal{G}(\{\mu_i\}, \beta, \{\nu_j\}) < \infty$, the optimal dual variables $\mu_i^\star$ must be positive for all $i = 1, \ldots, K$. Based on this fact, the following theorem presents the optimal energy transmit covariance matrices $\{\mathbf{W}_j^\star\}$ with fixed $\{\delta_j\}$.

Theorem 1: The optimal $\{\mathbf{W}_j^\star\}$ can be expressed as

$$\mathbf{W}_j^\star = \zeta_j \mathbf{u}_{\mathbf{B}_j,1} \mathbf{u}_{\mathbf{B}_j,1}^H \tag{26}$$

where $\mathbf{B}_j \triangleq \sum_{i=j}^{K} \mu_i^\star \eta_i \mathbf{G}_i^H \mathbf{G}_i$, $\mathbf{u}_{\mathbf{X},k}$ represents the unit-norm eigenvector of a Hermitian matrix $\mathbf{X}$ corresponding to the kth eigenvalue $\lambda_{\mathbf{X},k}$, and $\{\mu_i^\star\}$ stand for the optimal dual variables. Here, the power control factors $\{\zeta_j\}$ are given by

$$\zeta_j = \begin{cases} P_P \delta_j, & \text{for } 1 \leq j \leq L-1, \\ P_A - P_P \sum_{j=1}^{L-1} \delta_j, & \text{for } j = L, \\ 0, & \text{for } L+1 \leq j \leq K \end{cases} \tag{27}$$

where the integer $L \in \{2, 3, \ldots, K+1\}$ is defined as

$$L \triangleq \arg \max_{2 \leq l \leq K+1} l, \tag{28}$$

$$\text{s.t.} \sum_{j=1}^{l-1} \delta_j \leq \frac{P_A}{P_P}. \tag{29}$$

Proof: See Appendix A.

Theorem 1 reveals that the optimal energy transmission strategy at the PB is to make energy beams aligned to the maximum Eigenmode of the weighted sum of the channel covariance matrix $\mathbf{B}_j$. Unlike the synchronous WPCN [15], where a single energy beamforming is utilized for a single WET phase, the PB in the asynchronous protocol should dynamically change the energy beam directions for $K+1$ different phases.

Also, due to the asynchronous operations, the PB can further optimize the power control factors $\{\zeta_j\}$ in (27) for K phases in order to efficiently transfer energy to all users. We can see from (27) that the optimal power control policy at the PB is to consume the available average power P_A with peak power

[1] Although existing convex algorithm can optimally solve (P2), they do not give any insight on the optimal solution.

transmission strategy $\zeta_j = P_P\delta_j$ for the first $L-1$ phases, and the PB is turned off for the remaining phases.

B. Optimal Information Precoding

In this section, we first compute the optimal information transmit covariance matrices $\{\mathbf{V}_{ij}\}$ for given $\{\delta_j\}$ and the dual variables, and then provide the optimal algorithm for determining $\{\mu_i^\star\}$, $\beta^\star$, and $\{\nu_j^\star\}$. By collecting the relevant terms with respect to $\{\mathbf{V}_{ij}\}$, the problem for identifying the dual function in (24) is recast to

$$\max_{\{\mathbf{V}_{ij}\succeq\mathbf{0}\}} \sum_{j=2}^{K+1} \tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}\},\{\mu_i\}). \tag{30}$$

For fixed $\{\mu_i\}$, the function $\tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}\},\{\mu_i\})$ only depends on $\mathbf{V}_{ij}$ for $i=1,\dots,j-1$, and thus, the problem in (30) can be decoupled into K individual problems as

$$\text{(P3)}: \max_{\{\mathbf{V}_{ij}\succeq\mathbf{0}\}_{i=1}^{j-1}} \tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}\},\{\mu_i\}),\ \text{for } j=2,\dots,K+1. \tag{31}$$

One can check that (P3) is similar to the problem that finds the optimal information transmit covariance matrices for the synchronous MIMO WPCN with $j-1$ users [15], and thus, the optimal $\mathbf{V}_{ij}^\star$ for $i=1,\dots,j-1$ is obtained as

$$\mathbf{V}_{ij}^\star = \frac{\delta_j}{\mu_i}\mathbf{U}_{\mathbf{P}_{ij}}\mathbf{D}_{ij}\mathbf{U}_{\mathbf{P}_{ij}}^H \tag{32}$$

where $\mathbf{U}_\mathbf{X} \triangleq [\mathbf{u}_{\mathbf{X},1},\dots,\mathbf{u}_{\mathbf{X},r_\mathbf{X}}]$ indicates the Eigenvector matrix of a Hermitian matrix $\mathbf{X}$ with $r_\mathbf{X}$ being the rank of $\mathbf{X}$, $\mathbf{P}_{ij} \triangleq \mathbf{C}_{ij}^H\mathbf{C}_{ij}$, and $\mathbf{D}_{ij} \triangleq \text{diag}(d_{ij}^1,\dots,d_{ij}^{r_{\mathbf{P}_{ij}}})$. Here, we have defined

$$\mathbf{C}_{ij} \triangleq \sqrt{\frac{1}{\mu_i}}\left(\mathbf{I}_{N_B} + \sum_{k\neq i,k=1}^{j-1}\frac{1}{\delta_j}\mathbf{H}_k\mathbf{V}_{kj}^\star\mathbf{H}_k^H\right)^{-1/2}\mathbf{H}_i \tag{33}$$

$$d_{ij}^l \triangleq \left(1-\frac{1}{\lambda_{\mathbf{P}_{ij},l}}\right)^+. \tag{34}$$

In order to obtain $\mathbf{V}_{ij}^\star$ for $i=1,\dots,j-1$ satisfying (32), we sequentially calculate $\mathbf{V}_{ij}$ from $i=1$ to $j-1$ until converge. Since (P3) is jointly convex, it can be verified that such an iterative process converges to the globally optimal point [10].

Now, the remaining work for solving (P2) with given $\{\delta_j\}$ is to find the optimal dual variables $\{\mu_i^\star\}$, $\beta^\star$, and $\{\nu_j^\star\}$ for the dual problem $\min_{\{\mu_i\geq 0\},\beta\geq 0,\{\nu_j\geq 0\}}\mathcal{G}(\{\mu_i\},\beta,\{\nu_j\})$. To this end, we introduce the following lemma which provides closed-form expressions for $\beta^\star$ and $\{\nu_j^\star\}$ as a function of $\{\mu_i^\star\}$.

Lemma 1: For a given L, the optimal $\beta^\star$ and $\{\nu_j^\star\}$ are, respectively, computed as $\beta^\star = \lambda_{\mathbf{B}_L,1}$ and $\nu_j^\star = (\lambda_{\mathbf{B}_j,1} - \lambda_{\mathbf{B}_L,1})^+$ for $j=1,\dots,K$, where we define $\lambda_{\mathbf{B}_{K+1},1}\triangleq 0$.

Proof: See Appendix B.

Utilizing Lemma 1, we only need to identify the optimal $\{\mu_i^\star\}$, which can be attained by the subgradient method such as the ellipsoid method. The subgradient ρ_i of the dual function $\mathcal{G}(\{\mu_i\},\beta,\{\nu_j\})$ in (24) with respect to μ_i is calculated as

$$\rho_i = \eta_i\sum_{j=1}^{i}\text{tr}(\mathbf{G}_i\mathbf{W}_j^\star\mathbf{G}_i^H) - \sum_{j=i+1}^{K+1}\text{tr}(\mathbf{V}_{ij}^\star) + a\partial_{\mu_i}\beta + \sum_{j=1}^{K}b_j\partial_{\mu_i}\nu_j \tag{35}$$

where $\partial_x f$ indicates the subgradient of f with respect to x, and a and $\{b_j\}$ are defined by $a \triangleq P_A - \sum_{j=1}^K \text{tr}(\mathbf{W}_j^\star)$, and $b_j \triangleq P_P\delta_j - \text{tr}(\mathbf{W}_j^\star)$. Here, a and $\{b_j\}$ can be, respectively, obtained from Theorem 1 as $a=0$ for $2\leq L\leq K$, $a = P_A - P_P\sum_{j=1}^K\delta_j$ for $L=K+1$, and

$$b_j = \begin{cases} 0, & \text{for } 1\leq j\leq L-1,\\ P_P\sum_{j=1}^L\delta_j - P_A, & \text{for } j=L,\\ P_P\delta_j, & \text{for } L+1\leq j\leq K.\end{cases} \tag{36}$$

Also, it is easy to see from Lemma 1 that the subgradient $\partial_{\mu_i}\beta$ becomes zero when $L=K+1$ because $\beta^\star = \lambda_{\mathbf{B}_{K+1},1}=0$. In addition, since $\lambda_{\mathbf{B}_1,1} > \cdots > \lambda_{\mathbf{B}_K,1}$ as discussed in Appendix A, it follows $\nu_j=0$ and $\partial_{\mu_i}\nu_j = 0$ for $L\leq j\leq K$ and $2\leq L\leq K$. By exploiting these results, the subgradient ρ_i can be derived as

$$\rho_i = \eta_i\sum_{j=1}^{i}\text{tr}(\mathbf{G}_i\mathbf{W}_j^\star\mathbf{G}_i^H) - \sum_{j=i+1}^{K+1}\text{tr}(\mathbf{V}_{ij}^\star). \tag{37}$$

C. Optimal Time Duration

In this section, we address the optimal time durations $\{\delta_j^\star\}$ for (P2). Let $\mathcal{R}(\{\delta_j\})$ be the optimal value of (P2) for given $\{\delta_j\}$, i.e., $\mathcal{R}(\{\delta_j\}) = \sum_{j=2}^{K+1}\delta_j\log|\mathbf{I}_{N_B} + \frac{1}{\delta_j}\sum_{i=1}^{j-1}\mathbf{H}_i\mathbf{V}_{ij}^\star\mathbf{H}_i^H|$ with the optimal $\{\mathbf{V}_{ij}^\star\}$ attained with fixed $\{\delta_j\}$ in the previous sections. Then, the problem for finding $\{\delta_j^\star\}$ is expressed as

$$\text{(P4)}: \max_{\{\delta_j\geq 0\}}\mathcal{R}(\{\delta_j\}) \tag{38}$$

$$\text{s.t.} \sum_{j=1}^{K+1}\delta_j = 1. \tag{39}$$

Since the strong duality holds for (P2) with given $\{\delta_j\}$ [20], from (58) in Appendix A, we can rewrite $\mathcal{R}(\{\delta_j\})$ as

$$\mathcal{R}(\{\delta_j\}) = \min_{\{\mu_i\geq 0\},\beta\geq 0,\{\nu_j\geq 0\}}\mathcal{G}(\{\mu_i\},\beta,\{\nu_j\}) \tag{40}$$

$$= \sum_{j=2}^{K+1}\tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}^\star\},\{\mu_i^\star\}) + P_P\sum_{j=1}^{K}\nu_j^\star\delta_j + \beta^\star P_A. \tag{41}$$

By substituting the constraint $\delta_1 = 1 - \sum_{j=2}^{K+1}\delta_j$ into (41), we have

$$\mathcal{R}(\{\delta_j\}) = \mathcal{T}(\{\delta_j\}) + P_P\nu_1^\star + \beta^\star P_A \tag{42}$$

Algorithm 1: Optimal algorithm for (P2).

Initialize $\{\delta_j\}$.
Repeat *(update* $\{\delta_j\}$*)*
  Initialize $\{\mu_i\}$.
  Repeat *(update* $\{\mu_i\}$*)*
    Compute L and $\{\mathbf{W}_j^\star\}$ from Theorem 1.
    For $j = 2 : K+1$
      Initialize $\{\mathbf{V}_{ij}\}_{i=1}^{j-1}$.
      Repeat *(update* $\{\mathbf{V}_{ij}\}_{i=1}^{j-1}$*)*
        Compute $\mathbf{V}_{ij}$ from $i = 1$ to $j-1$ from (32).
      Until $\{\mathbf{V}_{ij}\}_{i=1}^{j-1}$ converge
    End
    Update $\{\mu_i\}$ based on the ellipsoid method.
  Until $\{\mu_i\}$ converge
  Update $\{\delta_j\}$ based on the ellipsoid method.
Until $\{\delta_j\}$ converge

where

$$\mathcal{T}(\{\delta_j\}) \triangleq \sum_{j=2}^{K+1} \tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}^\star\},\{\mu_i^\star\}) + P_P \sum_{j=2}^{K}(\nu_j^\star - \nu_1^\star) - P_P \nu_1^\star \delta_{K+1}. \quad (43)$$

As a result, (P4) can be reformulated as

$$\text{(P5)}: \quad \max_{\{\delta_j \geq 0\}} \mathcal{T}(\{\delta_j\}), \quad (44)$$

$$\text{s.t.} \quad \sum_{j=2}^{K+1} \delta_j \leq 1. \quad (45)$$

Due to the convexity of (P5), we can compute the optimal $\{\delta_j^\star\}$ by employing the subgradient method. The subgradient θ_j of the objective function in (P5) with respect to δ_j for $j = 2, \ldots, K+1$ is given by

$$\theta_j = \psi\left(\mathbf{I}_{N_B} + \frac{1}{\delta_j}\sum_{i=1}^{j-1}\mathbf{H}_i\mathbf{V}_{ij}^\star\mathbf{H}_i^H\right) + P_P(\nu_j^\star - \nu_1^\star) - N_B \quad (46)$$

where $\psi(\mathbf{X}) \triangleq \log|\mathbf{X}| + \mathrm{tr}(\mathbf{X}^{-1})$ and $\nu_{K+1}^\star \triangleq 0$. After obtaining $\delta_j^\star$ for $j = 2, \ldots, K$, the optimal $\delta_1^\star$ can be directly calculated as $\delta_1^\star = 1 - \sum_{j=2}^{K+1}\delta_j^\star$. Finally, we summarize the proposed optimal algorithm for (P2) below. It has been proved that the proposed algorithm is guaranteed to converge to the globally optimal point [21].

In practice, the optimal time durations $\{\delta_j^\star\}$ and the optimal transmit covariance matrices $\{\mathbf{W}_j^\star\}$ and $\{\mathbf{V}_{ij}^\star\}$ can be computed by the BS as follows: First, the PB and the BS obtain the CSI $\{\mathbf{G}_i\}$ and $\{\mathbf{H}_i\}$, respectively, by using the channel estimation methods in [14]. Second, the PB sends the CSI $\{\mathbf{G}_i\}$ to the BS via reliable feedback channels. Collecting all the CSI, the BS is now able to calculate the optimal solution from Algorithm 1.

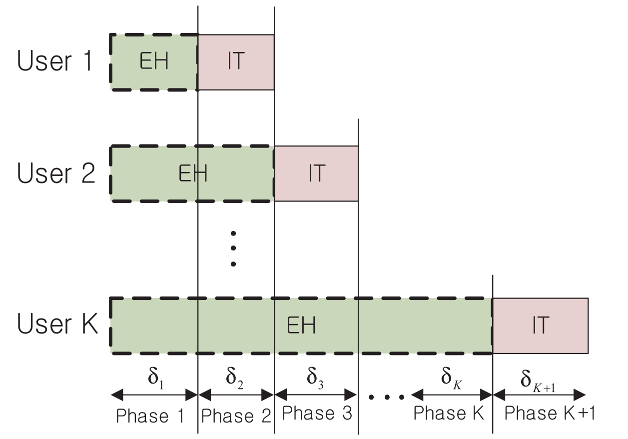

Fig. 3. Frame structure of TDMA WPCN.

Then, the BS can inform the solution to the PB and the users through downlink control channels.[2]

Now, we briefly discuss the computational complexity of the proposed algorithm. For given $\{\delta_j\}$ and $\{\mu_i\}$, the optimal covariance matrices $\{\mathbf{W}_j^\star\}$ and $\{\mathbf{V}_{ij}^\star\}$ can be determined in closed-form expressions. Thus, the complexity of Algorithm 1 is dominated by two ellipsoid methods, each of which finds the optimal $\{\delta_j^\star\}$ and $\{\mu_i^\star\}$, respectively, and the iterative procedures calculating $\{\mathbf{V}_{ij}^\star\}_{i=1}^{j-1}$ for $j = 2, \ldots, K+1$. Since the computational complexity of each ellipsoid loop is given by $\mathcal{O}(K^2)$ [21], the overall complexity of Algorithm 1 becomes $\mathcal{O}(IK^4)$, where $I \triangleq \sum_{j=2}^{K+1} I_j$ and I_j for $j = 2, \ldots, K+1$ denotes the number of iterations to compute $\{\mathbf{V}_{ij}^\star\}_{i=1}^{j-1}$.[3]

IV. Conventional WPCN

In this section, we discuss the TDMA and the synchronous protocols as conventional multiuser MIMO WPCN schemes and briefly provide the optimal solution for each protocol.

A. Time Division Multiple Access

In the TDMA protocol [9], [10], user i can only transmit its information signal in phase $i+1$ with covariance matrix $\tilde{\mathbf{S}}_i$, whereas the WET is still performed in an asynchronous manner as shown in Fig. 3. Note that the TDMA protocol is a special case of the proposed asynchronous WPCN with $\mathbf{S}_{ij} = \tilde{\mathbf{S}}_i$ for $j = i+1$ and $\mathbf{S}_{ij} = \mathbf{0}$ for $j = i+2, i+3, \ldots, K+1, \forall i$.

Let us define $\tilde{\mathbf{V}}_i \triangleq \delta_{i+1}\tilde{\mathbf{S}}_i$ for $i = 1, \ldots, K$. Then, the sum rate maximization problem (P2) for the TDMA protocol reduces to

$$\max_{\{\delta_j \geq 0\},\{\mathbf{W}_j \succeq \mathbf{0}\},\{\tilde{\mathbf{V}}_i \succeq \mathbf{0}\}} \sum_{i=1}^{K} \delta_{i+1} \log\left|\mathbf{I}_{N_B} + \frac{1}{\delta_{i+1}}\mathbf{H}_i\tilde{\mathbf{V}}_i\mathbf{H}_i^H\right| \quad (47)$$

[2]The proposed algorithm should be performed in each coherence time block, which is typically longer than one system block in slow-fading environments such as sensor networks, IoT, and RFID.

[3]We have checked from numerical simulations that $I = 8$ is enough for the convergence.

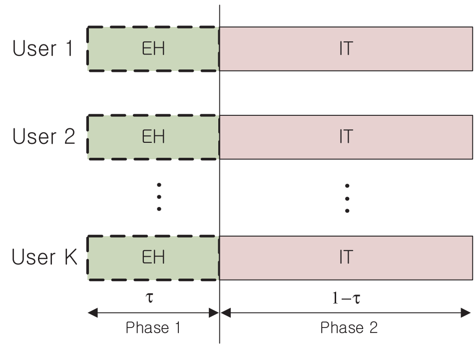

Fig. 4. Frame structure of a synchronous WPCN.

$$\text{s.t. } \operatorname{tr}(\tilde{\mathbf{V}}_i) \le \eta_i \sum_{j=1}^{i} \operatorname{tr}(\mathbf{G}_i \mathbf{W}_j \mathbf{G}_i^H), \text{ for } i = 1, \ldots, K, \tag{48}$$

$$(14), (17), (18). \tag{49}$$

In [9] and [10], the sum rate maximization problem was solved only for a SISO WPCN case with $N_P = N_B = N_U = 1$, and thus the existing algorithms in [9] and [10] cannot be applied to the MIMO case (47).

To optimally solve (47), we employ the approach proposed in Section III. First, for given $\{\delta_j\}$, the optimal energy covariance matrix $\{\mathbf{W}_j^\star\}$ can be directly obtained from Theorem 1. Next, similar to (32), the optimal $\{\tilde{\mathbf{V}}_i^\star\}$ with fixed $\{\delta_j\}$ is expressed as

$$\tilde{\mathbf{V}}_i^\star = \frac{\delta_{i+1}}{\mu_i} \mathbf{U}_{\tilde{\mathbf{P}}_i} \tilde{\mathbf{D}}_i \mathbf{U}_{\tilde{\mathbf{P}}_i}^H \tag{50}$$

where we define $\tilde{\mathbf{P}}_i \triangleq \frac{1}{\mu_i} \mathbf{H}_i^H \mathbf{H}_i, \tilde{\mathbf{D}}_i \triangleq \operatorname{diag}(\tilde{d}_i^1, \ldots, \tilde{d}_i^{r_{\tilde{\mathbf{P}}_i}})$, and $\tilde{d}_i^l \triangleq (1 - \frac{1}{\lambda_{\tilde{\mathbf{P}}_i,l}})^+$.

Then, the optimal $\{\mu_i^\star\}$ and $\{\delta_j^\star\}$ can be computed by the ellipsoid method, whose subgradients $\{\tilde{\rho}_i\}$ and $\{\tilde{\theta}_j\}$ are, respectively, calculated as

$$\tilde{\rho}_i = \eta_i \sum_{j=1}^{i} \operatorname{tr}(\mathbf{G}_i \mathbf{W}_j^\star \mathbf{G}_i^H) - \operatorname{tr}(\tilde{\mathbf{V}}_i^\star) \tag{51}$$

$$\tilde{\theta}_j = \psi\left(\mathbf{I}_{N_B} + \frac{1}{\delta_j} \mathbf{H}_{j-1} \tilde{\mathbf{V}}_{j-1}^\star \mathbf{H}_{j-1}^H\right) + P_P(\nu_j^\star - \nu_1^\star) - N_B \tag{52}$$

where the optimal dual variable $\{\nu_j^\star\}$ is attained from Lemma 1. As a result, problem (47) can be optimally solved by Algorithm 1, where the overall computational complexity equals $\mathcal{O}(\tilde{I}K^4)$ with $\tilde{I}$ being the number of iterations for calculating (50).

B. Synchronous Methods [15]

As shown in Fig. 4, the synchronous WPCN [15] considers a scenario where the WET and the WIT of the WPCN are perfectly synchronized. Hence, there are only two phases in the synchronous method. It is easy to check that when $\tau = \tau_i, \forall i$, $\hat{\mathbf{Q}} = \mathbf{Q}_j, \forall j$, and $\hat{\mathbf{S}}_i = \mathbf{S}_{ij}, \forall j$, the proposed asynchronous protocol boils down to the synchronous WPCN, where $\hat{\mathbf{Q}}$ and $\hat{\mathbf{S}}_i$ denote the energy covariance matrix at the PB and the information covariance matrix at user i, respectively.

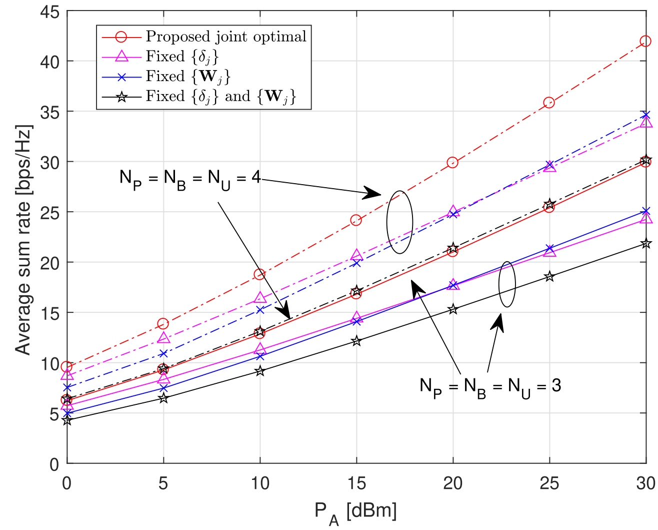

Fig. 5. Average sum rate performance as a function of P_A with $K = 2$.

Then, we can formulate the sum rate maximization problem for the synchronous WPCN as

$$\max_{\tau, \hat{\mathbf{W}} \succeq 0, \{\hat{\mathbf{V}}_i \succeq 0\}} (1-\tau) \log \left| \mathbf{I}_{N_B} + \frac{1}{1-\tau} \sum_{i=1}^{K} \mathbf{H}_i \hat{\mathbf{V}}_i \mathbf{H}_i^H \right| \tag{53}$$

$$\text{s.t. } \operatorname{tr}(\hat{\mathbf{V}}_i) \le \eta_i \operatorname{tr}(\mathbf{G}_i \hat{\mathbf{W}} \mathbf{G}_i^H), \text{ for } i = 1, \ldots, K, \tag{54}$$

$$\operatorname{tr}(\hat{\mathbf{W}}) \le P_A, \tag{55}$$

$$\operatorname{tr}(\hat{\mathbf{W}}) \le \tau P_P, \tag{56}$$

$$0 \le \tau \le 1 \tag{57}$$

where $\hat{\mathbf{W}} \triangleq \tau \hat{\mathbf{Q}}$ and $\hat{\mathbf{V}}_i \triangleq (1-\tau)\hat{\mathbf{S}}_i$ for $i = 1, \ldots, K$. The optimal solution for problem (53) is available in [15]. Since the optimal time duration $\tau^\star$ for (53) can be found via a one-dimensional line search method, the overall complexity for the synchronous WPCN becomes $\mathcal{O}(\hat{I}K^2)$, where $\hat{I}$ represents the number of iterations required for finding the optimal $\{\hat{\mathbf{V}}_i^\star\}$.

V. Numerical Results

In this section, we evaluate the average sum rate performance of the proposed algorithm in a Rayleigh fading setup. For the simulations, the noise variance at the BS is set to -60 dBm, and an average pathloss of 30 dB is assumed from the PB and the BS to all users, which corresponds to 1 m distance at a carrier frequency of 900 MHz [7], [10]. In other words, we consider a scenario where the PB and the BS are colocated as in [7]–[16]. Also, the EH efficiency and the peak power constraint are given by $\eta_i = 0.5, \forall i$ and $P_P = 2P_A$, respectively.

In Fig. 5, we exhibit the average sum rate of the asynchronous WPCN as a function of P_A with $K = 2$ for $N_P = N_B = N_U =$

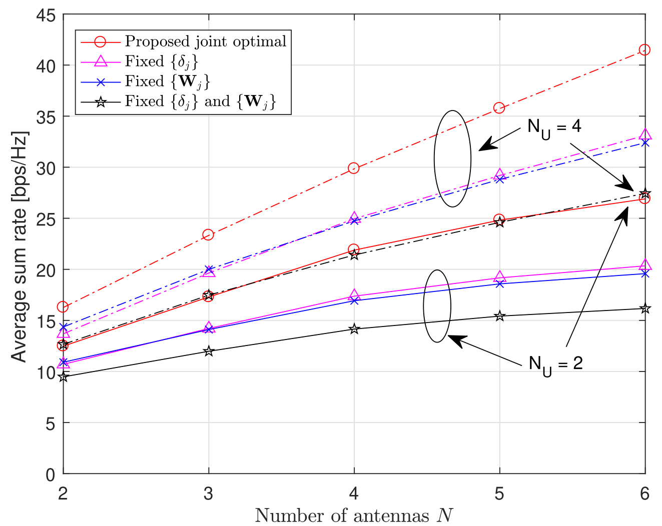

Fig. 6. Average sum rate performance as a function of the number of antennas N with $K = 2$ and $P_A = 20$ dBm.

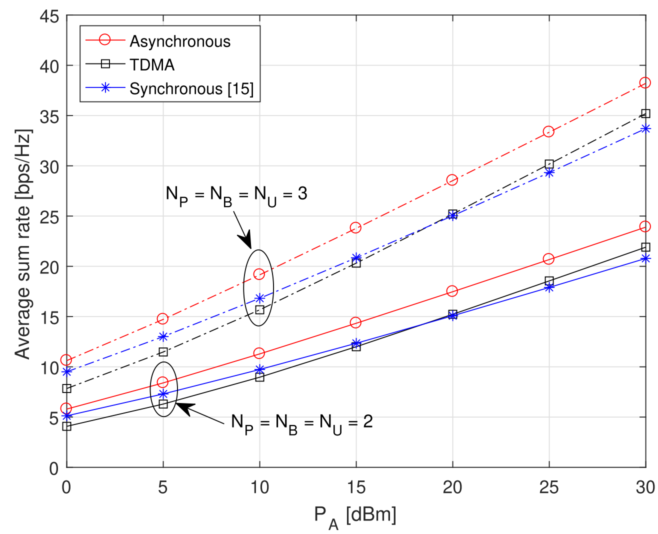

Fig. 7. Average sum rate performance as a function of P_A with $K = 6$.

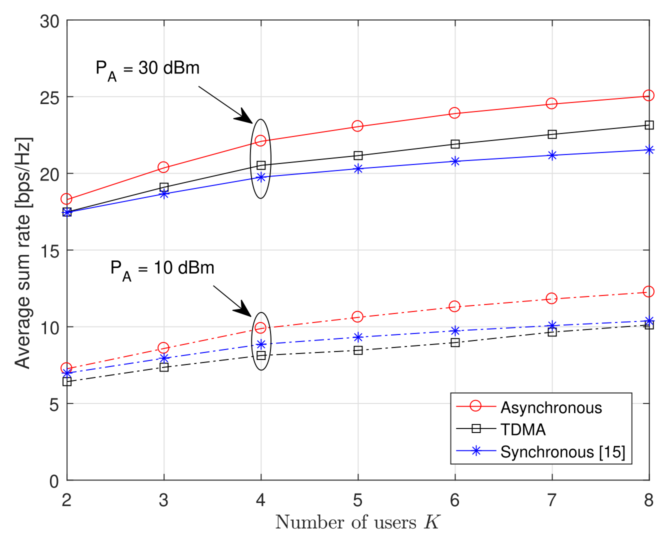

Fig. 8. Average sum rate performance as a function of K with $N_P = N_B = N_U = 2$.

3 and 4. To demonstrate the efficacy of the proposed optimal algorithm, the following three baseline schemes are considered.

1) *Fixed* $\{\delta_j\}$: The time duration variable $\{\delta_j\}$ is fixed as $\delta_j = \frac{1}{K+1}$ for $j = 1, \ldots, K+1$. Then, the energy and the information transmit covariance matrices $\{\mathbf{W}_j\}$ and $\{\mathbf{V}_{ij}\}$ are optimized via the proposed algorithm.
2) *Fixed* $\{\mathbf{W}_j\}$: We employ naive energy transmit covariance matrices $\mathbf{W}_j = \frac{P_A}{KN_B}\mathbf{I}_{N_B}$ for $j = 1, \ldots, K$, while the time durations and the information precoding matrices are computed from the proposed algorithm with additional constraints $\delta_j \geq \frac{P_A}{KP_P}$ in order to ensure the peak power constraint (18).
3) *Fixed* $\{\delta_j\}$ *and* $\{\mathbf{W}_j\}$: In this scheme, we simply set $\delta_j = \frac{1}{K+1}$ and $\mathbf{W}_j = \frac{P_A}{KN_B}\mathbf{I}_{N_B}$, $\forall j$. The information precoding matrices are then optimized with fixed uplink power constraints $\sum_{j=i+1}^{K+1} \text{tr}(\mathbf{V}_{ij}) \leq \frac{\eta_i P_A}{KN_B}\sum_{j=1}^{i} \text{tr}(\mathbf{G}_i^H \mathbf{G}_i)$ for $i = 1, \ldots, K$.

From the figure, we can first see that the proposed joint optimal algorithm outperforms the baseline schemes for all P_A regime. By comparing the curves of the proposed algorithm and "fixed $\{\delta_j\}$," it can be inferred that optimizing the time durations are crucial for the asynchronous WPCN. For a large P_A, "fixed $\{\mathbf{W}_j\}$" slightly performs better than "fixed $\{\delta_j\}$" regardless of the number of antennas. As expected, "fixed $\{\delta_j\}$ and $\{\mathbf{W}_j\}$" provides the worst average sum rate performance among the baseline schemes. At $P_A = 20$ dBm, the proposed joint optimal algorithm presents about 37% and 39% performance gains over "fixed $\{\delta_j\}$ and $\{\mathbf{W}_j\}$" for $N_P = N_B = 3$ and $N_P = N_B = 4$, respectively.

Fig. 6 shows the average sum rate performance of various schemes as a function of the number of antennas $N = N_P = N_B$ with $K = 2$ and $P_A = 20$ dBm for $N_U = 2$ and 4. We can check that the performance gap between the proposed algorithm and the baseline methods gets larger as N grows, implying the importance of joint optimization of the time durations and the transmit covariance matrices with a large N. It is shown that for $N_U = 2$ and 4, the proposed algorithm offers 66% and 51% average sum rate gains over "fixed $\{\delta_j\}$ and $\{\mathbf{W}_j\}$," respectively.

Next, in Figs. 7 and 8, we compare the performance of the proposed asynchronous WPCN with the conventional protocols discussed in Section IV. First, Fig. 7 depicts the average sum rate performance of various WPCN protocols as a function of the average power constraint P_A with $K = 6$ for $N_P = N_B = N_U = 2$ and 3. It is observed that the asynchronous WPCN performs better than the conventional synchronous and TDMA protocols. It is interesting to see that the average sum rate of the synchronous method is larger than that of the TDMA protocol at low P_A, but two performance curves are reversed in the high P_A regime. For the average sum rate of 20 bps/Hz, the proposed scheme with $N_P = N_B = N_U = 2$ offers 3 and 5 dB gains over the TDMA and the synchronous schemes, respectively.

In Fig. 8, we provide the average sum rate performance as a function of K for $P_A = 10$ and 30 dBm with $N_P = N_B = N_U = 2$. We can see that a performance gain of the proposed asynchronous WPCN over the conventional ones grows as the number of users gets larger. As observed from Fig. 7, the TDMA WPCN performs better than the synchronous scheme with a large P_A. At $K = 9$ and $P_A = 10$ dBm, the proposed scheme attains about 18% and 20% gains compared to the conventional synchronous and TDMA protocols, respectively.

VI. Conclusion

In this paper, we have studied a multiuser MIMO WPCN, which allows asynchronous operations among multiple users. We have proposed the optimal algorithm for the sum rate maximization problem by optimizing the energy beamformers, the information precoding matrices, and the time durations. First, the closed-form expressions for the optimal energy and information transmit covariance matrices have been determined with given time durations. Then, we can compute the optimal time durations by utilizing the ellipsoid method. Simulation results have demonstrated the efficiency of the proposed asynchronous WPCN over the baseline schemes. This paper have focused on investigating the theoretical performance of the proposed asynchronous protocol, which could be useful for practical WPCN designs. Implementation issues of the proposed algorithm such as the exact execution time analysis would be important and interesting future research plans.

Appendix A
Proof of Theorem 1

When $\delta_j = 0$, it is easy to check from the constraint in (18) that the optimal energy transmit covariance matrix $\mathbf{W}_j^\star$ is given by $\mathbf{W}_j^\star = \mathbf{0}$. Thus, for the rest of the proof, we consider a nontrivial case of $\delta_j > 0$ for $j = 1, \ldots, K$. The KKT conditions relevant to $\{\mathbf{W}_j^\star\}$ are expressed as [4], [20]

$$(\mathbf{B}_j - (\nu_j^\star + \beta^\star)\mathbf{I}_{N_P})\mathbf{W}_j^\star = \mathbf{0}, \text{ for } j = 1, \ldots, K, \quad (58)$$

$$\mathbf{B}_j - (\nu_j^\star + \beta^\star)\mathbf{I}_{N_P} \preceq \mathbf{0}, \text{ for } j = 1, \ldots, K, \quad (59)$$

$$\beta^\star \left(P_A - \sum_{j=1}^{K} \mathrm{tr}(\mathbf{W}_j^\star) \right) = 0, \quad (60)$$

$$\nu_j^\star (P_P \delta_j - \mathrm{tr}(\mathbf{W}_j^\star)) = 0, \text{ for } j = 1, \ldots, K \quad (61)$$

where $\{\mu_i^\star\}$, $\beta^\star$, and $\{\nu_j^\star\}$ are the optimal dual variables, (59) is obtained to guarantee $\mathcal{G}(\{\mu_i\}, \beta, \{\nu_j\}) < \infty$, and (60) and (61) stand for the complementary slackness conditions.

Since the optimal dual variables $\{\mu_i^\star\}$ are positive, the maximum Eigenvalue $\lambda_{\mathbf{B}_j,1}$ must be greater than 0 for $j = 1, \ldots, K$. Thus, we can show that (59) is equivalent to

$$\nu_j^\star + \beta^\star \geq \lambda_{\mathbf{B}_j,1} > 0, \text{ for } j = 1, \ldots, K. \quad (62)$$

Then, we consider two different cases $\beta^\star = 0$ and $\beta^\star > 0$ in the following. For the first case of $\beta^\star = 0$, it can be verified from (62) that the optimal dual variable $\nu_j^\star$ is bounded as $\nu_j^\star \geq \lambda_{\mathbf{B}_j,1} > 0$ for $j = 1, \ldots, K$. Suppose $\nu_j^\star > \lambda_{\mathbf{B}_j,1}$. Then, we have $\mathbf{W}_j^\star = \mathbf{0}$ from (58), which violates the complementary slackness condition (61). Hence, in the first case, the optimal $\nu_j^\star$ is given by $\nu_j^\star = \lambda_{\mathbf{B}_j,1}$.

From (58) and (61), we can compute the optimal $\mathbf{W}_j^\star$ in this case as

$$\mathbf{W}_j^\star = P_P \delta_j \mathbf{u}_{\mathbf{B}_j,1} \mathbf{u}_{\mathbf{B}_j,1}^H, \text{ for } j = 1, \ldots, K. \quad (63)$$

Combining (63) and the primal constraint in (17), it can be checked that the case of $\beta^\star = 0$ is equivalent to $\sum_{j=1}^{K} \delta_j \leq \frac{P_A}{P_P}$. Therefore, the optimal solution for the first case can be determined as in (27) with $L = K + 1$.

Second, we consider the case where the optimal $\beta^\star$ is positive, which occurs only when $\sum_{j=1}^{K} \delta_j > \frac{P_A}{P_P}$. In this case, the optimal $\nu_j^\star$ can be 0 for some $j \in \{1, \ldots, K\}$. Let us denote $\mathcal{J} \subseteq \{1, \ldots, K\}$ as the set of indices j such that $\nu_j^\star > 0$. Then, it follows $\nu_j^\star + \beta^\star = \lambda_{\mathbf{B}_j,1}$ for $j \in \mathcal{J}$, since otherwise, i.e., when $\nu_j^\star + \beta^\star > \lambda_{\mathbf{B}_j,1}$, the optimal $\mathbf{W}_j^\star$ would be calculated as $\mathbf{W}_j^\star = \mathbf{0}$ from (59). Because this solution disobeys the optimality conditions in (61), we always have $\nu_j^\star + \beta^\star = \lambda_{\mathbf{B}_j,1}$ for $j \in \mathcal{J}$, and the corresponding optimal solution is obtained as $\mathbf{W}_j^\star = P_P \delta_j \mathbf{u}_{\mathbf{B}_j,1} \mathbf{u}_{\mathbf{B}_j,1}^H, \forall j \in \mathcal{J}$.

Next, for $j \in \mathcal{J}^c$, we can show from (62) that $\beta^\star$ is greater than or equal to $\lambda_{\mathbf{B}_j,1}$. If $\beta^\star > \lambda_{\mathbf{B}_j,1}$ for some $j \in \mathcal{J}^c$, due to the condition in (58), the optimal energy transmit covariance matrix is given by $\mathbf{W}_j^\star = \mathbf{0}$. In contrast, when $\beta^\star = \lambda_{\mathbf{B}_j,1}$ for some $j \in \mathcal{J}^c$, the optimal $\mathbf{W}_j^\star$ can be represented as $\mathbf{W}_j^\star = \alpha_j \mathbf{u}_{\mathbf{B}_j,1} \mathbf{u}_{\mathbf{B}_j,1}^H$ with $\alpha_j \in [0, P_P \delta]$ such that $\sum_{j=1}^{K} \mathrm{tr}(\mathbf{W}_j^\star) = P_A$.

Now, we derive a closed-form expression for the set $\mathcal{J}$. By using the facts $\beta^\star = \lambda_{\mathbf{B}_j,1} - \nu_j^\star < \lambda_{\mathbf{B}_j,1}$ for $j \in \mathcal{J}$ and $\beta^\star \geq \lambda_{\mathbf{B}_j,1}$ for $j \in \mathcal{J}^c$, it follows

$$\max_{j \in \mathcal{J}^c} \lambda_{\mathbf{B}_j,1} \leq \beta^\star < \min_{j \in \mathcal{J}} \lambda_{\mathbf{B}_j,1}. \quad (64)$$

Recalling the definition $\mathbf{B}_j \triangleq \sum_{i=j}^{K} \mu_i^\star \eta_i \mathbf{G}_i^H \mathbf{G}_i$ and the fact $\mu_i^\star > 0$ for $i = 1, \ldots, K$, we can check that the maximum Eigenvalues $\{\lambda_{\mathbf{B}_j,1}\}$ are distinct and decrease with respect to j, i.e., $\lambda_{\mathbf{B}_1,1} > \cdots > \lambda_{\mathbf{B}_K,1} > 0$.[4] For this reason, there always exists an index $L \in \mathcal{J}^c$ such that

$$\lambda_{\mathbf{B}_K,1} < \cdots < \lambda_{\mathbf{B}_L,1} \leq \beta^\star < \lambda_{\mathbf{B}_{L-1},1} < \cdots < \lambda_{\mathbf{B}_1,1}. \quad (65)$$

Comparing (64) and (65), one can verify that the set $\mathcal{J}$ is given by $\mathcal{J} = \{1, \ldots, L-1\}$. Based on these results, the optimal solution $\mathbf{W}_j^\star$ for $j = 1, \ldots, K$ is obtained by $\mathbf{W}_j^\star = \zeta_j \mathbf{u}_{\mathbf{B}_j,1} \mathbf{u}_{\mathbf{B}_j,1}^H$, where

$$\zeta_j = \begin{cases} P_P \delta_j, & \text{for } j = 1, \ldots, L-1, \\ \alpha_j, & \text{for } j = L, \\ 0, & \text{for } j = L+1, L+2, \ldots, K. \end{cases} \quad (66)$$

[4]In random fading environments, the probability that any two channel matrices are orthogonal becomes zero, and thus the maximum Eigenvalues $\{\lambda_{\mathbf{B}_j,1}\}$ are not the same in general.

Here, α_L is chosen to fulfill $\sum_{j=1}^{K} \text{tr}(\mathbf{W}_j^\star) = P_A$, i.e., $\alpha_L = P_A - P_P \sum_{j=1}^{L-1} \delta_j$. Plugging this into the constraint $0 \leq \alpha_L \leq P_P \delta_L$, it is easy to see that L is the maximum index l satisfying $\sum_{j=1}^{l-1} \delta_j \leq \frac{P_A}{P_P}$. This completes the proof.

Appendix B
Proof of Lemma 1

It is straightforward to show Lemma 1 for $L = K + 1$, since it is equivalent to the case $\beta^\star = 0$ presented in Appendix A. For $L = 2, \ldots, K$, by exploiting the KKT condition in (58), the dual problem can be formulated as [20]

$$\min_{\{\mu_i \geq 0\}, \beta \geq 0, \{\nu_j \geq 0\}} \sum_{j=2}^{K+1} \tilde{\mathcal{L}}_j(\{\mathbf{V}_{ij}^\star\}, \{\mu_i\}) + P_P \sum_{j=1}^{K} \nu_j \delta_j + \beta P_A \tag{67}$$

$$\text{s.t.} \ (62), (65). \tag{68}$$

It is easy to check that the optimal $\beta^\star$ minimizing $\mathcal{G}(\{\mu_i\}, \beta, \{\nu_j\})$ is given by its minimum value $\lambda_{\mathbf{B}_L,1}$. Also, by arranging the results in Appendix A, we have $\nu_j^\star = \lambda_{\mathbf{B}_j,1} - \beta^\star = \lambda_{\mathbf{B}_j,1} - \lambda_{\mathbf{B}_L,1}$ for $j = 1, \ldots, L-1$ and $\nu_j^\star = 0$ for $j = L, L+1, \ldots, K$. Lemma 1 is thus proved.

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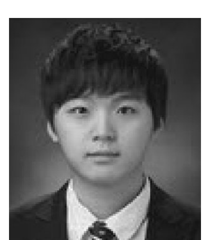

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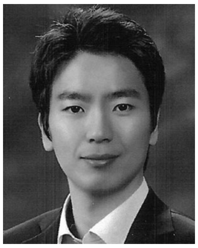

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