

Orthogonalized Spatial Multiplexing for MIMO systems

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Abstract— In this paper, we propose a new spatial multiplexing scheme for transmission over flat-fading multiple-input multiple-output (MIMO) channels, which allows a simple maximum-likelihood decoding at the receiver with small feedback information. We begin with a real-valued representation of the complex-valued system model and show that we can achieve orthogonality between transmitted signals by applying a proper rotation to transmitted symbols. Based on the minimum Euclidean distance between received vectors, we also present a simple antenna selection metric for the proposed spatial multiplexing systems. Simulation results demonstrate that our spatial multiplexing system performs close to the optimum closed loop system with much reduced complexity and feedback overhead.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems provide a very promising means to increase the spectral efficiency for wireless systems. Especially, spatial multiplexing schemes enable extremely high spectral efficiencies by transmitting independent streams of data simultaneously through multiple transmit antennas [1] [2]. Assuming perfect channel knowledge at the receiver, the potential gains of using the MIMO systems are well presented in [3] and [4].

In order to fully exploit the potential of multiple antennas and achieve the promised capacity, we can apply full channel state information (CSI) knowledge to the transmit side to optimize the transmission scheme according to current channel conditions. Most work on these closed-loop MIMO systems is carried out by obtaining singular value decomposition (SVD) of the channel transfer matrix [5] [6]. More realistic assumptions about CSI at the transmitter and receiver can impact the potential channel gains of MIMO systems [7] [8]. Another drawback of precoding systems is that the SVD operation requires high computational complexity and is known to be numerically sensitive [9].

To address these issues, the transmitter with limited feedback information in a communication system tries to utilize the system resources more efficiently [10]. Precoding based on the limited feedback has been proposed in [11], where the transmit precoder is chosen from a finite set of precoding matrices, called codebook, known to both the receiver and transmitter. The receiver selects the optimal precoder from the codebook with a selection criterion based on the current CSI and reports the index of this matrix to the transmitter over a limited feedback channel.

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In this paper, we propose a new spatial multiplexing scheme for a closed loop MIMO system which allows a simple maximum-likelihood (ML) receiver. Our interest is restricted to spatial multiplexing systems transmitting two independent data streams, which are important in practical wireless system designs. We first present a new orthogonalized spatial multiplexing scheme with reduced complexity and overhead. The ML decoding (MLD) is optimal for detecting symbols in MIMO spatial multiplexing (SM) systems. However, its computational complexity becomes exponential with the number of transmit antennas and the size of constellations. In order to reduce the processing complexity of an ML receiver, we propose a new SM system which requires only a single phase value from the receiver.

Next we extend the proposed spatial multiplexing scheme to systems with a larger number of transmit antennas. When there are more than two transmit antennas, our proposed scheme needs to choose the two best antennas to maximize the performance. We consider a criterion based on the minimum Euclidean distance for selecting the optimal subset of multiple transmit antennas in the proposed spatial multiplexing systems, since the Euclidean distance between received vectors accounts for the symbol error probability [12]. In the simulation section, we compare the performance of the proposed scheme with other closed loop systems such as the optimal unitary precoding [11] and the optimal linear precoding [6] over flat-fading quasistatic channels in terms of bit error rate (BER).

The organization of the paper is as follows: Section II presents the system model and reviews conventional precoding schemes. In Section III, we propose a new spatial multiplexing scheme and show that the proposed transmission scheme attains single-symbol decodability at the receiver. Section IV illustrates the antenna selection method based on the Euclidean distance between received vectors in the proposed spatial multiplexing system. In Section V, the simulation results are presented comparing the proposed method with other precoding schemes. Finally, the paper is terminated with conclusions in Section VI.

II. SYSTEM DESCRIPTIONS

In this section, we consider a spatial multiplexing system with M_t transmit and M_r receive antennas. We assume that the elements of the MIMO channel matrix are obtained from an independent and identically distributed (i.i.d) complex Gaussian distribution. Each channel realization is assumed to be known at the receiver. Throughout this paper, normal letters

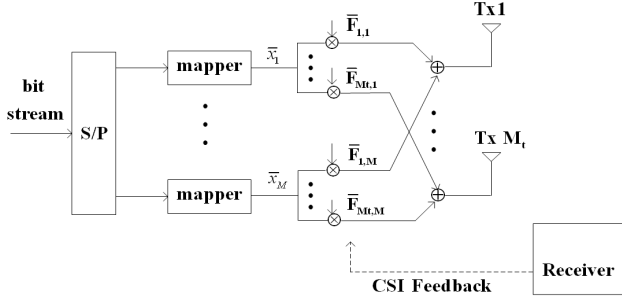


Fig. 1. Schematic diagram of transmission from M_t transmit antennas in closed-loop precoding MIMO systems

represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. With a bar accounting for complex variables, for any complex notation \bar{c} , we denote the real and imaginary part of \bar{c} by $\Re[\bar{c}]$ and $\Im[\bar{c}]$, respectively.

Let us define the M_t -dimensional complex transmitted signal vector $\bar{\mathbf{x}}$, and the M_r -dimensional complex received signal vector $\bar{\mathbf{y}}$. Then the complex received signal is given by

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}} \quad (1)$$

where $\bar{\mathbf{n}}$ is a complex Gaussian noise vector with covariance matrix $\sigma_n^2 \mathbf{I}_{M_r}$, and \mathbf{I}_d indicates an identity matrix of size d . Here the channel response matrix can be written as

$$\bar{\mathbf{H}} = \begin{bmatrix} \bar{h}_{11} & \cdots & \bar{h}_{1M_t} \\ \vdots & \ddots & \vdots \\ \bar{h}_{M_r1} & \cdots & \bar{h}_{M_r M_t} \end{bmatrix}$$

where \bar{h}_{ji} represents the channel response between the i th transmit and the j th receive antenna.

In what follows, we briefly review other two closed loop schemes. The system consists of a spatial demultiplexer that produces M independent data streams and a spatial precoder that maps these M streams to M_t transmit antennas ($M \leq M_t$), as shown in Figure 1. Here $\bar{\mathbf{F}}$ denotes the M_t by M precoding matrix and \bar{F}_{ij} indicates the (i, j) element of $\bar{\mathbf{F}}$. First, we consider the unitary precoding scheme proposed in [11] for limited feedback cases. Denoting $\bar{\mathcal{U}}(m, n)$ as a set of m by n matrices with orthogonal columns, the singular value decomposition of $\bar{\mathbf{H}}$ is given by

$$\bar{\mathbf{H}} = \bar{\mathbf{U}}\bar{\Sigma}\bar{\mathbf{V}}^* \quad (2)$$

where $\bar{\mathbf{U}} \in \bar{\mathcal{U}}(M_r, M_r)$, $\bar{\mathbf{V}} \in \bar{\mathcal{U}}(M_t, M_t)$, and $\bar{\Sigma}$ denotes an M_r by M_t diagonal matrix with the k th singular value of $\bar{\mathbf{H}}$ at entry (k, k) .

Then the optimal precoder $\bar{\mathbf{F}}_{opt} \in \bar{\mathcal{U}}(M_t, M)$ is given by $\bar{\mathbf{F}}_{opt} = \bar{\mathbf{V}}_M$ [11], where $\bar{\mathbf{V}}_M$ is a matrix constructed from the first M columns of $\bar{\mathbf{V}}$. We refer to this precoder as *Optimal Unitary Precoding* (OUP).

Second, for the case of full CSI at the transmitter, we consider the optimum linear precoder using the minimum mean squared error (MMSE) criterion subject to a transmitted

power constraint. Then the optimal linear precoder can be described by $\bar{\mathbf{F}}_{opt} = \bar{\mathbf{V}}_M \Phi_f$ with the diagonal matrix Φ_f given by [6]

$$\Phi_f = \left(\frac{\sigma_n}{\sqrt{E_s} \mu} \Sigma_M^{-1} - \frac{\sigma_n^2}{E_s} \Sigma_M^{-2} \right)_+^{1/2} \quad (3)$$

where $(\cdot)_+$ indicates that negative elements of the matrix are replaced by zero, μ is a parameter computed according to the total transmit power constraint, and Σ_M represents the $M \times M$ upper-left matrix of the diagonal matrix Σ . We denote this precoder as *Optimal Linear Precoding* (OLP).

The main problem with these approaches in (2) and (3) is that the conventional precodings require high complexity processing associated with SVD and high feedback overhead in sending information on the channel or precoding matrices. In the following sections, we propose a new spatial multiplexing scheme with reduced complexity and overhead.

III. NEW SPATIAL MULTIPLEXING SCHEME

In this section, we present an orthogonal spatial multiplexing (OSM) scheme based on a single phase value and show how to simplify the ML detection for the spatial multiplexing systems with two transmit antennas ($M_t = 2$).

Let Q denote a signal constellation of size M_c . Given the channel matrix $\bar{\mathbf{H}}$, the ML estimate of the transmitted vector $\bar{\mathbf{x}}$ is given by

$$\hat{\bar{\mathbf{x}}} = [\hat{x}_1 \ \hat{x}_2]^t = \arg \min_{\bar{\mathbf{x}} \in Q^2} \|\bar{\mathbf{y}} - \bar{\mathbf{H}}\bar{\mathbf{x}}\|^2 \quad (4)$$

where $[\cdot]^t$ indicates the transpose of a vector or matrix and $\|\cdot\|$ denotes the Euclidean norm. Note that the ML decoding problem is exponential in the number of constellation points.

Equivalently, the real-valued representation of the system (1) can be written as [13]

$$\mathbf{y} = \begin{bmatrix} \Re[\bar{\mathbf{y}}] \\ \Im[\bar{\mathbf{y}}] \end{bmatrix} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (5)$$

where $\mathbf{x} = [\Re[\bar{\mathbf{x}}^t] \ \Im[\bar{\mathbf{x}}^t]]^t$, $\mathbf{n} = [\Re[\bar{\mathbf{n}}^t] \ \Im[\bar{\mathbf{n}}^t]]^t$, and

$$\mathbf{H} = \begin{bmatrix} \Re[\bar{\mathbf{H}}] & -\Im[\bar{\mathbf{H}}] \\ \Im[\bar{\mathbf{H}}] & \Re[\bar{\mathbf{H}}] \end{bmatrix} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \mathbf{h}_3 \ \mathbf{h}_4]. \quad (6)$$

Here \mathbf{n} is a real Gaussian noise vector with covariance matrix $\frac{\sigma_n^2}{2} \mathbf{I}_{2M_r}$.

From the real-valued representation of the channel matrix in (6), it is easy to see that the column vectors \mathbf{h}_1 and \mathbf{h}_2 are orthogonal to \mathbf{h}_3 and \mathbf{h}_4 , respectively ($\mathbf{h}_1 \perp \mathbf{h}_3$ and $\mathbf{h}_2 \perp \mathbf{h}_4$). We also notice that $\mathbf{h}_1 \cdot \mathbf{h}_2 = \mathbf{h}_3 \cdot \mathbf{h}_4$ and $\mathbf{h}_1 \cdot \mathbf{h}_4 = -\mathbf{h}_2 \cdot \mathbf{h}_3$, where $\mathbf{h}_i \cdot \mathbf{h}_j$ denotes the inner (dot) product between vectors \mathbf{h}_i and \mathbf{h}_j . For the rest of this section, we will see that these properties are essential to the development of the new spatial multiplexing scheme.

Based on the real-valued representation in (5), the ML solution $\hat{\bar{\mathbf{x}}}$ to (4) can be alternatively obtained by

$$\hat{\bar{\mathbf{x}}} = [\hat{x}_1 \ \hat{x}_2]^t = \arg \min_{\bar{\mathbf{x}} \in Q^2} \left\| \mathbf{y} - \mathbf{H} \begin{bmatrix} \Re[\bar{\mathbf{x}}] \\ \Im[\bar{\mathbf{x}}] \end{bmatrix} \right\|^2. \quad (7)$$

Note that the ML estimation metric (4) and (7) require the same amount of computation.

In what follows, we present the OSM to simplify the ML decoding. To achieve this goal, we encode the two transmitted symbols as

$$\mathcal{F}(\bar{\mathbf{x}}, \theta) = \begin{bmatrix} 1 & 0 \\ 0 & \exp(j\theta) \end{bmatrix} \mathbf{s}(\bar{\mathbf{x}}) \quad (8)$$

where θ is the rotation phase angle applied to the second antenna and

$$\mathbf{s}(\bar{\mathbf{x}}) \triangleq \begin{bmatrix} \Re[\bar{x}_1] + j\Re[\bar{x}_2] \\ \Im[\bar{x}_1] + j\Im[\bar{x}_2] \end{bmatrix}.$$

With the above precoding, the original system model in (1) is transformed into

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\mathcal{F}(\bar{\mathbf{x}}, \theta) + \bar{\mathbf{n}} = \bar{\mathbf{H}}_\theta \mathbf{s}(\bar{\mathbf{x}}) + \bar{\mathbf{n}} \quad (9)$$

where

$$\bar{\mathbf{H}}_\theta = \bar{\mathbf{H}} \begin{bmatrix} 1 & 0 \\ 0 & \exp(j\theta) \end{bmatrix}.$$

Here $\bar{\mathbf{H}}_\theta$ accounts for the effective channel matrix for $\mathbf{s}(\bar{\mathbf{x}})$.

Then, the real-valued system model corresponding to (9) can be represented as

$$\begin{aligned} \mathbf{y} = \begin{bmatrix} \Re[\bar{\mathbf{y}}] \\ \Im[\bar{\mathbf{y}}] \end{bmatrix} &= \begin{bmatrix} \Re[\bar{\mathbf{H}}_\theta] & -\Im[\bar{\mathbf{H}}_\theta] \\ \Im[\bar{\mathbf{H}}_\theta] & \Re[\bar{\mathbf{H}}_\theta] \end{bmatrix} \begin{bmatrix} \Re[\mathbf{s}(\bar{\mathbf{x}})] \\ \Im[\mathbf{s}(\bar{\mathbf{x}})] \end{bmatrix} + \begin{bmatrix} \Re[\bar{\mathbf{n}}] \\ \Im[\bar{\mathbf{n}}] \end{bmatrix} \\ &= [\mathbf{h}_1^\theta \ \mathbf{h}_2^\theta \ \mathbf{h}_3^\theta \ \mathbf{h}_4^\theta] \begin{bmatrix} \Re[\bar{x}_1] \\ \Im[\bar{x}_1] \\ \Re[\bar{x}_2] \\ \Im[\bar{x}_2] \end{bmatrix} + \mathbf{n} \end{aligned} \quad (10)$$

where the real column vector \mathbf{h}_i^θ of length $2M_r$ denotes the i th column of the effective real-valued channel matrix. Recall that the column vectors \mathbf{h}_1^θ and \mathbf{h}_2^θ are orthogonal to \mathbf{h}_3^θ and \mathbf{h}_4^θ , respectively, regardless of θ . In this case, the spatial multiplexing scheme becomes fully orthogonal if and only if $\mathbf{h}_1^\theta \perp \mathbf{h}_4^\theta$ and $\mathbf{h}_2^\theta \perp \mathbf{h}_3^\theta$.

Denoting \bar{h}_{ij}^θ as the (i, j) th entry of $\bar{\mathbf{H}}_\theta$, we obtain the inner product between \mathbf{h}_1^θ and \mathbf{h}_4^θ as

$$\begin{aligned} \mathbf{h}_1^\theta \cdot \mathbf{h}_4^\theta &= (\mathbf{h}_1^\theta)^T \mathbf{h}_4^\theta \\ &= -\sum_{m=1}^{M_r} \Re[\bar{h}_{m1}^\theta] \Im[\bar{h}_{m2}^\theta] + \sum_{m=1}^{M_r} \Im[\bar{h}_{m1}^\theta] \Re[\bar{h}_{m2}^\theta]. \end{aligned} \quad (11)$$

Since $\mathbf{h}_2^\theta \cdot \mathbf{h}_3^\theta = -\mathbf{h}_1^\theta \cdot \mathbf{h}_4^\theta$, the orthogonality of the spatial multiplexing can be achieved as long as Equation (11) becomes zero. After trigonometric computations on (11), the rotation angle for the orthogonality between \mathbf{h}_1^θ and \mathbf{h}_4^θ (or \mathbf{h}_2^θ and \mathbf{h}_3^θ) can be written as

$$\theta = \tan^{-1} \left(\frac{B}{A} \right) \pm \frac{\pi}{2} \quad (12)$$

where $A = \sum_{m=1}^{M_r} |\bar{h}_{m1}| |\bar{h}_{m2}| \sin(\angle \bar{h}_{m2} - \angle \bar{h}_{m1})$ and $B = \sum_{m=1}^{M_r} |\bar{h}_{m1}| |\bar{h}_{m2}| \cos(\angle \bar{h}_{m2} - \angle \bar{h}_{m1})$. Here $|\cdot|$ and \angle indicate the magnitude and the phase of a complex number, respectively.

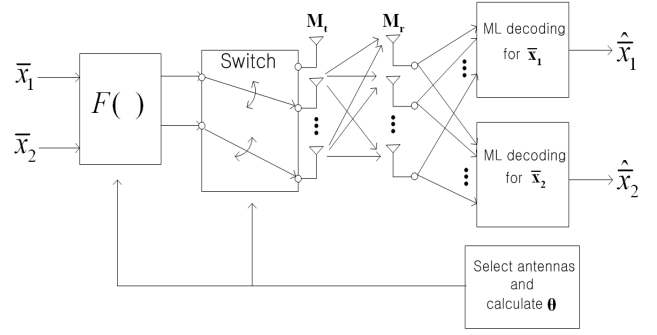


Fig. 2. Block diagram of a limited feedback MIMO system

Using the rotation angle of (12), we can achieve the orthogonality between transmitted signals in (10) where the subspace spanned by \mathbf{h}_1^θ and \mathbf{h}_2^θ becomes orthogonal to that spanned by \mathbf{h}_3^θ and \mathbf{h}_4^θ . As shown in [14], utilizing this orthogonality, the ML solution $\hat{\bar{\mathbf{x}}} = [\hat{x}_1 \ \hat{x}_2]^T$ in Equation (7) can be individually given by

$$\hat{x}_1 = \arg \min_{\bar{x} \in Q} \left\| \mathbf{y} - [\mathbf{h}_1^\theta \ \mathbf{h}_2^\theta] \begin{bmatrix} \Re[\bar{x}] \\ \Im[\bar{x}] \end{bmatrix} \right\|^2 \quad (13)$$

and

$$\hat{x}_2 = \arg \min_{\bar{x} \in Q} \left\| \mathbf{y} - [\mathbf{h}_3^\theta \ \mathbf{h}_4^\theta] \begin{bmatrix} \Re[\bar{x}] \\ \Im[\bar{x}] \end{bmatrix} \right\|^2. \quad (14)$$

Note that in determining \hat{x}_1 and \hat{x}_2 in (13) and (14), the size of the search set reduces to Q . These ML decoding equations show that with the proposed transmission scheme, the ML decoding at the receiver can be done by searching for a single symbol (called *single-symbol decodable*), while the traditional ML decoding in (4) requires searching a pair of symbols. Therefore, in our proposed spatial multiplexing system, the decoding complexity reduces from $\mathcal{O}(M_c^2)$ to $\mathcal{O}(M_c)$, where the complexity accounts for the number of search candidates in the ML decoding.

IV. ANTENNA SELECTION SCHEME

In this section, we will extend the proposed scheme to systems with more than two transmit antennas. To this end, we introduce a simplified antenna selection method for the proposed spatial multiplexing system. We assume a spatial multiplexing system with M_t transmit antennas and M_r receive antennas.

The general data path for the proposed MIMO transmission is shown in Figure 2. Two input symbols are precoded by the function $\mathcal{F}(\bar{\mathbf{x}}, \theta)$ as in (8), and are transmitted over two transmit antennas out of M_t transmit antennas. The optimal selection of two transmit antennas is made based on the minimum Euclidean distance.

Let $\mathbf{P}(M_t, 2)$ denote the set of all possible $\binom{M_t}{2} = \frac{M_t(M_t-1)}{2}$ subsets out of M_t transmit antennas. For a subset $P \in \mathbf{P}(M_t, 2)$, the receive constellation is defined as $\{\bar{\mathbf{H}}_P^\theta \mathbf{s}(\bar{\mathbf{x}}) | \bar{\mathbf{x}} \in Q^2\}$ [12] where $\bar{\mathbf{H}}_P^\theta$ denotes the $M_r \times 2$ virtual channel matrix corresponding to the transmit antenna

subset P . Then, we need to determine the optimum subset P whose squared minimum distance $d_{min}^2(P)$ between transmitted vectors $\bar{\mathbf{x}}_c$ and $\bar{\mathbf{x}}_e$ is the greatest. We compute $d_{min}^2(P)$ as

$$\begin{aligned} d_{min}^2(P) &= \min_{\bar{\mathbf{x}}_c, \bar{\mathbf{x}}_e \in Q^2} \left\| \bar{\mathbf{H}}_P^\theta (\bar{\mathbf{x}}_c - \bar{\mathbf{x}}_e) \right\|^2 \\ &= \min_{\bar{\mathbf{x}}_c, \bar{\mathbf{x}}_e \in Q^2} \left\| \begin{bmatrix} \mathbf{h}_{P,1}^\theta & \mathbf{h}_{P,2}^\theta & \mathbf{h}_{P,3}^\theta & \mathbf{h}_{P,4}^\theta \end{bmatrix} \begin{bmatrix} \Re[\bar{x}_{1,c} - \bar{x}_{1,e}] \\ \Im[\bar{x}_{1,c} - \bar{x}_{1,e}] \\ \Re[\bar{x}_{2,c} - \bar{x}_{2,e}] \\ \Im[\bar{x}_{2,c} - \bar{x}_{2,e}] \end{bmatrix} \right\|^2 \end{aligned} \quad (15)$$

where $\mathbf{h}_{P,i}^\theta$ is the i th column of the real-valued representation of $\bar{\mathbf{H}}_P^\theta$. Since the computation of $d_{min}^2(P)$ involves all possible pairs of $\bar{\mathbf{x}}_c$ and $\bar{\mathbf{x}}_e$, the conventional spatial multiplexing systems require a search over $\binom{M_c}{2} = \frac{M_c(M_c-1)}{2}$ vectors.

In the following, we show that the proposed spatial multiplexing allows us to obtain the minimum distance in a much simpler form. Note that the subspace spanned by $\mathbf{h}_{P,1}^\theta$ and $\mathbf{h}_{P,2}^\theta$ is orthogonal to that spanned by $\mathbf{h}_{P,3}^\theta$ and $\mathbf{h}_{P,4}^\theta$. In this case, assuming that two symbols \bar{x}_1 and \bar{x}_2 are independent of each other, Equation (15) can be rewritten as

$$\begin{aligned} d_{min}^2(P) &= \min_{\bar{x}_{1,c}, \bar{x}_{1,e} \in Q} \left\| \begin{bmatrix} \mathbf{h}_{P,1}^\theta & \mathbf{h}_{P,2}^\theta \end{bmatrix} \begin{bmatrix} \Re[\bar{x}_{1,c} - \bar{x}_{1,e}] \\ \Im[\bar{x}_{1,c} - \bar{x}_{1,e}] \end{bmatrix} \right\|^2 \\ &\quad + \min_{\bar{x}_{2,c}, \bar{x}_{2,e} \in Q} \left\| \begin{bmatrix} \mathbf{h}_{P,3}^\theta & \mathbf{h}_{P,4}^\theta \end{bmatrix} \begin{bmatrix} \Re[\bar{x}_{2,c} - \bar{x}_{2,e}] \\ \Im[\bar{x}_{2,c} - \bar{x}_{2,e}] \end{bmatrix} \right\|^2 \end{aligned} \quad (16)$$

Furthermore, we note that the first term on the right-hand side of equation (16) has the same minimum distance as the second term since the geometrical relationship between $\mathbf{h}_{P,1}^\theta$ and $\mathbf{h}_{P,2}^\theta$ remains the same as that between $\mathbf{h}_{P,3}^\theta$ and $\mathbf{h}_{P,4}^\theta$ (i.e., $\|\mathbf{h}_{P,1}^\theta\| = \|\mathbf{h}_{P,3}^\theta\|$, $\|\mathbf{h}_{P,2}^\theta\| = \|\mathbf{h}_{P,4}^\theta\|$, and $\mathbf{h}_{P,1}^\theta \cdot \mathbf{h}_{P,2}^\theta = \mathbf{h}_{P,3}^\theta \cdot \mathbf{h}_{P,4}^\theta$). This symmetry means that, in the computation of the minimum distance, we need to consider only one of the two terms in (16) while assuming that the other term is zero. In other words, we can set $\bar{x}_{2,c} = \bar{x}_{2,e}$ while $d_{min}^2(P)$ is computed with $\bar{x}_{1,c} \neq \bar{x}_{1,e}$. Let us define a difference vector as $\mathbf{e}(\bar{x}_c, \bar{x}_e) = [\Re[\bar{x}_c - \bar{x}_e] \Im[\bar{x}_c - \bar{x}_e]]^T$ with $\bar{x}_c \neq \bar{x}_e$. Then, Equation (16) can be simplified as

$$d_{min}^2(P) = \min_{\bar{x}_c, \bar{x}_e \in Q} \left\| \begin{bmatrix} \mathbf{h}_{P,1}^\theta & \mathbf{h}_{P,2}^\theta \end{bmatrix} \mathbf{e}(\bar{x}_c, \bar{x}_e) \right\|^2. \quad (17)$$

It is clear that the computation of $d_{min}^2(P)$ in (17) requires a search over $\binom{M_c}{2} = \frac{M_c(M_c-1)}{2}$ vectors.

Now we will illustrate a way to reduce the computational complexity even further. Considering the symmetries in uniform QAM constellations, we can significantly reduce the set of difference vectors to search in Equation (17). For illustrative purposes, we consider 16QAM as shown in Figure 3. Note that, among all possible $\binom{16}{2} = 120$ difference vectors $\mathbf{e}(\bar{x}_c, \bar{x}_e)$, there exist many equal and collinear difference vectors. For example, pairs (\bar{x}_4, \bar{x}_6) and $(\bar{x}_{12}, \bar{x}_{14})$ yield the same difference vectors ($\mathbf{e}(\bar{x}_4, \bar{x}_6) = \mathbf{e}(\bar{x}_{12}, \bar{x}_{14})$) while pairs $(\bar{x}_4, \bar{x}_{10})$ and (\bar{x}_4, \bar{x}_7) are related as collinear difference vectors ($\mathbf{e}(\bar{x}_4, \bar{x}_{10}) = 2\mathbf{e}(\bar{x}_4, \bar{x}_7)$). Then, by excluding these

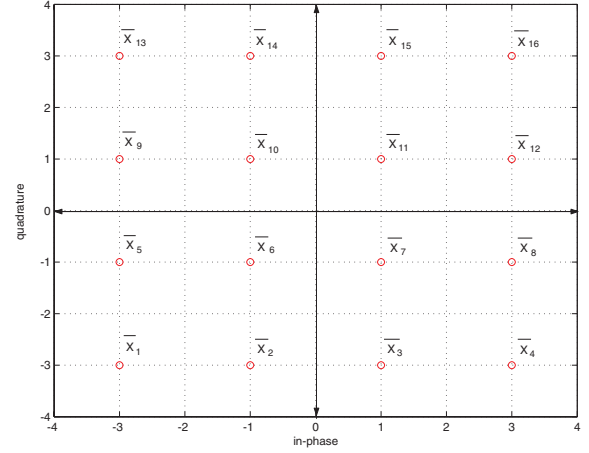


Fig. 3. 16QAM constellation

TABLE I
CANDIDATE PAIRS OF (\bar{x}_c, \bar{x}_e) FOR THE SET $\hat{\chi}_{c-e}$ OF $\mathbf{e}(\bar{x}_c, \bar{x}_e)$

Case	Mod	$(\Re[\bar{x}_c], \Im[\bar{x}_c])$	$(\Re[\bar{x}_e], \Im[\bar{x}_e])$
$\mathbf{h}_{P,1}^\theta \cdot \mathbf{h}_{P,2}^\theta \geq 0$, $\ \mathbf{h}_{P,1}^\theta\ \leq \ \mathbf{h}_{P,2}^\theta\ $	QPSK	(1, -1)	(-1, ± 1)
	16QAM	(3, -3)	(-3, ± 1), (± 1 , -1), (1, -3)
$\mathbf{h}_{P,1}^\theta \cdot \mathbf{h}_{P,2}^\theta \geq 0$, $\ \mathbf{h}_{P,1}^\theta\ > \ \mathbf{h}_{P,2}^\theta\ $	QPSK	(1, -1)	(± 1 , 1)
	16QAM	(3, -3)	(± 1 , 3), (1, ± 1), (3, -1)
$\mathbf{h}_{P,1}^\theta \cdot \mathbf{h}_{P,2}^\theta < 0$, $\ \mathbf{h}_{P,1}^\theta\ > \ \mathbf{h}_{P,2}^\theta\ $	QPSK	(-1, -1)	(± 1 , 1)
	16QAM	(-3, -3)	(± 1 , 3), (-1, ± 1), (-3, -1)
$\mathbf{h}_{P,1}^\theta \cdot \mathbf{h}_{P,2}^\theta < 0$, $\ \mathbf{h}_{P,1}^\theta\ \leq \ \mathbf{h}_{P,2}^\theta\ $	QPSK	(-1, -1)	(1, ± 1)
	16QAM	(-3, -3)	(3, ± 1), (± 1 , -1), (-1, -3)

equal and collinear difference vectors and utilizing the channel's geometrical properties such as the norm and the inner product of $\mathbf{h}_{P,1}^\theta$ and $\mathbf{h}_{P,2}^\theta$, we can determine the set $\hat{\chi}_{c-e}$ of $\mathbf{e}(\bar{x}_c, \bar{x}_e)$ which is actually used in the computation of $d_{min}^2(P)$ without any performance degradation. We generalize the candidate pairs (\bar{x}_c, \bar{x}_e) for the set $\hat{\chi}_{c-e}$ as listed in Table I. This approach can be easily extended to higher order M_c -QAM schemes.

Finally, $d_{min}^2(P)$ can be expressed as

$$d_{min}^2(P) = \min_{\mathbf{e}(\bar{x}_c, \bar{x}_e) \in \hat{\chi}_{c-e}} \left\| \begin{bmatrix} \mathbf{h}_{P,1}^\theta & \mathbf{h}_{P,2}^\theta \end{bmatrix} \mathbf{e}(\bar{x}_c, \bar{x}_e) \right\|^2 \quad (18)$$

which facilitates a search for the optimal antenna subset P^* .

As a consequence, the optimal subset P^* from the entire set $\mathbf{P}(M_t, 2)$ is obtained as

$$P^* = \arg \max_{P \in \mathbf{P}(M_t, 2)} d_{min}^2(P).$$

It is important to note that the proposed spatial multiplexing scheme reduces the size of the set of candidate vectors in computing $d_{min}^2(P)$ from 120, 32640, and 8386560 to 2, 5, and 19 for QPSK, 16QAM and 64QAM, respectively, as Equation (15) for the minimum Euclidean distance is equivalent to Equation (18).

V. SIMULATION RESULTS

In this section, we provide simulation results that illustrate the performance of our proposed OSM compared with the

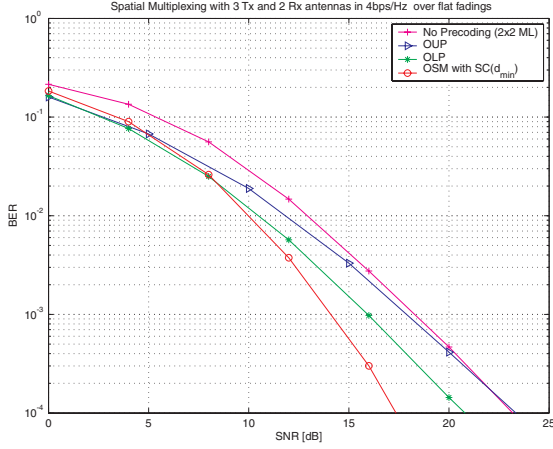


Fig. 4. BER performance of the spatial multiplexing schemes with 4QAM.

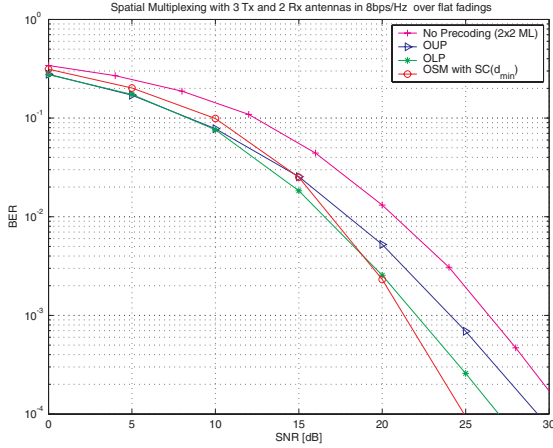


Fig. 5. BER performance of the spatial multiplexing schemes with 16QAM.

closed loop schemes OUP and OLP in Section II.

In Figures 4 and 5, we depict the BER comparison of the OSM and two optimal precodings OUP and OLP with $M_t = 3$ and $M_r = 2$. For comparison purposes, we also plot the performance of the 2x2 spatial multiplexing with ML decoding at the receiver where the number of the search candidates for the ML decoding is M_c^2 without any precoding. We employ the proposed antenna selection method based on Equation (18) (denoted by $SC(d_{min})$). For the 4QAM case presented in Figure 4, we can see that the OSM provides a 4 dB gain at a BER of 10^{-3} over the no precoding case. More importantly, Figure 4 shows that the OSM outperforms both the OUP and OLP cases by 1.8 dB and 3.8 dB, respectively. It should be noted that in the conventional precoding systems we need to solve (2) and (3) to derive the precoding matrix by computing the SVD operation and the power allocation matrix that determines the power distribution among the spatial modes. As for the feedback overhead, our proposed scheme needs only a single phase value feedback, while the conventional precoding schemes require much larger feedback information in sending back the entire channel or precoding

matrix, especially including the power allocation matrix for the OLP. Finally, for the case of 16QAM in Figure 5, we see a similar trend as for 4QAM in Figure 4.

VI. CONCLUSION

In this paper, we presented a new orthogonalized spatial multiplexing scheme for MIMO systems which minimizes the overall complexity. The primary goal of this work is to maximize the system performance with a low complexity at the receiver while maintaining the optimal ML decoding. By taking a single phase value feedback, simple orthogonal ML decoding is achieved. Also, we presented a simple antenna selection method for choosing the subset of transmit antennas that maximizes the minimum Euclidean distance of the receive constellation in the proposed spatial multiplexing system. The simulation results confirm that the proposed orthogonal spatial multiplexing scheme is quite effective in approaching the performance of the optimal linear precoding with a significantly reduced complexity and feedback amount.

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