Abstract—This paper considers a three-way wireless communication system employing physical layer network coding (PNC), where each user desires to transmit independent data to the other users via relay. However, one difficulty with this system is that the performance is limited by the worst channel. To overcome this problem, we adopt a scheduling system. Since channel characteristics vary over time in wireless communications, a scheduling technique employing network coding where users are selected based on the instantaneous signal-to-noise ratio was proposed for the broadcast channel (BC) phase. In this paper, we extend the scheduling technique to the multiple access channel phase as well as the BC phase. We propose two criteria for selecting users based on the channel norm and the minimum distance criterion. Also, an efficient method to compute the minimum distance is introduced. The proposed scheduling for PNC provides a significant improvement over the conventional scheme.

I. INTRODUCTION

Network coding has been one of subjects which attracts interests in the research area over several years. The network coding was first proposed in [1] as a network-layer technique in wired networks. The basic concept is that the transmitter combines packets intended for different destination and broadcasts the combined packets. Due to its advantage of throughput improvement, the network coding was adopted to a wireless relaying scenario.

Most of works on wireless relaying system with the network coding is related with two-way traffic [2]–[6]. Thus, In this paper, we consider a three-way communication system utilizing network coding where a relay node conveys the communications among three user nodes. By adopting the network coding at the relay, we can reduce the number of time slots required to transmit data to users. There are several previous works on the three-way relay channels employing the network coding. In [7], joint network coding and superposition coding was proposed for the three-way relay channels. Also, an iterative three-way network coding optimization algorithm was introduced in [8].

However, one of difficulties with the network coding involving multiple users is that the performance is limited by the channel with the worst signal-to-noise ratio (SNR) [9]. This problem should be properly addressed in designing practical communication systems over wireless channels where the links have time-varying characteristics. To overcome this problem, opportunistic scheduling for wireless network coding has been proposed in [9], where the relay combines the data only for the limited number of users with higher instantaneous SNR at each time. However, the scheduling technique in [9] considers only the broadcast channel (BC) phase where the relay transmits the data to the users.

On the other hand, it is well known that we can improve the performance of communication systems with network coding by adopting physical layer network coding (PNC) [10]. With the PNC, we can further reduce the required number of time slots for the communication from the users to the relay, since the relay node receives the signals from multiple user simultaneously during the multiple access channel (MAC) phase. In this paper, we extend the scheduling scheme to the MAC phase as well as the BC phase for physical layer network coding with BPSK and QPSK. More specifically, we propose a new scheme where we select a pair of users out of three users when exchanging data by means of the PNC.

We present new criteria for selecting users to implement the scheduling. Instead of using the instantaneous SNR of channels for each user as in [9], we adopt two other criteria; one is channel norm, and the other one is minimum distance between the signals of the superposed constellation at the
relay. To this end, we employ an efficient method which obtains the minimum distance between the signals for QPSK without any search. Simulation results show that the proposed techniques outperform the conventional schemes.

The rest of this paper is organized as follows: In Section II, we describe the three-way relay channel model used in this paper. Section III proposes two criteria for the user selection over fading channels. Section IV presents an efficient method to obtain the minimum distance of the constellation of the PNC for QPSK. In Section V, we provide the numerical results which compare with other schemes. Section VI concludes the paper with further research directions.

II. SYSTEM DESCRIPTIONS

Fig. 1 illustrates the three-way relay channel model considered in this paper. Each node wants to transmit different information to the other two nodes through the relay. For example, $s_{AB}$ represents the information to be sent from user A to user B. We assume that there is no direct communication among user nodes. Without network coding, this requires six time slots for the MAC phase and six additional time slots for the BC phase to complete the three-way communication. If the relay employs the network coding, the required time slots for the BC phase is reduced to three, since the relay can broadcast the network coded data $s_{AB} \oplus s_{BA}, s_{BC} \oplus s_{BC}$ and $s_{CA} \oplus s_{AC}$ instead of delivering individual data. Then each user can decode the data from the other users by using the XOR operation between the received data and its own data. Thus, it requires nine time slots in total to complete the communication.

To further reduce the number of required time slots, we consider a scheme where the three users transmit to the relay simultaneously during the MAC phase. However, the received signal at the relay may suffer from multiple access interference. Thus, in this paper, we employ a pairwise PNC where during the MAC phase, two out of three users transmit simultaneously as shown in Fig. 2 (a). Then the relay generates a network coded data using the received data, and broadcasts the network coded data to users. When the relay generates the network coded data, the noise impact on the relay is removed by maximum-likelihood (ML) detector and network code mapper, which is referred to as denoising. We perform this process for all three node pairs. The time slots required for the MAC phase is then reduced to three and the total required time slots become six. We can view this system as the two-way relay channel (TWRC) operating in an alternating fashion among three different channel pairs.

Now we describe the TWRC system between users $p$ and $q \in \{A, B, C\}$. Let us denote $s_{pq} \in \mathbb{Z}_n$ as the transmitted data from node $p$, where $\mathbb{Z}_n$ is defined as a non-negative integer set with cardinality $n$ and $n$ indicates the modulation level. For example, we have $\mathbb{Z}_4 = \{0, 1, 2, 3\}$. The constellation mapper $\mathcal{M}$ generates the transmitted signal as $x_p = \mathcal{M}(s_{pq})$. The transmitted signals are assumed to have a unit energy, i.e., $E[|x_p|^2] = 1$. We assume that the channel coefficients do not change during the MAC and the BC operation for the selected channel pair. We also assume that perfect channel estimation is available at the receiver.

During the MAC phase, a selected node pair $[p, q] \in U = \{(A, B), (B, C), (C, A)\}$ transmits its signals to the relay simultaneously. Then, the received signal at the relay is given by

$$y_{[p,q]} = h_p x_p + h_q x_q + z$$

(1)

where the channel gains $h_A$, $h_B$ and $h_C$ are independent complex Gaussian random variables with unit variances and $z \sim \mathcal{N}(0, \sigma^2)$ indicates the complex Gaussian noise.

The relay detects $(\hat{s}_p, \hat{s}_q)$ based on the received signal $y_{[p,q]}$ of (1) by employing an ML detector as

$$(\hat{s}_p, \hat{s}_q) = \arg \min_{(s_p, s_q)} |y_{[p,q]} - (h_p x_p + h_q x_q)|^2.$$  

(2)

Then, the denoising mapper $\mathcal{D}(\cdot)$ generates the denoising codeword $s_{R,pq} = \mathcal{D}(\hat{s}_p, \hat{s}_q)$ using ML output of (2). The most conventional denoising mapper is the XOR operation, i.e., $\mathcal{D}_{\text{XOR}}(s_p, s_q) = s_p \oplus s_q$. For the case where QPSK is employed at the users, the adaptive denoising mapper $\mathcal{D}_{\text{AD}}$ proposed in [5] can be also considered which outputs either 4-ary or 5-ary alphabets.

During the BC phase, the relay $R$ broadcasts $x_{R,pq} = \mathcal{M}_R(s_{R,pq})$ to the user $p$ and $q$ as illustrated in Fig. 2 (b), where $\mathcal{M}_R$ represents the constellation mapper at the relay. For the case of QPSK, the cardinality of $\mathcal{D}_{\text{XOR}}$ is 4, while that of $\mathcal{D}_{\text{AD}}$ is either 4 or 5. Thus, $\mathcal{M}_R$ with the denoising mapper $\mathcal{D}_{\text{AD}}$ is either QPSK or 5QAM as in [5]. The received signal at the node $p$ is obtained by

$$y_p = g_p x_{R,pq} + w_p$$

where the channel gains $g_p$ is the complex Gaussian random variable with unit variance and $w_p \sim \mathcal{N}(0, \sigma^2)$ indicates the complex Gaussian noise. Then, node $p$ and $q$ can decode $s_q$ and $s_p$ using their own data $s_p$ and $s_q$, respectively, if the denoising mapper is appropriately designed.

Once the MAC and the BC phases are completed for a given user pair, we repeat this process for different user pairs. We may coordinate this scheduling among the user pairs sequentially in a round-robin fashion. However, when
the channel coefficients change over time, we can improve the performance by opportunistically selecting the user pair based on the channel condition. In the next section, we present the selection schemes proposed in this paper.

III. OPPORTUNISTIC SELECTION CRITERIA

In this section, we propose two criteria for selecting a user pair for given instantaneous channels $h_A, h_B$ and $h_C$. Since the distribution of the channels are assumed to be identical, all node pairs have an equal opportunity to communicate on average. Our goal is to find the pair of users which generates the largest rate among three user pairs. However, computing the exact instantaneous rate of PNC for TWRC would be very complicated. Thus, we propose two simple criteria with manageable complexity for selecting the user pair in this section.

A. Channel Norm Criterion

One simple criterion is to find two users with two largest channel norm. The relay identifies the user with the smallest channel norm and selects the other two users. Although this criterion provides a good performance improvement over the fixed selection scheme, choosing the user pair based only on the channel norm is suboptimal. Thus, we consider other criteria which provide better performance in the following.

B. Minimum Distance Criterion

A better selection criterion is to identify the user pair based on the bit error rate (BER) of PNC for TWRC. Since the BER depends on the phase between the two channel coefficients as well as the channel norms, we need to compute the exact instantaneous BER of PNC for TWRC for given channels.\(^1\)

However, the computation of the BER for the PNC may not be tractable. Thus we employ a criterion based on the minimum distance between signals at the relay. To obtain the minimum distance for node $p$ and $q$, denoted as $d_{\text{min}}[p,q]$, we first calculate the Euclidean distance between two different points $(s_p, s_q)$ and $(s_p', s_q')$ as

$$d_{(s_p, s_q)}(p, q) = |h_p - h_q| |M(s_p) - M(s_q)|.$$  

(3)

The distances between the signal points mapped to the same codeword are excluded since this does not contribute to the BER. Then the minimum distance associated with node $[p,q]$ is given by

$$d_{\text{min}}[p,q] = \min_{[p,q] \in U} d_{(s_p, s_q)}[p, q].$$

(4)

We compute the minimum distances of (4) for all possible three pairs of users and select the user pair with the largest minimum distance as

$$[\hat{p}, \hat{q}] = \arg\max_{[p,q]} d_{\text{min}}[p, q].$$

In the following section, we will describe how to calculate $d_{\text{min}}[p,q]$.

\(^1\)A method to obtain the exact instantaneous BER is recently proposed in [11] for the BPSK modulation.

IV. COMPUTATION OF MINIMUM DISTANCE

In this section, we propose a technique for obtaining the minimum distance of the superposed constellation for given channels. For the case of BPSK, it is easy to show that the minimum distance is simply equal to the magnitude of the smaller channel gain multiplied by two, i.e. $d_{\text{min}}[p,q] = 2 \min(|h_p|, |h_q|)$. Therefore, the channel norm and the minimum distance criteria become equivalent for BPSK. In contrast, for QPSK, there are 16 possible signal points since the QPSK signals from two users are superposed at the relay during the MAC phase. Hence, the search size of $20 \times 15 = 120$ is required to find the minimum distance for a given user pair. For three pairs, we need to search 360 distances in total. In this section, we propose a method to compute the minimum distance without any search.

The QPSK constellation mapper at user nodes is represented as

$$\mathcal{M}_{QPSK}(s_p) = \left\{ \frac{1+j}{2}, \frac{1-j}{2}, \frac{-1+j}{2}, \frac{-1-j}{2} \right\}.$$  

(5)

We consider both $D_{\text{XOR}}$ and $D_{\text{AD}}$ as the constellation mapper at the relay. Fig. 3 illustrates the constellation at the relay for QPSK, where the symbol $(m,n)$ represents the signal for $s_p = m$ and $s_q = n$.

Due to symmetry, there are several signal pairs which generate the same distance. For example, we can see that $d_{(1,1)}[p,q] = d_{(3,3)}[p,q]$ from the figure. Thus, there are only 20 distinct distances. Also, we can exclude $d_{(3,0)}[p,q]$ from the candidate for the minimum distance since $d_{(3,0)}[p,q]$ equals $\sqrt{2}d_{(0,0)}[p,q]$ regardless of the channel condition. Among the 20 distinct distances, there exist 6 distances which establish such relation. As a result, we can reduce the number of candidates for the minimum distance to 14 from 120.

To obtain the minimum distance, we need to determine which pair of the superposed signals has the smallest distance. The pair of the superposed signals with the smallest distance
depends on the channel ratio \( h_q/h_p \triangleq \gamma \exp(j\theta) \) and the denoising mapper at the relay. Fig. 4 shows several regions according to the channel ratio for two different denoising mappers. Each region decides the signal pair which contributes to the minimum distance. The regions in Fig. 4 are obtained through the computer simulations. Table I lists the signal pairs \((s_p, s_q)\) and \((s'_p, s'_q)\) that contribute to the minimum distance and the corresponding signal difference vectors for each region. Since there exist many signal pairs with the same distance, only one signal pair is shown for simplicity. The signal difference vector for the signal pair is defined as

\[
\Delta_{(s_p, s_q) - (s'_p, s'_q)} = \begin{bmatrix} M(s_p) - M(s'_p) \\ M(s_q) - M(s'_q) \end{bmatrix}.
\]

Utilizing the region map and Table I, we can determine \( d_{\min}[p, q] \) directly without any search. For instance, if the channel ratio falls into region 4 in Fig. 4, the corresponding vector \( \Delta_{(s_p, s_q) - (s'_p, s'_q)} \) is \( \sqrt{2}[1 + j]^T \). Inserting this vector into (3), we obtain \( d_{\min}[p, q] = \sqrt{2}|h_p + (1 + j)h_q| \).

Note that the number of regions equals 12 for the case where the relay adopts \( D_{XOR} \) as illustrated in Fig. 4 (a). The signal points for \((0, 0)\) and \((1, 1)\) are mapped to the same symbol and so are the signal points \((0, 1)\) and \((1, 0)\). Thus, regions 13 and 14 which correspond to \( d^{(0, 0)}[p, q] \) and \( d^{(1, 1)}[p, q] \), respectively, are eliminated in \( D_{XOR} \). Meanwhile, for \( D_{AD} \), since different denoising code is employed adaptively depending on the channel ratio \( \gamma \exp(j\theta) \), Fig. 4 (b) shows all 14 regions.

We also note that the regions 1 and 2 in Fig. 4 whose \( d_{\min}[p, q] \) contains the vector \([0\ 1]^T\) and \([1\ 0]^T\), respectively, have a special property. In these regions, \( d_{\min}[p, q] \) is calculated as \( d_{\min}[p, q] = \sqrt{2}\min(|h_p|, |h_q|) \). Thus, the minimum distance criterion results in the same user pair as the channel norm criterion in these regions. Since regions 1 and 2 in Fig. 4 (a) are smaller than those in Fig. 4 (b), the performance gap between the channel norm criterion and the minimum distance criterion would become smaller with \( D_{AD} \) than with \( D_{XOR} \), and this will be confirmed in the simulation section.

V. SIMULATION RESULTS

In this section, we evaluate the end-to-end throughput performance of the proposed selection schemes. We assume reciprocity between the MAC and the BC phases, i.e. \( h_p = g_p \). The average SNR at the relay is defined as \( 1/\sigma^2 \). Given channel conditions, the relay selects a user pair with two

<table>
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<tr>
<th>Region</th>
<th>1</th>
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<th>3</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
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<tbody>
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<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(0, 1)</td>
<td>(0, 1)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(0, 2)</td>
<td>(0, 1)</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>((s'_p, s'_q))</td>
<td>(0, 1)</td>
<td>(1, 0)</td>
<td>(1, 2)</td>
<td>(1, 3)</td>
<td>(2, 1)</td>
<td>(2, 3)</td>
<td>(3, 1)</td>
<td>(3, 2)</td>
<td>(1, 2)</td>
<td>(3, 0)</td>
<td>(2, 1)</td>
<td>(3, 0)</td>
<td>(1, 1)</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>(\Delta_{(s_p, s_q) - (s'_p, s'_q)})</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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TABLE I

POINT PAIRS AND THE CORRESPONDING VECTORS \( \Delta_{(s_p, s_q) - (s'_p, s'_q)} \)

![Fig. 4. Region maps (a) XOR denosing (b) Adaptive denosing](image-url)
criteria. The selected users transmit data packets of 256 symbols simultaneously. Then the relay detects the data and generates the transmitted data with either $D_{XOR}$ or $D_{AD}$. Each user decodes the message from the other user and check if the packets are received without an error.

Fig. 5 shows the end-to-end throughput for the BPSK case. From this figure, it is confirmed that the proposed selection schemes have significant advantages over the round-robin selection system. All two criteria generate the same performance, and achieve a gain of 5dB at 0.8bps/Hz over the system without scheduling.

Fig. 6 illustrates the QPSK case, where the minimum distance depends not only on the channel norm but also on the channel phase difference. In this case, the channel norm criterion and the minimum distance criterion result in different performance. Fig. 6 shows that when the relay employs $D_{XOR}$, the channel norm criterion and the minimum distance criterion exhibit the gains of 2.8dB and 4.7dB at 1.6 bps/Hz, respectively, over the round-robin selection system. In contrast, for the case of $D_{AD}$, a 3.7dB gain at 1.6bps/Hz is observed for both the channel norm criterion and the minimum distance criterion. Since the neighboring signal points are clustered into the same group by $D_{AD}$, the minimum distance is dominantly determined by the channel norm. Therefore, the simpler channel norm criterion is sufficient for the system with $D_{AD}$. Also we can see that a scheduling gain with the minimum distance criterion is bigger in the $D_{XOR}$ system than in the $D_{AD}$ system.

VI. CONCLUSION

In this paper, we have proposed an opportunistic user selection scheme for the three-way relay channel. We have introduced two selection criteria based on the channel norm and the minimum distance. Moreover, we have studied an efficient method for computing the minimum distance between the signal points for the superposed QPSK signals. The proposed schemes are shown to significantly outperform systems without selection. The schemes proposed in this paper can be easily applied to multi-way relay channels. It would be an interesting future research topic to find a method for the minimum distance computation for higher order modulations. Also, it is important to derive the regions analytically for computing the minimum distance.

REFERENCES