Block Diagonalization Approach for Amplify-and-Forward Relay Systems in MIMO Multi-User Channels

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Abstract—In this paper, we consider a relay system in multiple-input multiple-output (MIMO) multi-user channels where a single MIMO relay is engaged in communication between multiple source-destination terminal pairs. We propose two amplify-and-forward (AF) relaying schemes which maximize the sum-rate for the interference channel with multiple source-destinations. First, we introduce an iterative scheme which provides a local optimal solution. The proposed scheme iteratively searches an optimum relay matrix by deriving the gradient of the sum-rate and applying the gradient descent algorithm. Next, in order to reduce the computational complexity of the iterative scheme, we propose a block diagonalization (BD) method which utilizes the minimum mean-square error (MMSE) criterion with singular value decomposition (SVD) based rate maximization. Simulation results show that the proposed iterative scheme achieves a near-optimal sum-rate for the given channel model, and the MMSE based BD with the rate maximization approaches the local optimal sum-rate at high signal-to-noise ratio (SNR) regime.

I. INTRODUCTION

A wireless relay scheme is a promising technology which exhibits advantages of increasing the system capacity and extending the coverage. Recently, with the increasing interests in relay networks, researchers have studied various relay systems. The capacity analysis for single antenna relay systems has been studied in [1], [2], and [3]. By natural requirements for the enhanced capacity, research of multiple-input multiple-output (MIMO) relay systems was introduced in [4] with the consideration of the decode-and-forward (DF) structure. In practical wireless relay networks, however, amplify-and-forward (AF) relay systems achieve low computational complexity compared to DF systems since the relay node in the AF relay systems linearly processes the received signal without decoding the information. In [5], an local optimal AF relaying scheme for single user two-way MIMO AF relay systems was proposed by adopting sum-rate maximization criterion.

A MIMO relay communication between multiple source-destination pairs was investigated in [6] while prior studies for relay networks focused on a single source-destination pair. Since this system model consists of multiple users, the inter-user interference is a significant factor of the sum-rate impairment in this system. In [6], the zero-forcing (ZF) and linear minimum mean-square error (MMSE) relay matrices were proposed to combat the inter-user interference.

In this paper, we study an optimal relay matrix which maximizes the sum-rate of MIMO multi-user relaying networks, and propose two linear processing schemes. Since the formulated maximization problem is not a convex problem in general, it cannot be solved analytically. Therefore, an iterative searching scheme is first introduced. We derive the gradient of the sum-rate and apply a gradient descent algorithm [7] to obtain the optimal relay matrix.

In order to reduce the computational complexity of the proposed iterative scheme, we also propose an non-iterative scheme which exploits the block diagonalization (BD) method [8]. In MIMO multi-user systems, it is well-known that the BD method outperforms the conventional ZF and MMSE method since the BD method can exploit the intra-user channel gain. Hence, we utilize the BD method to eliminate the multi-user interference and then maximize the sum-rate by applying the optimal singular value decomposition (SVD) based rate-maximization technique to each block diagonalized channel. Since the conventional BD method suffers from the noise enhancement problem at low signal-to-noise ratio (SNR), we adopt an enhanced BD method based on the MMSE proposed in [9].

Simulation results show that both the proposed schemes outperform existing schemes in terms of the sum-rate performance. Also, we demonstrate that the proposed MMSE based BD scheme with the rate maximization approaches the iterative gradient descent scheme at high SNR.

This paper is organized as follows: Section II describes the system model for MIMO multi-user relaying networks. In Section III, we propose two linear processing schemes to maximize the sum-rate. Section IV presents the simulation results. Finally, the paper is terminated with conclusions in Section V.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices and lowercase boldface for vectors. For a certain matrix $\mathbf{A}$, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and the conjugate transpose, respectively. $\text{Tr}(\mathbf{A})$ indicates the trace, $\text{vec}(\mathbf{A})$ represents the stacked columns of a matrix $\mathbf{A}$, and $\|\mathbf{A}\|_F$ stands for the Frobenius norm which equals to $\text{Tr}(\mathbf{A}^H\mathbf{A})$. For $m \times m$ matrices $\mathbf{A}, \mathbf{B}$, $\mathbf{A} = \text{diag}\{\mathbf{A}_1, \cdots, \mathbf{A}_n\}$ denotes an $mn \times mn$ block diagonal matrix. $\mathbb{E}(\cdot)$ denotes the expectation operator.
II. SYSTEM MODEL

We consider a MIMO multi-user relaying system where there are $2K+1$ terminal nodes with $K$ active source-destination pairs and single relay node as shown in Fig. 1. In this system, the $k$th source terminal node $S_k$ wants to send information to the $k$th destination terminal node $D_k$, and the relay node $R$ helps the communication between each pair. The terminal nodes and the relay node are equipped with $M$ and $N$ antennas, respectively. We assume that a direct link between the source and destination terminal nodes can be ignored due to a large path loss, and the relay node operated in time-division duplex (TDD) mode. Also, it is assumed that the relay and the destination nodes have full channel state information (CSI) while the source nodes have no CSI.

We define the transmitted data symbol vector $s$, and the channel matrices $H$ and $G$ as

$$s = \begin{bmatrix} s_1^T \ s_2^T \ \cdots \ s_K^T \end{bmatrix}^T,$$

$$H = \begin{bmatrix} H_1 \ H_2 \ \cdots \ H_K \end{bmatrix},$$

$$G = \begin{bmatrix} G_1^T \ G_2^T \ \cdots \ G_K^T \end{bmatrix}^T,$$

where $s_j$ is a $M$ dimensional transmitted signal vector from $S_j$, $H_j$ denotes the $N \times M$ complex channel matrix from $S_j$ to $R$, and $G_j$ represents the $M \times N$ complex channel matrix from $R$ to $D_j$. Here, $s_j$ satisfies $E(s_j s_j^H) = \frac{P}{KM} I_M$ where $P$ indicates the total transmission power of $K$ source terminal nodes.

In the first time slot, $K$ source terminal nodes $\{S_k\}$ for $k = 1, 2, \cdots, K$, transmit their signals $\{s_k\}$ to the relay node $R$ simultaneously. Then, the $N$ dimensional received signal vector $r$ at the relay node is given by

$$r = \sum_{k=1}^{K} H_k s_k + n$$

(1)

where $n$ denotes the additive complex Gaussian noise vector with zero mean and $E(n n^H) = \sigma_n^2 I_N$.

In the second time slot, assuming that a linear processing is employed at the relay node, the received signal $r$ is multiplied by the $N \times N$ relay matrix $F$. Then, the signal vector $x$ transmitted from the relay node is written by

$$x = \eta F r = \sum_{k=1}^{K} \eta F H_k s_k + \eta F n$$

where $\eta$ is the power normalizing coefficient. In order to satisfy the relay power constraint $E(\|x\|^2_F) = P_R$, $\eta$ is given as

$$\eta = \sqrt{\frac{P_R K M}{\text{Tr}(F (P_T H H^H + KM \sigma_n^2 I_N) F^H)}}.$$

Finally, the received signal vector at the $k$th destination node $D_k$ can be expressed as

$$y_k = \eta G_k F H_k s_k + \sum_{j=1,j\neq k}^{K} \eta G_k F H_j s_j + \eta G_k F n + z_k$$

(2)

where $z_k$ is the additive complex Gaussian noise vector with zero mean and $E(z_k z_k^H) = \sigma_n^2 I_M$. Here, the first term of the right hand side of (2) includes the signal vector originally transmitted from $S_k$ to $D_k$ while the second term denotes the interference from the other source nodes.

III. SUM-RATE MAXIMIZATION

In this section, we investigate the sum-rate maximization for MIMO multi-user relay systems. The sum-rate of the MIMO multi-user relaying system model (2), denoted by $R_{\text{sum}}$, can be expressed as

$$R_{\text{sum}} = \sum_{k=1}^{K} R_k = \sum_{k=1}^{K} \frac{1}{2} I(y_k; s_k)$$

where $R_k$ represents the rate of the $k$th source destination pair, and $I(x; y)$ is the mutual information between $x$ and $y$. Here, the factor of $\frac{1}{2}$ comes from the half-duplex operation of the relay. The information rate of each user, $R_k$, is given by

$$R_k = \frac{1}{2} \log_2 \left( \rho_T \sum_{j=1}^{K} G_k F H_j H_j^H F^H G_k^H + \Psi_k \right)$$

where $\rho_T = \frac{P_T}{K M \sigma_n^2}$ and $\Psi_k = G_k FF^H G_k^H + \frac{\sigma_n^2}{\eta^2 \sigma_n^2} I_M$.

To obtain the optimum relay matrix to maximize the sum-rate, our problem can be formulated as

$$F_{\text{opt}} = \arg \max_F R_{\text{sum}}.$$  (3)

Since $R_{\text{sum}}$ is not a convex or concave function with respect to $F$ in general, the preceding optimization problem is difficult to solve analytically. Hence, we first iteratively identify the local optimal solution by using the gradient descent algorithm, and also propose an alternative scheme with the reduced computational complexity which utilizes the MMSE based BD method.

A. Iterative scheme using a gradient descent algorithm

In this subsection, we introduce a gradient descent algorithm which optimizes the relay matrix. The gradient descent algorithm exploits the fact that a real-valued function $f(X)$ increases the fastest from an initial point $X_0$ if $X_0$ moves in the direction of the gradient of $f(X)$. Therefore, we derive the gradient $R_{\text{sum}}$, and then apply an iterative algorithm to obtain the optimal relay matrix.

Since the sum-rate $R_{\text{sum}}$ is a real-valued function, the gradient of the sum-rate is given as

$$\nabla R_{\text{sum}} = 2 \partial R_{\text{sum}} / \partial F^*.$$
Therefore, we first compute the differential of \( R_{\text{sum}} \) to derive \( \partial R_{\text{sum}} / \partial \mathbf{F} \). For simplicity, let us define \( \Pi_k \) and \( \Omega_k \) as

\[
\Pi_k = \left( \rho_T \sum_{j=1}^{K} G_k F H_j H_j^H F^H G_k^H + \Psi_k \right)^{-1},
\]
\[
\Omega_k = \left( \rho_T \sum_{j=1, j \neq k}^{K} G_k F H_j H_j^H F^H G_k^H + \Psi_k \right)^{-1}.
\]

Then, exploiting \( \text{Tr}(X^T Y) = \text{Tr}(XY^T) \), \( d\{\text{Tr}(X)\} = \text{Tr}(d(X)) \), \( d\{\ln |X|\} = \text{Tr}(\ln(X^{-1}) d(X)) \), and \( \text{Tr}(X^T Y) = \text{vec}(X)^T \text{vec}(Y) \) \[11\], the gradient of the sum-rate \( \nabla R_{\text{sum}} \) is derived as (4) at the top of this page. In (4), \( \rho_R \) is defined as \( P_R / (\sigma_n^2) \).

With the derived gradient expression, we can solve the optimization problem (3) as follows:

**Given:**
1. Set \( \mathbf{F} \) as a randomly chosen \( N \times N \) matrix
2. Compute the gradient \( \nabla R_{\text{sum}}(\mathbf{F}) \)
3. Update \( \mathbf{F} \leftarrow \mathbf{F} + \delta \nabla R_{\text{sum}}(\mathbf{F}) \)
4. If \( \|\nabla R_{\text{sum}}(\mathbf{F})\|_F^2 < \epsilon \), stop the loop
   Otherwise go back to step 2

In this algorithm, \( \epsilon \) is the tolerance factor for terminating the iteration. To find a proper step size \( \delta \), we employ Armijo’s Rule \[12\] which guarantees a non-decreasing sum-rate value in an iteration loop.

In order to increase the probability that the resulted local optimum sum-rate is identical to the global optimum, we can repeat the proposed algorithm for randomly chosen multiple initial matrices and select one which achieves the largest sum-rate. Therefore, the proposed scheme can achieve a near-optimum sum-rate capacity for the MIMO multi-user relaying network.

**B. Noniterative scheme using MMSE based block diagonalization**

In this subsection, we propose an alternative scheme to obtain the relay matrix by adopting the MMSE based BD method and SVD based rate maximization. The proposed noniterative scheme obtains the relay matrix through a two stage process. In the first stage, the BD method is adopted to eliminate the multi-user interference. Then, since the subblock of the resulting block diagonalized channel matrices becomes the effective channel for each source-destination pair, we can consider the \( K \)-user relay system as \( K \) parallel relay systems which have an independent relay channel, respectively. In the second stage, we identify the weighting matrix which maximizes each information rate in \( K \) single user relay channels. Since it was shown in \[13\] that the SVD based linear processing achieves the capacity, we similarly derive the optimal rate-maximizing matrix for corresponding block diagonalized channels.

For a multi-user broadcasting channel, the key idea of a BD method is to eliminate the multi-user interference by placing all data streams of the other users at nullspace of the intended user’s channel. In \[9\], based on a new interpretation that the conventional BD scheme can be considered as an extension of zero-forcing channel inversion (ZF-CI), an alternative BD method based on the MMSE was introduced. Since this MMSE based BD method outperforms the ZF based conventional BD method by balancing the interference and noise for each user, we adopt the MMSE based BD method for calculating the BD matrices.

We define the relay matrix \( \mathbf{F} \) as

\[
\mathbf{F} = \mathbf{P}_G \mathbf{F} \mathbf{P}_H
\]

where \( \mathbf{P}_G \) and \( \mathbf{P}_H \) are the BD matrices for the relaydestination channel and the source-relay channel, respectively, and \( \mathbf{F} \) denotes the rate-maximizing matrix. Here, we use the terminologies of BD matrices for the matrices which make the block diagonalized channels and a rate-maximizing matrix for the weighting matrix which maximizes the sum-rate in \( K \) single user relay channels. As our relay-destination channel can be regarded as a multi-user broadcasting channel, we first apply the MMSE based BD method into the channel \( \mathbf{G} \). In order to compute the BD matrix \( \mathbf{P}_G \), we first define \( \mathbf{G} \) as

\[
\mathbf{G} = \mathbf{G}^H \left( \mathbf{G} \mathbf{G}^H + \frac{1}{\rho_R} \mathbf{I}_M \right)^{-1} = [\mathbf{G}_1 \mathbf{G}_2 \cdots \mathbf{G}_K]
\]

where \( \mathbf{G}_j \) is the \( N \times M \) matrix which represents the \( j \)th subblock of the MMSE channel inversion matrix \( \mathbf{G} \).

Then, for identifying the orthonormal basis of column space of \( \mathbf{G}_j \), we consider the QR decomposition of \( \mathbf{G}_j \) as

\[
\mathbf{G}_j = \mathbf{Q}_j \mathbf{R}_j
\]

where \( \mathbf{Q}_j \) is an \( N \times M \) matrix whose columns form an orthonormal basis for \( \mathbf{G}_j \) and \( \mathbf{R}_j \) represents an \( M \times M \) upper triangular matrix. Finally, \( \mathbf{P}_G \) can be expressed as

\[
\mathbf{P}_G = [\mathbf{Q}_{G_1}, \mathbf{Q}_{G_2}, \cdots, \mathbf{Q}_{G_K}].
\]

By exploiting the symmetry, we can also compute the BD matrix \( \mathbf{P}_H \) for the source-relay channel \( \mathbf{H} \). Defining \( \mathbf{H} \) as

\[
\mathbf{H} = \left( \mathbf{H}^H \mathbf{H} + \frac{1}{\rho_T} \mathbf{I}_M \right)^{-1} \mathbf{H}^H = [\mathbf{H}_1^H \mathbf{H}_2^H \cdots \mathbf{H}_K^H]^H,
\]

maximizes each information rate in \( K \) single user relay channels. Since it was shown in \[13\] that the SVD based linear processing achieves the capacity, we similarly derive the optimal rate-maximizing matrix for corresponding block diagonalized channels.
P_H is calculated as

\[ P_H = \begin{bmatrix} Q_{H_1}^H & Q_{H_2}^H & \cdots & Q_{H_K}^H \end{bmatrix}^H \]  

(10)

where \( Q_{H_j}^H \) is an \( N \times M \) matrix whose columns form an orthonormal basis for \( H_j^H \) which can be obtained from the QR decomposition \( H_j^H = Q_{H_j}^H R_{H_j}. \) As we can see in (6) and (9), the MMSE based BD solution approaches the ZF based BD scheme at high \( \rho_R \) and \( \rho_T. \)

Multiplied by \( P_H, \) the source-relay channel \( H \) is transformed to

\[ P_H H = \begin{bmatrix} Q_{H_1}^H H_1 & Q_{H_2}^H H_2 & \cdots & Q_{H_K}^H H_K \\ Q_{H_1}^H H_1 & Q_{H_2}^H H_2 & \cdots & Q_{H_K}^H H_K \\ \vdots & \vdots & \ddots & \vdots \\ Q_{H_1}^H H_1 & Q_{H_2}^H H_2 & \cdots & Q_{H_K}^H H_K \end{bmatrix} \]  

(11)

where \( Q_{H_j}^H H_k \) for all \( j \neq k \) and \( 1 \leq j, k \leq K \) are \( M \times M \) matrices which represents residual interference. Also, \( G P_{G_k} \) can be expressed similarly. Note that the MMSE based BD generates the residual interference while the ZF based BD scheme provides complete elimination of residual terms. Due to the residual interference, we cannot derive the closed form solution of the optimal rate-maximizing matrix. Therefore, we identify the rate-maximizing matrix with an assumption of high SNR.

Now, applying (5) to (2) with the high SNR assumption, the received signal vector at the \( k \)th destination can be expressed as

\[ y_k = G_k \hat{F}_k H_k s_k + G_k \hat{F}_k Q_{H_k}^H n + z_k \]  

(12)

where \( G_k = G_k Q_{G_k} \) and \( H_k = Q_{H_k}^H H_k \) represent the effective channels for the \( k \)th relay-destination link and the \( k \)th source-relay link, respectively. Here, to exploit the blockwise property of the effective channels, the rate-maximizing matrix \( \hat{F} \) is given as \( \text{diag}(\hat{F}_1, \hat{F}_2, \cdots, \hat{F}_K) \) where \( \hat{F}_k \) indicates the sub-block matrix for maximizing the rate of the \( k \)th source-destination pair.

In order to obtain the optimal rate-maximizing matrix for each single user relay system given in (12), we first compute the SVD of \( \hat{H}_k \) and \( \hat{G}_k \) as

\[ \hat{H}_k = U_{H_k} \Sigma_{H_k} V_{H_k}^H, \hat{G}_k = U_{G_k} \Sigma_{G_k} V_{G_k}^H \]  

(13)

where \( \Sigma_{H_k} = \text{diag} \{ \sqrt{\alpha_{k_1}}, \sqrt{\alpha_{k_2}}, \cdots, \sqrt{\alpha_{k_M}} \} \) and \( \Sigma_{G_k} = \text{diag} \{ \sqrt{\beta_{k_1}}, \sqrt{\beta_{k_2}}, \cdots, \sqrt{\beta_{k_M}} \} \) are singular value matrices. Then, the optimum \( \hat{F}_k \) is represented as [13]

\[ \hat{F}_k = V_{G_k} \Phi_k U_{H_k}^H \]  

(14)

where \( \Phi_k = \text{diag} \{ \sqrt{\phi_{k_1}}, \sqrt{\phi_{k_2}}, \cdots, \sqrt{\phi_{k_M}} \} \) denotes the diagonal power-loading matrix which maximizes the information rate of the \( k \)th user.

Using the equations (13) and (14), the received signal vector for the \( k \)th destination (12) can be rewritten as

\[ y_k = U_{G_k} \Sigma_{G_k} \Phi_k U_{H_k}^H s_k + U_{G_k} \Sigma_{G_k} \Phi_k U_{H_k}^H Q_{H_k}^H n + z_k. \]

Then, the instantaneous rate between the \( k \)th source and the corresponding destination is given as

\[ R_k = \frac{1}{2} \log_2 \left| \frac{P_R k M U_{G_k} \Sigma_{G_k} \Phi_k \Sigma_{H_k} U_{H_k}^H + \Xi_k}{|H_k|} \right| \]

(15)

where \( \Xi_k = \sigma_n^2 U_{G_k} \Sigma_{G_k} \Phi_k^2 U_{H_k}^H + \sigma_n^2 I_M. \)

Assuming that the relay distributes equal power to each source-destination link, the relay transmit power constraint for each pair can be computed by

\[ E \left( \| F_k H_k s_k + F_k n \|^2 \right) = \text{Tr} \left( G_k V_k \left( \frac{P_T}{K M} \Phi_k^2 H_k^H + \sigma_n^2 \Phi_k^2 \right) V_k^H G_k^H \right) \]

\[ = \sum_{m=1}^M \left( \frac{P_T}{K M} \alpha_{k_m} + \sigma_n^2 \right) \phi_{k_m} \leq \frac{P_R}{K}. \]  

(16)

The problem of maximizing \( R_k \) under the relay transmit power constraint can be written as

\[ \text{arg} \max_{\phi_{k_m}} \sum_{m=1}^M \log_2 \left( 1 + \rho_T \alpha_{k_m} - \frac{\rho_T \alpha_{k_m}}{1 + \frac{\sigma_n^2}{\sigma_R^2} \beta_{k_m} \phi_{k_m}} \right) \]

subject to \( \sum_{m=1}^M \left( (1 + \rho_T \alpha_{k_m}) \phi_{k_m} \right) \leq \frac{P_R}{K \sigma_n^2}. \)

Since the instantaneous rate \( R_k \) (15) is concave with respect to \( \phi_{k_m}, \) the optimization of \( \phi_{k_m} \) is easily solved by using a Lagrange multiplier. The cost function to maximize \( R_k \) can be expressed as

\[ C = \frac{1}{2} \sum_{k=1}^M \log_2 \left( 1 + \rho_T \alpha_{k_m} - \frac{\rho_T \alpha_{k_m}}{1 + \frac{\sigma_n^2}{\sigma_R^2} \beta_{k_m} \phi_{k_m}} \right) \]

\[ + \lambda \left( \sum_{k=1}^M \left( (1 + \rho_T \alpha_{k_m}) \phi_{k_m} \right) - \frac{P_R}{K \sigma_n^2} \right). \]

Now, we take a derivative of the cost function \( C \) with respect to \( \phi_{k_m} \) and set it to zero. As a result, \( \phi_{k_m} \) is derived as

\[ \left[ \frac{\sigma_n^2}{\sigma_R^2} - 2 \rho_T \alpha_{k_m} + 4 \log_2 \left( \frac{\rho_T \alpha_{k_m} \beta_{k_m} \sigma_n^2}{\sigma_R^2} \right) \right] + \left( 1 + \rho_T \alpha_{k_m} \right) \beta_{k_m} \phi_{k_m} \]

(17)

where \( [A]^+ = \max(0,A) \) and \( \lambda \) should be computed to meet the relay power constraint (16).

In summary, from (8), (10), (14), and (17), the proposed MMSE based BD relay matrix is given as

\[ F = P_G \text{diag} \{ \hat{F}_1, \hat{F}_2, \cdots, \hat{F}_K \} P_H. \]

(18)
with 5 random initial matrices and set the tolerance factor variables with zero mean and unit variance. Also, we start and identically distributed (i.i.d.) complex Gaussian random vectors produced in [6]. For all simulations, we use spatially uncorrelated MIMO multi-user relaying channel with $M=2$, $N=4$, and $K=2$. From this plot, we can observe that our proposed schemes outperform the conventional ZF and MMSE schemes at all SNR region.

IV. SIMULATION RESULTS

In this section, we present the sum-rate performance of the proposed schemes comparing with the existing schemes introduced in [6]. For all simulations, we use spatially uncorrelated MIMO channels with the elements generated by independently and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Also, we start with 5 random initial matrices and set the tolerance factor $\epsilon$ to $10^{-5}$ for the gradient descent algorithm. In the plots, ZF and MMSE represent systems with the relay matrix using the conventional ZF and MMSE algorithms [6], and MMSE-BD and GD denote the systems with the proposed MMSE based BD and gradient descent schemes, respectively.

Fig. 2 shows the cumulative distribution function (CDF) of the ergodic sum-rate for various schemes proposed for the MIMO multi-user relaying channel with $M=2$, $N=4$, and $K=2$. From this plot, we can observe that our proposed schemes outperform the conventional ZF and MMSE schemes at all SNR region.

With the same system configuration, we plot the ergodic sum-rate as a function of $\rho_T$ with fixed $\rho_R$ in Fig. 3 where the SNRs are defined as $\rho_T = \frac{P_T}{K \cdot M^2 \cdot \sigma^2}$ and $\rho_R = \frac{P_R}{N \cdot \sigma^2}$, respectively. It is clear that the average sum-rates obtained by the proposed gradient descent method and MMSE-BD method are higher than that of the conventional MMSE scheme. Also, we can see that the proposed MMSE based BD method approaches the gradient descent algorithm as the SNR increases. Although we do not present here, the sum-rate according to $\rho_T$ with fixed $\rho_R$ shows similar curve shapes. Furthermore, a sum-rate gain of the proposed schemes over the conventional ZF and MMSE schemes can become larger as the dimension of $M$, $N$, and $K$ increases.

V. CONCLUSION

In this paper, we have developed the relay matrix which maximizes the sum-rate in multi-user MIMO AF relaying networks. Since the formulated optimization problem is not a convex or concave in general, we first propose an iterative scheme by utilizing the gradient descent algorithm which provides a local optimal solution. In order to reduce the computational complexity, we also propose a noniterative method by using the MMSE based BD method combined with SVD and the power-loading technique. Simulation results show that the proposed noniterative scheme approaches the sum-rate performance of the iterative scheme at high SNR regime, and both the proposed schemes outperform the conventional schemes in MIMO multi-user AF relaying systems at all SNR region.

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