Distributed Precoding Techniques for Weighted Sum Rate Maximization in MIMO Interfering Broadcast Channels

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Abstract—In this paper, we propose a linear precoding technique for weighted sum rate (WSR) maximization in multiple-input multiple-output interfering broadcast channels. In multi-cell environments, the WSR can be jointly maximized through centralized processing which causes a large amount of channel state information (CSI) exchange. In order to reduce the overhead associated with CSI, we focus on a distributed precoding scheme utilizing local CSI at each base station (BS). First, applying a high signal-to-interference-plus-noise ratio assumption, we decouple the WSR maximization problem into distributed problems. Then we solve this distributed WSR maximization problem for each BS by using a zero-gradient based algorithm which converges to a local maximum point. Unlike conventional distributed schemes which require additional information, our proposed scheme at each base station utilizes only the local CSI to compute its precoding matrices. Through the Monte-Carlo simulation, we show that our proposed algorithm exhibits the performance almost identical to the centralized scheme requiring the global CSI.

I. INTRODUCTION

In cellular systems, as needs for the high rate data service are increased, techniques to enhance spectral efficiency become important. In order to satisfy these needs, multiple-input multiple-output (MIMO) transmission methods have attracted a lot of attention in the last decade due to their great potential to achieve high throughput in wireless communication systems [1]–[3]. In MIMO broadcast schemes, a linear increase of the system throughput can be achieved with respect to the number of transmit antennas by transmitting different data streams to multiple users simultaneously. Although such MIMO broadcast schemes enhance the throughput of users, the performance of cell-edge users may be poor due to inter-cell interference caused by adjacent base stations (BSs). In order to enhance the sum rate for cell-edge users, we consider the MIMO interfering broadcast channel (IFBC). A promising candidate to mitigate the inter-cell interference is the network MIMO [4]–[6], which coordinates the data transmission utilizing all users’ message and channel state information (CSI) among BSs according to the BS cooperation level. However, in this network MIMO system which requires centralized processing to jointly optimize an objective function, the overhead for information exchange may be too large to be implemented in practical cellular systems as the number of BSs and users increases. Motivated by this reason, we focus on distributed approaches which utilize only local CSI to mitigate the inter-cell interference.

In [7], the authors proposed an algorithm to maximize the weighted sum rate (WSR) based on a recent result [8] which showed a relation between the minimum weighted sum mean squared error and the maximum WSR for MIMO interference channels. However, the proposed algorithm in [7] utilized global CSI at all BSs, which led to a large amount of CSI exchange. On the other hand, the authors in [9] proposed the scheme to reduce the exchange overhead by exploiting the decreasing rate of sum rate according to the increase of interference at the receiver. However, this scheme requires additional information such as the amount of change in the interference measured by its receiver at each iteration.

In this paper, we propose a fully distributed precoding technique to maximize the WSR for the MIMO IFBC systems. This is different from our previous work [10] where we considered K-user multiple-input single-output interfering channels, i.e. each BS only serves a single user with single antenna. Under the assumption of single-user detection (SUD) in which receivers treat the interference signals as noise [11]–[14], we restrict our scheme to a linear precoding method.

In multi-cell environments, the WSR maximization is quite complicated due to the inter-cell interference which affects all related BSs. Hence, applying high signal-to-interference-plus-noise ratio (SINR) approximation as in [13] and [15], we decouple the WSR maximization problem into distributed problems which involve only local CSI. Then we find the solution for the distributed WSR maximization problem at each BS by adopting zero-gradient (ZG) conditions. Simulation results show that the WSR performance of our proposed scheme with significantly reduced overhead is almost identical to that of the centralized scheme utilizing global CSI.

Throughout this paper, boldface uppercase and lowercase letters indicate matrices and vectors, respectively. The conjugate and the conjugate transpose are denoted by $X^*$ and $X^H$, respectively. $\text{tr}(X)$ and $|X|$ stand for the trace and determinant operation for a matrix $X$, respectively.

II. SYSTEM MODEL

In this section, we provide a system model for the MIMO IFBC. Let $M$, $K$, $N_r$, and $N_b$ be the number of BSs, users per BS, transmit antennas and receive antennas, respectively. Each BS serves $K$ users with $N_r$ receive antennas as shown in Fig. 1. The $k$-th user served by BS $m$ is denoted as user $(k, m)$. We
will refer to this configuration as the \([M, K, N_t, N_r]\) MIMO IFBC system. Although we assume that all BSs and users have the same number of users and antennas, respectively, for simplicity, they can be easily generalized to arbitrary numbers.

At the \(m\)-th BS, the transmitted signal is expressed as \(\mathbf{x}^{[m]} = \sum_{k=1}^{K} \mathbf{V}_{k}^{[m]} \mathbf{s}_{k}^{[m]}\), where \(\mathbf{s}_{k}^{[m]} \sim \mathcal{CN}(0, I)\) and \(\mathbf{V}_{k}^{[m]} \in \mathbb{C}^{N_t \times N_r}\) indicate the complex symbol vector and the precoding matrix for user \((k, m)\), respectively. In order to satisfy the per-BS power constraint as in [7], [8], we assume that

\[
\sum_{k=1}^{K} \text{tr} \left( \mathbf{V}_{k}^{[m]} \mathbf{V}_{k}^{[m]^H} \right) = P^{[m]},
\]

where \(P^{[m]}\) equals the total transmit power at the \(m\)-th BS.

Then, the received signal at user \((k, m)\) can be written as

\[
\mathbf{y}_{k}^{[m]} = \mathbf{H}_{k}^{[mm]} \mathbf{V}_{k}^{[m]} \mathbf{s}_{k}^{[m]} + \sum_{i=1, i \neq k}^{K} \mathbf{V}_{i}^{[m]} \mathbf{s}_{i}^{[m]} + \sum_{j=1, j \neq m}^{M} \mathbf{H}_{k}^{[mj]} \mathbf{V}_{i}^{[j]} \mathbf{v}_{i}^{[j]} + \mathbf{n}_{k}^{[m]},
\]

where \(\mathbf{H}_{k}^{[mj]} \in \mathbb{C}^{N_r \times N_t}\) represents the channel matrix from the \(j\)-th BS to user \((k, m)\) and \(\mathbf{n}_{k}^{[m]} \sim \mathcal{CN}(0, \sigma_{n}^2 I)\) stands for the additive white Gaussian noise vector at user \((k, m)\). In (2), the first term of the right hand side (RHS) is the desired signal, while the second and the third term of the RHS indicate intra-cell interference at the \(m\)-th BS and inter-cell interference, respectively.

At the receiver side, approaches to optimize the WSR may involve joint detection of the signals for both other intra-cell users and inter-cell users. However we only consider a linear receiver by restricting both intra-cell interference and inter-cell interference as noise. In other words, SUD is assumed, the individual rate of user \((k, m)\) is given by

\[
R_{k}^{[m]}(\mathbf{y}_{i}^{[m]}) = \log_2 \left| \mathbf{I} + \mathbf{V}_{k}^{[m]} \mathbf{H}_{k}^{[mm]} \mathbf{V}_{i}^{[m]} \mathbf{H}_{k}^{[mm] \mathbf{H}} \mathbf{R}_{k}^{[m]} \mathbf{H}_{k}^{\mathbf{H}} \right|^{-1},
\]

where \(\mathbf{R}_{k}^{[m]}\) represents the effective noise covariance matrix defined as

\[
\mathbf{R}_{k}^{[m]} = \sigma_{n}^2 \mathbf{I} + \sum_{i=1, i \neq k}^{K} \mathbf{H}_{k}^{[mm]} \mathbf{V}_{i}^{[m]} \mathbf{V}_{i}^{[m]^H} \mathbf{H}_{k}^{[mm] \mathbf{H}}.
\]

In this work, our goal is to find the precoding matrices which maximize the WSR

\[
\sum_{m=1}^{M} \mathbf{R}_{k}^{[m]}.
\]

WSR \(\mathbf{R}_{k}^{[m]}\) is given by

\[
\mathbf{R}_{k}^{[m]} = \sum_{k=1}^{K} \mathbf{w}_{k}^{[m]} \mathbf{H}_{k}^{[mm]} \mathbf{V}_{k}^{[m]^H} \mathbf{H}_{k}^{[mm] \mathbf{H}}.
\]

Here \(\mathbf{w}_{k}^{[m]}\) indicates the weight of user \((k, m)\) which is a predetermined value depending on the required quality for applications.

Then, the problem can be mathematically formulated as

\[
\begin{align*}
\max_{\mathbf{V}_{i}^{[j]}} & \quad \sum_{m=1}^{M} \mathbf{R}_{k}^{[m]} \\
\text{s.t.} & \quad \sum_{k=1}^{K} \text{tr} \left( \mathbf{V}_{k}^{[m]} \mathbf{V}_{k}^{[m]^H} \right) \leq P^{[m]} \quad \forall m. \tag{3}
\end{align*}
\]

This problem is non-convex and involves global CSI and precoding matrices for all related BSs. In the following section, after decoupling the WSR problem into distributed problems, we will propose a WSR maximization algorithm utilizing only local CSI. From a viewpoint of BS \(m\), local CSI means the channel matrices from BS \(m\) to all users, i.e., \(\mathbf{H}_{k}^{[m]}\) for all \(k = 1, \ldots, K\) and all \(j = 1, \cdots, M\).

III. PROPOSED DISTRIBUTED BEAMFORMING

In this section, a precoding scheme which can operate in a fully distributed manner is proposed for the MIMO IFBC. We assume that the number of transmit antennas is equal or greater than the sum of all receiver antennas, i.e. \(N_t \geq MN_r\). Under this assumption, because each BS has sufficient spatial dimensions to nullify the interference affecting both other intra-cell users and inter-cell users, the zero-forcing (ZF) precoding strategy is always feasible. Note that while our proposed algorithm works for any antenna configuration, this assumption \(N_t \geq MN_r\) is needed only for a new cost function derivation which will be defined as the effective WSR.

In what follows, the objective function in (3) with the constraint (1) will be decoupled into \(M\) WSR maximization problems which involve only local CSI, and then a distributed WSR maximization algorithm will be proposed.

A first goal of our paper is to derive a metric function for the WSR maximization where only local CSI and the information of precoding matrices on one cell are required. To this end, for the design of the precoder at BS \(m\), all other BSs are assumed to employ the ZF precoding strategy to remove the inter-cell interference at user \((k, m)\). Denoting the effective noise covariance at user \((k, m)\) \(\tilde{\mathbf{R}}_{k}^{[m]}\) as

\[
\tilde{\mathbf{R}}_{k}^{[m]} = \sigma_{n}^2 \mathbf{I} + \sum_{i=1, i \neq k}^{K} \mathbf{H}_{k}^{[mm]} \mathbf{V}_{i}^{[m]} \mathbf{V}_{i}^{[m]^H} \mathbf{H}_{k}^{[mm] \mathbf{H}},
\]
the WSR of BS $m$ is given by
\[ R[m]_\Sigma = \sum_{k=1}^K u[k][m] \log_2 |I + V[k][m] H[m][m] H^{-1}[m][m] V[k][m]|. \]

which involves only intra-cell interference.

Based on the ZF assumption at all other BSs ($\neq m$), the WSR for the $p$-th BS can be obtained as
\[ R[p]_\Sigma = \sum_{q=1}^K u[q][p] \log_2 |I + V[q] H[p][p] H^{-1}[p][p] V[q]|, \]

where $\hat{R}[p]$ is the effective noise covariance at user $(q,p)$ defined as
\[ \hat{R}[p] = \sigma[p]^2 I + \sum_{i=1}^K H[q][p][i][p][i] H[q][p][i][p][i]. \]

Here, because of the ZF assumption at all the other BSs, the effective noise covariance of the $p$-th BS only involves precoding matrices at BS $m$ and the channel between the $m$-th BS and user $(q,p)$.

In order to decouple CSI related to BS $m$ from $R[p]_\Sigma$, we adopt a high SINR assumption to get
\[
\begin{align*}
R[p]_\Sigma &\approx \sum_{q=1}^K u[q][p] \log_2 |V[q][p][p] H[q][p][p] H^{-1}[p][p] V[q]| \\
&= \sum_{q=1}^K u[q][p] \log_2 |H[q][p][p] V[q][p][p] H[q][p][p]| - \sum_{q=1}^K u[q][p] \log_2 |\hat{R}[p]|, \quad p \neq m. (4)
\end{align*}
\]

Observing (4), the second term of the RHS accounts for a rate loss at the $p$-th BS caused by the precoding matrices of the $m$-th BS. Aggregating the rate losses of all BSs except BS $m$, the total rate loss due to the $m$-th BS precoding matrices $V[1][m], \ldots, V[K][m]$ is given by

\[ L[m](V[m]) = \sum_{p=1}^M \sum_{q=1}^K u[q][p] \log_2 |\hat{R}[p]|. \]

We are now ready to define the effective WSR of the $m$-th BS which involves only local CSI and the precoding matrices for the $m$-th BS as
\[ f[m](V[m]) = R[m]_\Sigma(V[m]) - L[m](V[m]), \]

where the first and second terms of the RHS represent the constructive and destructive effects to the WSR in (3) caused by the precoding matrices of BS $m$. Combining the effective WSR of the $m$-th BS (5) and the equality constraint (1), the distributed WSR maximization problem for the $m$-th BS is derived as
\[
\begin{align*}
\max_{V[m]} f[m](V[m]) \\
\text{s.t.} \quad \sum_{k=1}^K \text{tr}(V[k][m] H[k][m][m] H[k][m][m] V[k][m]) = P[m]. \quad (6)
\end{align*}
\]

Although the problem (6) involves only the local CSI compared with the problem (3), it is still non-convex. Then, we propose an algorithm to solve (6) using the ZG method introduced in [16] which utilizes the fact that the precoder at a stationary point always satisfies the ZG condition. By expressing $V[k][m] = \Sigma[m] / P[m] / \sum_{k=1}^K V[k] V[k][m][m] H[k][m][m] H[k][m][m] V[k][m] - 1$, we obtain an unconstrained problem with respect to $(V[1][m], \ldots, V[K][m])$ as
\[
\max_{V[m]} f[m](V[m]). \quad (7)
\]

Letting $N[m]$ be $N[m] = \sum_{k=1}^K \text{tr}(\Sigma[m] - \Sigma[m][m])$, we can define $D[p][m]$ and $C[p][m]$ as
\[
\begin{align*}
D[p][m] &= \sigma[p]^2 N[m] I + P[m] \sum_{k=1}^K H[k][p][m] V[k][m][m] H[k][p][m], \\
C[p][m] &= \sigma[p]^2 N[m] I + P[m] \sum_{k=1, k \neq m}^K H[k][p][m] V[k][m][m] H[k][p][m],
\end{align*}
\]

respectively. Then, the cost function in (7) can be written as
\[
\begin{align*}
f[m](V[m]) &= \sum_{q=1}^K u[q][m] \log_2 |D[q][m]| - \sum_{q=1}^K u[q][m] \log_2 |C[q][m]| \\
&= \sum_{q=1}^K u[q][m] \log_2 |D[q][m]| - \sum_{p=1, p \neq m}^K \sum_{q=1}^M u[q][m] \log_2 |D[p][m]|.
\end{align*}
\]

Since a local optimal occurs at $\nabla V_k[m] f[m](V_k[m]) = 0$, we need to compute the gradient of $f[m](V_k[m])$ with respect to $V_k[m]$ as
\[
\nabla V_k[m] f[m] = \frac{2}{\ln 2} \left( \sum_{q=1}^K u[q][m] \sigma^2 \text{tr} \left( (C[q][m])^{-1} \right) I + P[m] \Sigma[q][m] \\
- u[k][m] \sigma^2 \text{tr} \left( (C[q][m])^{-1} \right) I + P[m] \Sigma[q][m] \\
- \sum_{p=1, p \neq m}^K \sum_{q=1}^M u[q][m] \sigma^2 \text{tr} \left( (D[p][m])^{-1} \right) I + P[m] \Sigma[p][m] \\
+ (N[m] - 1) \sum_{p=1, p \neq m}^K \sum_{q=1}^M u[q][p] \right) V_k[m].
\]

where we have used the fact that $\nabla V_k[m] f[m] = 2 \frac{\partial f[m]}{\partial V_k[m]}. \quad (8)$.
respectively, we realize that \( \nabla \mathbf{V}_k^{[m]} f^{[m]}(\mathbf{V}_k^{[m]}) = 0 \) is satisfied when

\[
\begin{align*}
(\alpha_k^{[m]} \mathbf{I} + \mathbf{H}_{km}^{[m]} \mathbf{H}_{km}^{[m]}^{-1} \mathbf{H}_{km}^{[m]}) \mathbf{V}_k^{[m]} &= \mathbf{O}_k^{[m]} \\
&= \left( \beta_k^{[m]} \mathbf{I} + \sum_{q=1}^{K} \frac{u_q^{[m]}}{w_k^{[m]}} (\mathbf{\Omega}_q^{[m]} - \mathbf{\Psi}_q^{[m]}) \right) + \sum_{p=1, p \neq m}^{M} \frac{u_q^{[p]}}{w_k^{[m]}} \mathbf{\Psi}_q^{[m]} \mathbf{V}_k^{[m]},
\end{align*}
\]

(9)

In (9), the positive scalar values \( \alpha_k^{[m]} \) and \( \beta_k^{[m]} \) are given as

\[
\begin{align*}
\alpha_k^{[m]} &= \frac{1}{P^{[m]}} \left( \mathbf{N}^{[m]} \right)^{-1} \sum_{q=1}^{K} \frac{u_q^{[m]}}{w_k^{[m]}}, \\
\beta_k^{[m]} &= \sigma_n^2 \frac{2}{P^{[m]}} \left( \sum_{q=1}^{K} \frac{u_q^{[m]}}{w_k^{[m]}} \right)^{-1} \mathbf{I} + \mathbf{\Psi}_q^{[m]},
\end{align*}
\]

and the \( \mathbf{\Psi}_q^{[ps]} \) and \( \mathbf{\Omega}_q^{[ps]} \) are defined by

\[
\begin{align*}
\mathbf{\Psi}_q^{[ps]} &= \sigma_n^2 \frac{2}{P^{[s]}} \mathbf{I} + \mathbf{\Psi}_q^{[ps]}, \\
\mathbf{\Omega}_q^{[ps]} &= \sigma_n^2 \frac{2}{P^{[s]}} \left( \sum_{q=1}^{K} \frac{u_q^{[m]}}{w_k^{[m]}} \right)^{-1} \mathbf{I} + \mathbf{\Psi}_q^{[ps]}.
\end{align*}
\]

Since it is not possible to solve the equation (9) explicitly, the precoding matrix \( \mathbf{W}_k^{[m]} \) at the \( m \)-th BS is updated iteratively in our proposed scheme. At each iteration, let us denote \( \mathbf{W}_k^{[m]} \) as

\[
\mathbf{W}_k^{[m]} = \left( \mathbf{A}_k^{[m]} \right)^{-1} \left( \alpha_k^{[m]} \mathbf{I} + \mathbf{\Xi}_k^{[mm]} \right) \mathbf{V}_k^{[m]} \tag{10}
\]

where \( \mathbf{A}_k^{[m]} \) is defined as

\[
\mathbf{A}_k^{[m]} = \beta_k^{[m]} \mathbf{I} + \frac{1}{u_k^{[m]}} \sum_{q=1, q \neq m}^{K} \frac{u_q^{[m]}}{w_k^{[m]}} (\mathbf{\Omega}_q^{[mm]} - \mathbf{\Psi}_q^{[mm]})
\]

\[
+ \sum_{p=1, p \neq m}^{M} \frac{u_q^{[m]}}{w_k^{[mm]}} \mathbf{\Psi}_q^{[m]}. \tag{11}
\]

Then we update a new precoding vector \( \mathbf{\hat{V}}_k^{[m]} \) as

\[
\mathbf{\hat{V}}_k^{[m]} \leftarrow \mu \mathbf{\hat{V}}_k^{[m]} + (1 - \mu) \mathbf{V}_k^{[m]} \tag{12}
\]

where \( \mu \) is determined from the line search

\[
\mu = \arg \max_{\nu \in [0,1]} f^{[m]}(\ldots, \nu \mathbf{W}_k^{[m]} + (1 - \nu) \mathbf{V}_k^{[m]}, \ldots) \tag{13}
\]

Note that line search in (13) is not necessary when \( \mu = 1 \) as in \([16]\). The proposed iterative algorithm at the \( m \)-th BS with \( \mu = 1 \) is summarized as Table I.

The following theorem will verify the convergence property of the proposed algorithm.

**Theorem 1:** At the \( m \)-th BS, the iterative process (12) always converges to a local maximum point.

**TABLE I**

**PROPOSED DISTRIBUTED ALGORITHM AT THE m-TH BS**

<table>
<thead>
<tr>
<th>Initialize ( \mathbf{V}_k^{[m]} ) for ( k = 1, \ldots, K ).</th>
<th>Compute ( \mathbf{A}_k^{[m]} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Update ( \mathbf{V}_k^{[m]} \leftarrow \left( \mathbf{A}_k^{[m]} \right)^{-1} \left( \alpha_k^{[m]} \mathbf{I} + \mathbf{\Xi}_k^{[mm]} \right) \mathbf{V}_k^{[m]} ).</td>
<td>Compute ( f^{[m]}(\mathbf{V}_k^{[m]}) ).</td>
</tr>
<tr>
<td>Repeat until convergence.</td>
<td></td>
</tr>
</tbody>
</table>

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**IV. SIMULATION RESULTS**

In this section, the WSR performance of our proposed algorithm is evaluated through the Monte-Carlo simulation. In all simulations, we assume uncorrelated Rayleigh fading channels with unit variance. Also the noise variance \( \sigma_n^2 \) is fixed to 1 for all BSs. For fair comparison, we compare the ZF scheme utilizing the local CSI with our scheme. In order to identify a performance upper bound, we present the performance of the gradient-descent (GD) algorithm which utilizes global CSI in \([17]\). In GD algorithm, the maximum-ratio transmission precoding, \( \mathbf{V}_k^{[mm]} = \mathbf{H}_k^{[mm]} \left( P^{[m]} / (K \mathbf{tr} (\mathbf{H}_k^{[mm]} \mathbf{H}_k^{[mm]} H_k^{[mm]}) \right) \), and the ZF precoding are employed as initial points, and the maximum value is selected. For our proposed scheme, only the ZF initial point is applied.

In Figure 2, the WSR performance is plotted as a function of the signal-to-noise ratio for the [3, 1, 6, 2] MIMO IFBC systems with weights \([w_1^{[1]}, w_1^{[2]}, w_1^{[3]}] = [4; 2; 1] \) and \([10; 5; 1] \). The proposed scheme exhibits the performance almost identical to the GD method with the global CSI. As the difference in weights increases, a performance gap from the GD decreases.
Algorithm is effective in adaptively adjusting the precoding matrices according to various weights and system configurations.

V. Conclusions

In this paper, we have proposed a linear precoding scheme to maximize the WSR for the MIMO IFBC systems. Especially, we focus on the distributed techniques using only the local CSI, which is an important issue in practical cellular systems. In order to derive decoupled problems for each BS from the WSR maximization which requires centralized processing, we have applied an assumption of the ZF strategy and high SINR. The distributed WSR maximization algorithm which considers both the constructive and destructive effects of the precoding matrices is solved iteratively using the ZG condition. Finally we show from the simulation results that the WSR performance of our proposed scheme with substantially lower CSI requirement is almost identical to that of the GD method which utilize global CSI.

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