An MMSE based Block Diagonalization for Multiuser MIMO Downlink Channels with Other Cell Interference

Hakjea Sung, Kyung-Jae Lee and Inkyu Lee
School of Electrical Eng., Korea University, Seoul, Korea
Email: {jaysung, kyoungjae, inkyu}@korea.ac.kr

Abstract—In this paper, we develop a minimum mean-squared error (MMSE) based block diagonalization (BD) algorithm for multiuser multi-input multi-output (MIMO) broadcast systems where each user has more than one antenna in the presence of other cell interference (OCI). Unlike the conventional BD based multiuser MIMO transmission schemes which suffer from the noise enhancement problem as eliminating all multi-user interference (MUI) completely, the proposed scheme attempts to suppress the MUI with a consideration of the OCI plus noise and employs an additional residual interference suppression process based on an MMSE criterion. As a result, the proposed scheme improves the signal-to-interference-plus-noise ratio (SINR) at each user’s receiver compared to conventional BD based schemes. Simulation results demonstrate that the sum rate performance of the proposed algorithm is always better than that of the conventional BD based algorithms for various OCI configurations.

I. INTRODUCTION

In multiuser multi-input single-output broadcast channels systems, a zero-forcing channel inversion (ZF-CI) method [1] is a well-known precoding technique. However, its performance is rather poor due to a transmit power boost issue. Although a minimum mean-squared error channel inversion (MMSE-CI) scheme [1] overcomes the drawback of the ZF-CI, it is still confined to the single receive antenna case. For the multiuser multi-input multi-output (MIMO) case where users in the network have multiple antennas, block diagonalization (BD) and modified channel inversion algorithms have been introduced as a linear precoding algorithm in [2] and [3]. However, in a multicell MIMO network, users located at the edge of the cell experience the interference from neighboring cells. Thus, the performance of these schemes can be degraded significantly when there exits other cell interference (OCI), since the schemes in [2] and [3] have been studied and designed for a single cell environment.

In conventional cellular systems, the OCI can be reduced by radio resource management techniques such as power control, frequency reuse and spreading code assignments. However, these techniques limit the achievable spectral efficiency. Recently, it has been shown that the base station cooperation, which allows information sharing among base stations, can significantly improve the overall spectral efficiency [4]. However, it is difficult in practical systems to jointly operate all base stations. As an alternative, the OCI aware multiuser transmission schemes have been introduced [5] [6]. In these works, the OCI plus noise covariance are estimated at each user and, this covariance information is utilized for signal processing method.

In [6], the authors proposed a modified BD algorithm which accounts for the OCI. Employing the whitening filter at the receiver side, this technique mitigates the effect of the OCI. Even though this scheme exhibits a performance gain over the conventional BD [2] in the presence of OCI, a noise enhancement issue is still present, since it completely eliminates multi-user interference (MUI) by placing all unintended users at the nullspace of the intended user’s channels without any consideration on the noise and the OCI. Also, the constraint on antenna configurations still holds for this scheme as in the conventional BD, i.e., the number of transmit antennas must be greater than the total number of receive antennas.

In this paper, we propose an MMSE based multiuser transmission algorithm in the presence of OCI as a non-iterative linear processing scheme. Based on the MMSE-CI which takes the OCI plus noise into account, we identify an orthonormal vector set of each user’s precoding matrix. Employing the transmit combining matrix obtained by exploiting a minimum total mean-squared error (MSE) criterion, the proposed scheme is able to increase the signal-to-interference-plus-noise ratio (SINR) at each user’s receiver. In addition, the proposed scheme eliminates the dimensional constraint on the number of antennas as opposed to the conventional BD based multiuser transmission schemes. From simulation results, we show that the sum rate performance of the proposed scheme outperforms the conventional BD based scheme over all OCI configurations.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices and lowercase boldface for vectors. For any general matrix \( A \), \( A^T \) and \( A^H \) denote the transpose and the conjugate transpose, respectively. \( \text{Tr}(A) \) indicates the trace of a matrix \( A \). For \( m \times n \) matrices \( A \), \( A = \text{diag}\{A_1, \cdots, A_k\} \) denotes an \( mk \times nk \) block diagonal matrix.

II. SYSTEM MODEL

We consider multiuser MIMO downlink systems where the base station is transmitting to \( K \) independent users simultaneously, and assume that each user experiences the interference transmitted from neighboring cells. In this system, the base
station is equipped with \( N_t \) transmit antennas and user \( j \) has \( n_j \geq 1 \) receive antennas, referred to in the following as \( \{n_1, \cdots, n_K\} \times N_t \). The total number of receive antennas at all users is defined as \( N_r = \sum_{j=1}^{K} n_j \). In the discrete-time complex band MIMO case, the channel from the base station to the \( j \)th user is modeled by the \( n_j \times N_t \) channel matrix \( \mathbf{H}_j \). We assume that \( \mathbf{H}_j \) has full row rank and independently and identically distributed (i.i.d.) entries according to \( N_c(0,1) \). Also, we assume that the network channel matrix \( \mathbf{H}_s = [\mathbf{H}_{1}^T \mathbf{H}_{2}^T \cdots \mathbf{H}_{K}^T]^T \) is known perfectly at the base station.

We define the transmitted data symbol vector \( \mathbf{s}_j \), the noise vector \( \mathbf{z}_j \) and the precoding matrix \( \mathbf{P}_j \) for all users as
\[
\mathbf{s}_j = [s_{j,1}^T s_{j,2}^T \cdots s_{j,n_j}^T]^T, \\
\mathbf{z}_j = [z_{j,1}^T z_{j,2}^T \cdots z_{j,n_j}^T]^T, \\
\mathbf{P}_j = [\mathbf{P}_1 \mathbf{P}_2 \cdots \mathbf{P}_K]
\]
where \( s_{j,i} \) is the \( n_j \times 1 \) OCI signal vector for the \( j \)th user from \( n_j \) interferers with average power \( P_j \), i.e., \( \mathbb{E}]\|s_{j,i}\|^2 = P_j \) and \( \mathbf{H}_{i}^j \) is the \( n_j \times N_t \) OCI channel matrix for the \( j \)th user. Then, the total received signal in the presence of OCI can be expressed as
\[
\mathbf{y}_s = \mathbf{H}_s \mathbf{P}_s \mathbf{s}_s + \mathbf{H}_s^1 \mathbf{s}_j^1 + \mathbf{z}_s.
\]
(1)

Let \( \mathbf{s}_s = \mathbf{P}_s \mathbf{s}_s \) denote the signal vector transmitted at the base station, which satisfies the total power constraint \( \mathbb{E}\|\mathbf{s}_s\|^2 \leq P_t \). Since each data symbol has unit variance, the total transmit power constraint can be expressed as \( \text{Tr}(\mathbf{P}_s^H \mathbf{P}_s) \leq P_t \).

From (1), denoting the OCI plus noise vector as
\[
\mathbf{w}_s = [\mathbf{w}_1^T \mathbf{w}_2^T \cdots \mathbf{w}_K^T]^T = \mathbf{H}_s^1 \mathbf{s}_s + \mathbf{z}_s
\]
where \( \mathbf{w}_j = \mathbf{H}_s^j \mathbf{s}_j^1 + \mathbf{z}_s \) and assuming that the interfering signal is independent with the noise, the \( j \)th user’s OCI plus noise covariance matrix is given by
\[
\mathbf{H}_j^O \mathbf{H}_j^O + \sigma^2 \mathbf{I}_{n_j}
\]
(2)
where \( \mathbf{O}_j = \mathbb{E}[s_{j,i}^1 s_{j,i}^H] \) and \( \text{Tr}(\mathbf{O}_j) = P_j \). There are various methods of estimating the OCI plus noise covariance \([7][8]\).

In this paper, we assume that each user can perfectly estimate the OCI plus noise covariance matrix (2).

Next, we consider the receiver operation. Denoting the overall receive filter \( \mathbf{M}_s \) as
\[
\mathbf{M}_s = \text{diag}\{\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_K\}
\]
where \( \mathbf{M}_j \) represents the \( j \)th user’s receive filter, the receive filter output vector of all users \( \mathbf{x}_s \) can be written from (1) as
\[
\mathbf{x}_s = \mathbf{M}_s \mathbf{H}_s \mathbf{P}_s \mathbf{s}_s + \mathbf{M}_s \mathbf{w}_s
\]
(3)

where \( \mathbf{x}_j = [x_{j,1}^T x_{j,2}^T \cdots x_{j,n_j}^T]^T \) and \( \mathbf{x}_j = [x_{j,1} \cdots x_{j,n_j}]^T \in \mathbb{C}^{n_j} \).

### III. PROPOSED MMSE BASED BLOCK DIAGONALIZATION

In this section, we illustrate an MMSE based linear processing algorithm for multiuser MIMO systems in the presence of OCI. We outline a procedure for identifying the proposed scheme through the three stage process. In the first stage, based on the MMSE-CI which takes the OCI plus noise into account, we find a set of orthonormal basis vectors of the precoding matrix. In the second stage, the transmit combining matrix multiplied to the orthonormal vector set is obtained with the MMSE criterion. Applying these at the transmitter, the interference suppressed block channel is created for each user. Finally, this block channel is decoupled into \( n_j \) parallel subchannels in order to allow single symbol decodable receiver for each user.

In the proposed scheme, the orthonormal vector set of the precoding matrix for each user can be determined from the MMSE-CI introduced in [1]. From (2), we can see that the OCI plus noise is correlated. Thus, in order to apply the MMSE-CI, we first whiten the OCI plus noise term. Since the matrix \( \mathbf{H}_j^O \mathbf{H}_j^O + \sigma^2 \mathbf{I}_{n_j} \) in (2) is Hermitian and positive definite, we can decompose this matrix using Cholesky factorization as
\[
\mathbf{H}_j^O \mathbf{H}_j^O + \sigma^2 \mathbf{I}_{n_j} = \mathbf{L}_j^H \mathbf{L}_j
\]
Then, denoting the total receive whitening filter \( \mathbf{M}_s \) as
\[
\mathbf{M}_s = \text{diag}\{\mathbf{L}_1^{-H}, \mathbf{L}_2^{-H}, \cdots, \mathbf{L}_K^{-H}\}
\]
the corresponding received signal vector with the uncorrelated noise of all users can be written from (3) as
\[
\tilde{\mathbf{x}}_s = \mathbf{H}_s^H \tilde{\mathbf{M}}_s \mathbf{s}_s + \mathbf{w}_s^w
\]
where \( \mathbf{H}_s^H = \mathbf{M}_s \mathbf{H}_s \) and \( \mathbf{w}_s^w = \mathbf{M}_s \mathbf{w}_s \).

Now, we denote the \( N_t \times N_r \) MMSE-CI matrix \( \mathbf{F}_s \) as
\[
\mathbf{F}_s = [\mathbf{F}_1 \mathbf{F}_2 \cdots \mathbf{F}_K] = (\mathbf{H}_s^H \mathbf{H}_s^H + \alpha \mathbf{I})^{-1} \mathbf{H}_s^H
\]
(5)
where \( \mathbf{F}_j \) is an \( N_t \times n_j \) submatrix of \( \mathbf{F}_s \) and \( \alpha \) represents the ratio of the total noise variance to the total transmit power, i.e., \( \alpha = \mathbb{E}\|\mathbf{w}_s^w\|^2 / P_t = N_r / P_t \) [1].

Let us denote \( f_{k,j} \) as the \( k \)th column vector of \( \mathbf{F}_j \) where \( k = 1, \cdots, n_j \). Here, \( f_{k,j} \) not only mitigates other users’ interference, but also it attempts to suppress the signal between each antenna of the \( j \)th user. Thus, if we use the MMSE-CI matrix in (5) as a precoding matrix, there should be a performance loss when each user has multiple receive antennas.

Hence, by applying the orthogonalization procedure to \( \mathbf{F}_j \), we compensate the suppression of each antenna signal of the \( j \)th user. In this way, we can expand the conventional MMSE-CI solution in (5) to the case where each user has
multiple antennas. For orthogonalization, we employ the QR decomposition as
\[
F_j = Q_j R_j \quad \text{for} \quad j = 1, \cdots, K
\]
where \(R_j\) is an \(n_j \times n_j\) upper triangular matrix and the \(N_j \times n_j\) matrix \(Q_j\) is composed of \(n_j\) orthonormal basis vectors of \(F_j\). Then, the \(j\)th user’s precoding matrix \(P_j\) of the proposed scheme can be constructed by a linear combination of columns of \(Q_j\).

Note that, if there is no consideration of OCI signals, \(\tilde{M}_s\) can be denoted as \(I_{N_j}\). Also, when we set \(\alpha = 0\), equation (5) becomes equivalent to the conventional ZF-CI matrix as
\[
F_s = [F_1^s, F_2^s, \cdots, F_K^s] = H_s^H (H_s H_s^H)^{-1}
\]
and \(F_s^\dagger\) can be decomposed as \(F_s^\dagger = Q_s^\dagger R_s^\dagger\). In this case, we have \(H_j F_s^\dagger = H_j Q_s^\dagger R_s^\dagger = 0\) from (6), where other users’ channel \(H_j\) is defined as
\[
\tilde{H}_j = [H_{1,j}^T, H_{2,j}^T, \cdots, H_{j-1,j}^T, H_{j+1}^T, \cdots, H_{K,j}^T]^T.
\]
Here, since \(R_j^\dagger\) is invertible, it follows \(\tilde{H}_j Q_j^\dagger = 0\). Thus, we can see that the columns of \(Q_j^\dagger\) form an orthonormal basis for the nullspace of \(\tilde{H}_j\) so that the \(j\)th user’s precoder constructed by a linear combination of \(Q_j^\dagger\) is equivalent to the conventional BD method in [2].

In contrast, we exploit the OCI plus noise covariance to form the MMSE-CI solution in (5) and identify the orthonormal vector sets of the proposed scheme based on this. Thus, the proposed scheme is able to balance the MUI and the OCI plus noise, and this enables to overcome the noise enhancement issue observed in the conventional BD based scheme. However, unlike the conventional BD based algorithm, the \(j\)th user’s precoder of the proposed scheme constructed by a linear combination of \(Q_j\) generates residual interference. Thus, a proper residual interference suppression process is needed additionally. In the following, we introduce a way of computing the transmit combining matrix under a total minimum MSE criterion.

First, denoting the \(n_j \times n_j\) transmit combining matrix applied to \(Q_j\) as \(T_j\), the corresponding received signal vector of the \(j\)th user can be written from (4) as
\[
x_j = H_j^H Q_j T_j s_j + H_j^H \sum_{k \neq j} Q_k T_k s_k + w_j
\]
where \(H_j^\dagger = L_j^{-H} H_j\) and \(w_j = L_j^{-H} w_j\).

Next, we define \(U_j^H\) and \(V_j\) as the matrices employed to receiver and precoder in order to decompose the block channel \(H_j^\dagger Q_j T_j\) in (7) into parallel subchannels, respectively. Then, applying these matrices to (7), the single symbol decodable signal vector of the \(j\)th user can be expressed as
\[
x_j = U_j^H H_j^\dagger \sum_{k=1}^{K} Q_k T_k s_k + U_j^H w_j.
\]

Here, similar to the conventional BD algorithm [2], \(U_j^H\) and \(V_j\) are computed by the SVD of \(H_j^\dagger Q_j T_j\). Note that, we do not know the values of these matrices’ entries, since \(T_j\) is not determined yet. However, we can assume that \(U_j^H\) and \(V_j\) are unitary, and we use this assumption in the following derivation.

Let us again define the \(j\)th user’s combining matrix \(T_j\) as \(\beta T_j\), where the scaling parameter \(\beta\) is a positive real number. Then, the MSE of the \(j\)th user is represented as [9][10]
\[
E[||U_j^H \Omega_j V_j s_j - \frac{1}{\beta} x_j||^2]
\]
where \(\Omega_j\) is the \(n_j \times n_j\) target channel matrix based on the MMSE criterion. From (8) and (9), the problem of minimizing the total MSE under the total transmit power constraint can be written as
\[
\min_{T_j} \sum_{j=1}^{K} E[||U_j^H \Omega_j V_j s_j - \frac{1}{\beta} x_j||^2]
\]
subject to \(\sum_{j=1}^{K} \text{Tr}(V_j^H T_j^H Q_j^H T_j V_j) = P_t\). (10)

In order to solve the above equation, we first convert this problem into an unconstrained minimization problem. In the constraint function of the above equation, we substitute \(\beta T_j\) for \(T_j\). Then, since \(V_j\) is unitary and \(Q_j^H Q_j = I_{n_j}\), the scaling parameter \(\beta\) is given by
\[
\beta = \sqrt{P_t} \left[ \sum_{j=1}^{K} \text{Tr}(T_j^H T_j) \right]^{-\frac{1}{2}}. \tag{11}
\]

Applying (11) to the cost function in (10), the unconstrained total MSE minimization problem on \(T_j\) is written as
\[
\min_{T_j} \sum_{j=1}^{K} \text{Tr}(U_j^H H_j^\dagger \sum_{k=1}^{K} Q_k T_k V_k s_k + U_j^H w_j^2) - \frac{1}{\sqrt{P_t}} \sum_{j=1}^{K} \text{Tr}(T_j^H T_j) \text{Tr}(T_k^H T_k) U_j^H w_j^2. \tag{11}
\]

Also, this problem can be formulated as
\[
\min_{T_j} \sum_{j=1}^{K} \text{Tr}(U_j^H H_j^\dagger \sum_{k=1}^{K} Q_k T_k V_k s_k + U_j^H w_j^2) - \frac{1}{\sqrt{P_t}} \sum_{j=1}^{K} \text{Tr}(T_j^H T_j) \text{Tr}(T_k^H T_k) U_j^H w_j^2. \tag{11}
\]

Now, we take a derivative of the above equation with respect to \(T_j^H\) and set it to zero. Then, this results in
\[
\tilde{T}_j = (Q_j^H \sum_{k=1}^{K} H_k^H H_k^\dagger Q_j + \alpha I_{n_j})^{-1} Q_j^H H_j^\dagger \Omega_j Q_j. \tag{12}
\]

Here, we employ \(H_j^\dagger Q_j T_{\text{high},j}\) as the target channel matrix of the \(j\)th user \(\Omega_j\) where \(T_{\text{high},j}\) is denoted as the \(j\)th user’s
combining matrix at high signal-to-noise-ratio (SNR), and we set \( T_{\text{high},j} \) to \( I_{s,j} \). Note that, from (5) and (12), it is obvious that \( T_j \) converges to an identity matrix as SNR increases. Finally, the \( j \)th user’s transmit combining matrix \( T_j \) which minimizes the total MSE is given by \( T_j = \beta T_j \) where \( \beta \) is determined by the equation (11).

After finding \( Q_j \) and \( T_j \), we now make each user’s received signal vector single symbol decodable. The term \( H_j^t Q_j T_j \) in (7) represents the interference suppressed block channel of the \( j \)th user. In order to decompose this channel into parallel subchannels, we apply the SVD of \( H_j^t Q_j T_j \) as

\[
H_j^t Q_j T_j = U_j \Lambda_j V_j.
\]

Then, the precoding matrix and the overall receive filter of the proposed scheme are obtained as

\[
P_s = [Q_1 T_1 V_1 \ Q_2 T_2 V_2 \ \cdots \ Q_K T_K V_K],
\]

\[
M_s = \text{diag}\{U_1^H L_1^{-H}, U_2^H L_2^{-H}, \cdots, U_K^H L_K^{-H}\}.
\]

Finally, after applying the above solutions to (3), the receive filter output signal vector at the \( j \)th user can be written as

\[
x_j = A_j s_j + M_j H_j \sum_{k \neq j} P_k s_k + M_j w_j.
\]  

(13)

We now briefly address the sum rate of the proposed scheme. For the conventional BD based schemes, the water-filling (WF) solution is utilized to maximize the sum rate. However, for the proposed scheme, it is not needed to allocate different power to each stream, since the transmit combining matrix has already been computed to meet its corresponding criterion. For instance, if we apply an additional power loading matrix to the precoding matrix, the minimum total MSE condition is no longer satisfied. Thus, we do not consider the WF algorithm when we compute the sum rate of the proposed scheme.

Denoting the \( i \)th diagonal element of \( A_j \) as \( \lambda_{j,i} \), each received signal in (13) contains in part the signal of interest with the channel gain \( \lambda_{j,i}^2 \), and in part the interference from the other users plus Gaussian noise. Then, the SINR of each stream can be expressed as

\[
\text{SINR}_{j,i} = \frac{\lambda_{j,i}^2}{|m_{j,i} w_j|^2 + \sum_{k \neq j} |m_{j,i} H_j P_k|^2}
\]

where \( m_{j,i} \) is the \( i \)th row vector of \( M_j \). Then, the sum rate of the proposed scheme is given by

\[
R = \sum_{j=1}^{K} \sum_{i=1}^{n_j} \log_2 (1 + \text{SINR}_{j,i}).
\]

IV. NUMERICAL RESULTS

In this section, we present the sum rate of the proposed scheme comparing with the conventional BD based multiuser transmission schemes in [2] and [6] under the various OCI configurations. All simulations are evaluated for \( \{2, 2, 2\} \times 6 \) case. In order to generate the OCI, we assume that the OCI channel matrix \( H_j^t \) is an uncorrelated MIMO Rayleigh fading channel. Also, we define the OCI-to-noise ratio of the \( j \)th user’s as \( \text{INR}_j \) = \( P_j / \sigma_j^2 \) in dB. We denote \( \text{INR}_s \) and \( N_j^t \) as \( \text{INR}_s = [\text{INR}_1, \text{INR}_2, \cdots, \text{INR}_K] \) and \( N_j^t = [N_1^t, N_2^t, \cdots, N_K^t] \), respectively.

In Fig. 1, we present the comparison of the complementary cumulative distribution functions (CCDFs) of the sum rate at SNR=15dB in the presence of the OCI with \( \text{INR}_s = [10, 10, 10] \) and \( N_j^t = [2, 2, 2] \). From Fig. 1, we observe that the sum rate of the proposed scheme is greater than that of the conventional BD based schemes with or without the WF algorithm.

Fig. 2 illustrates the sum rates as a function of SNR for \( \{2, 2, 2\} \times 6 \) multiuser MIMO systems with various \( \text{INR}_j \) and \( N_j^t \).
scheme in [6] grows as the OCI increases.

In Fig. 3, we compare the sum rates of the proposed scheme and conventional BD based schemes in terms of INR, for systems with $N_1^i = [2, 2, 2]$ at SNR=25dB. For this simulation, all INR$_j$ for $j = 1, 2, 3$ are set to be equal. In this plot, it is shown that the sum rates of conventional BD based schemes approach that of the proposed scheme when the OCI is small. However, it is clear from the plot that, as the OCI increases, the average sum rates of all schemes decrease but the performance gaps between the proposed scheme and the conventional BD based schemes grow even at high SNR.

Finally, in Fig. 4, we evaluate an effect of the number of interferers on the sum rate performance of the proposed scheme and conventional BD based schemes. For this simulation, all $N_1^i$ for $j = 1, 2, 3$ are set to be the same. We plot the sum rates of these schemes at SNR=15dB in the presence of the OCI with INR$_s = [10, 10, 10]$ as a function of the $N_1^i$. As the number of interferers increases, the OCI at each user becomes uncorrelated with each other. Since the scheme in [6] only considers a whitening filter to make the OCI uncorrelated at each user’s receiver, it can be seen in the figure that the sum rate of this scheme converges to that of the conventional BD when there are a large number of interferers. Unlike the scheme in [6], the proposed scheme also utilizes the OCI plus noise for identifying the precoding matrix. Thus, it is observed that the proposed scheme still outperforms the conventional BD even at large number of interferers contrasted with the scheme in [6].

V. Conclusion

In this paper, we have proposed an MMSE based BD algorithm as a non-iterative precoder design method for a multiuser multicell downlink network with users equipped with multiple receive antennas. Unlike the conventional BD based schemes which eliminate MUI completely, the proposed scheme identifies an orthonormal vector set of each user’s precoding matrix using the MMSE-CI which takes the noise and OCI into account. Also, an optimal combining matrix is employed at the orthonormal basis matrix of each precoder in order to mitigate residual interference. Through simulations, we have verified that the proposed scheme can provide the improved sum rate performance over the conventional BD based transmission schemes for multiuser MIMO downlink channels with various configurations of OCI.

REFERENCES