Unitary Precoding Techniques based on Transmit-MRC for MIMO Wireless Systems

Seok-Hwan Park, Heunchul Lee, Sang-Rim Lee, and Inkyu Lee
School of Electrical Eng., Korea University, Seoul, Korea
Email: {shpark, heunchul, srlee}@wireless.korea.ac.kr and inkyu@korea.ac.kr

Abstract—This paper proposes a low complexity unitary precoding scheme for multiple-input multiple-output (MIMO) systems. The singular-value decomposition (SVD) based transmission is capable of maximizing the system throughput when combined with power allocation and bit loading, and is known to be optimum in terms of capacity. This paper focuses on a system which attains the same optimality as the SVD-based system with low complexity utilizing transmit maximum-ratio combining (TMRC) techniques. The TMRC scheme is the optimum structure for single beamforming systems in terms of received signal-to-noise ratio (SNR) in multiple-input single-output (MISO) channels. In this paper, we generalize the TMRC scheme to multiple beamforming MIMO systems which supports more than one data stream in coded systems. Simulation results demonstrate that the proposed scheme achieves the almost identical link performance as the SVD precoding system for arbitrary configurations with reduced complexity.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless systems have been widely studied to increase the communication reliability and spectral efficiency. The capacity analysis of MIMO systems has shown significant gains over single-input single-output (SISO) systems [1]. In [2], it is suggested that an additional performance gain can be extracted from multiple antennas in the presence of channel state information (CSI) at the transmitter. In such closed-loop systems, the optimization of linear precoding and decoding has been presented in [3]. Most work on these closed-loop MIMO systems has been carried out by performing singular value decomposition (SVD) of the channel transfer matrix. Among unitary precoders, the SVD-based precoding is shown to be optimum in terms of capacity [4]. When the constraint of unitary precoders is relaxed, the optimality in terms of channel capacity can be obtained when the transmission based on the SVD is combined with the water-filling power allocation. Besides, it is shown in [5] that the SVD-based transmission combined with bit interleaved coded modulation (BICM) can achieve both full spatial multiplexing and full spatial diversity gain for any antenna configurations. However, the computational complexity of the SVD operation becomes problematic with the increased number of transmit and receive antennas, due to the iterative nature of the SVD computation.

The goal of this paper is to design a simple transmission scheme which achieves the same optimality as SVD-based schemes. We focus on coded systems since MIMO techniques combined with a powerful channel coding can provide an effective means to counteract severe impairments of fading channels. We will start with a review of transmit maximal ratio (TMRC) introduced in [6], which is the optimal solution for single beamforming multiple-input single-output (MISO) systems. Although the TMRC scheme provides quite a good link performance gain with low complexity, the number of data streams is restricted to one since the TMRC can be applied only to systems with single receive antenna.

In this paper, we propose a new unitary precoding system which can transmit multiple data streams simultaneously in MIMO channels by generalizing the TMRC technique. We first represent beamforming vectors as a linear combination of TMRC vectors and then the coefficients of linear combination are optimized successively. Adopting the conception of the gradient ascent algorithm, the vector coefficients are optimized in a non-iterative way to achieve a near-optimal performance with a proper choice of initialization vectors. Motivated by the fact that the beamforming vectors of the SVD-based system are fully orthogonal to each other, we orthogonalize the basis TMRC vectors to previously computed beamforming vectors. As we shall see later, the proposed one-shot approach provides the performance almost identical to the optimum unitary precoding scheme with reduced complexity.

For clarity, the following notations are used for description throughout this paper. Normal letters represent scalar quantities, boldface letters indicate vectors and boldface uppercase letters designate matrices. The superscripts $(\cdot)^\dagger$, $(\cdot)^T$ and $(\cdot)^*$ represent the complex conjugate, transpose and Hermitian transpose, respectively.

II. SYSTEM DESCRIPTION

In this section, we present a general description of coded unitary precoding systems combined with BICM. We consider a MIMO link with $M_t$ transmit and $M_r$ receive antennas where $M$ independent data streams are transmitted simultaneously $(M \leq \min(M_t, M_r))$.

At the transmitter, the information bits are encoded with a code rate $R_c$, bit-interleaved and symbol-mapped to yield $M$-dimensional symbol vector $s_k = [s_{1,k}, \cdots, s_{M,k}]^T$, where the subscript $k$ indicates the $k$-th time slot. The unitary precoder
matrix $\mathbf{W}$ of size $M_r$-by-$M$ receives the symbol vector $s_k$ and generates the linearly precoded signal vector $x_k$ of length $M_t$ as

$$
x_k = \mathbf{W}s_k = \sum_{m=1}^{M} w_ms_{m,k} \tag{1}
$$

where $w_m$ denotes the $m$-th column vector of $\mathbf{W}$. As we concentrate on unitary precoders in this paper, $w_m$ should satisfy the condition $w_m^*w_n = \delta_{mn}$ where $\delta_{mn}$ represents Kronecker delta. The resulting precoded symbol vector $x_k$ is then transmitted through $M_t$ transmit antennas. Throughout the paper, we assume the flat fading channel model where the fading coefficients are static over a frame of transmitted symbols and independent over frames.

Then, the $M_r$-dimensional received signal vector can be written as

$$
y_k = \mathbf{H}x_k + \mathbf{n}_k
$$

where $n_k$ is the complex Gaussian noise vector with the covariance matrix $\sigma_n^2 \mathbf{I}_{M_r}$. Here $\mathbf{I}_d$ indicates an identity matrix of size $d$. Also, the $M_t$-by-$M_r$ channel matrix $\mathbf{H}$ is given by

$$
\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \cdots \ \mathbf{h}_{M_r}]^T
$$

where $\mathbf{h}_i^T$ denotes the $i$-th row vector of $\mathbf{H}$. The elements of $\mathbf{H}$ are obtained from an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero-mean and unit variance and assumed to be perfectly known at the receiver.

Without loss of generality, we assume that

$$
||\mathbf{h}_1||^2 \geq ||\mathbf{h}_2||^2 \geq \cdots \geq ||\mathbf{h}_{M_r}||^2. \tag{2}
$$

At the receiver, the received signal vector $y_k$ is post-filtered by the receive matrix $\mathbf{G}$ of size $M$-by-$M_r$ as

$$
z_k = \mathbf{G}y_k = \mathbf{GHW}s_k + \tilde{\mathbf{n}}_k \tag{3}
$$

where $\tilde{\mathbf{n}}_k = \mathbf{Gn}_k$ is the filter output noise with the covariance matrix $E[\tilde{\mathbf{n}}_k\tilde{\mathbf{n}}_k^H] = \sigma_n^2 \mathbf{G}\mathbf{G}^H$. The $M$ output streams $z_k = [z_{1,k}, \cdots, z_{M,k}]^T$ are converted to soft log-likelihood ratio values in the soft demapper to be processed in the decoder.

In what follows, we briefly review the optimal unitary precoding structure [4]. Defining $\mathcal{U}(m,n)$ as a set of $m$-by-$n$ matrices with orthonormal columns, the SVD of $\mathbf{H}$ is given by $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$ where $\boldsymbol{\Sigma}$ denotes an $M_r$-by-$M_t$ diagonal matrix with the $i$-th largest singular value of $\mathbf{H}$ at entry $(i,i)$ and $\mathbf{U}$ and $\mathbf{V}$ are represented as $\mathbf{U} \in \mathcal{U}(M_r,M_t)$ and $\mathbf{V} \in \mathcal{U}(M_t,M_r)$. Here, we denote $v_i$ and $u_i$ as the $i$-th column of $\mathbf{V}$ and $\mathbf{U}$. It was shown in [4] that the optimal unitary precoder $\mathbf{W}_{\text{opt}} \in \mathcal{U}(M_t,M)$ in terms of capacity is given by $\mathbf{W}_{\text{opt}} = \mathbf{V}_M$, where $\mathbf{V}_M$ is a matrix constructed from the first $M_t$ columns of $\mathbf{V}$. We refer to this precoder as SVD-based Unitary Precoding (SVD-UP). The SVD-UP technique combined with adaptive bit-loading was evaluated via throughput simulations for adaptive MIMO systems in [7]. It was known that the SVD-UP suffers from its high computational complexity and numerical sensitivity, since the SVD computation is an inherently iterative process.

### III. New Unitary Precoding Schemes

In this section, we present a new unitary precoding scheme, which is called generalized TMRC (G-TMRC), achieving the optimality of SVD-UP system. For the MISO case, the channel matrix reduces to the vector channel $\mathbf{H} = h_k^T$, and thus the linear precoder can support only one data stream due to the rank-deficient MISO channel matrix. In this case, the optimal SVD-UP solution simply becomes

$$
w_1 \propto h_1^*.
$$

This solution is known as the TMRC technique [6]. A limitation of the TMRC is that it can be applied only to single beamforming in MISO systems ($M_t = M_r = 1$). To overcome this problem, we extend this TMRC technique to multiple beamforming systems ($M \geq 2$) with general configurations ($M_t \geq 2$) in this section.

#### A. Single Beamforming ($M=1$)

In this section, the TMRC scheme is extended to single beamforming systems with multiple receive antennas. We set the beamforming vector $w_1$ in (1) as the linear combination of TMRC vectors as

$$
w_1 = \sum_{i=1}^{M_r} c_i t_i \tag{4}
$$

where the TMRC vectors $t_i$ $(i = 1, \cdots, M_r)$ are defined as

$$
t_i = h_i^*.
$$

Then, (4) can be rewritten in a matrix form as $w_1 = \mathbf{T}\mathbf{c}$ where the TMRC matrix $\mathbf{T}$ and the weighting vector $\mathbf{c}$ are defined as

$$
\mathbf{T} = [t_1 \ \cdots \ t_{M_r}] \text{ and } \mathbf{c} = [c_1 \ \cdots \ c_{M_r}]^T.
$$

In the single beamforming case, $\mathbf{T}$ and the channel matrix $\mathbf{H}$ are related by $\mathbf{T} = \mathbf{H}^T$.

Assuming that the MRC is applied at the receiver, the output signal-to-noise ratio (SNR) of the MRC combiner becomes

$$
\text{SNR} = \frac{E_s||\mathbf{H}w_1||^2}{\sigma_n^2}
$$

where $E_s$ denotes the average constellation energy. Thus, maximizing the output SNR is equivalent to maximization of $||\mathbf{H}w_1||^2 = ||\mathbf{HTc}||^2$. Then, our problem can be formulated as follows:

$$
\max_{\mathbf{c}} : F(\mathbf{c}) = ||\mathbf{HTc}||^2 \tag{5}
$$

subject to: $||\mathbf{Tc}||^2 = 1$.

This constrained optimization problem is convex, but becomes complex to obtain the closed-form solution when $M_r \geq 3$.

One of the simplest, yet suboptimum solution is to choose $\mathbf{c}$ as the prefixed vector as

$$
\mathbf{c} = \mathbf{c}_0 \triangleq \frac{1_1}{||1_1||} \tag{6}
$$

where $1_1$ denotes an $M_r$-by-1 unit vector of 0's with one at the $i$-th position. This choice of (6) is based on the observation...
made in [8], where it is shown that employing the TMRC vector matched to only one row vector $h_i^T$ (out of $H$) provides the near-optimal performance of the SVD-UP by properly selecting $j \in \{1, \ldots, M_r\}$. Because of the assumption in (2), the choice of $j = 1$ in (6) means that the prefixed beamforming vector $w_1 = TC_{(0)}$ is matched to the channel row vector with the largest energy. Note that this choice of $j$ becomes the optimum selection in [8] only when $M_r = 2$. In the case of $M_r \geq 3$, the performance loss of this energy-based selection of $j$ is not significant as will be shown in Figure 1. We call the proposed scheme with this simple prefixed coefficient as “G-TMRC Type-I”.

The performance of the proposed G-TMRC Type-I is not optimum due to its simple choice of $c$. We can further improve the proposed scheme by incorporating the idea of the gradient ascent algorithm [9]. The original gradient ascent method is an iterative process. However, we propose a method which does not require iterations. The gradient ascent algorithm is based on the observation that the objective function $F(c)$ in (5) increases fastest if one goes from $c_{(0)}$ in the direction of $\nabla F(c_{(0)})$, where $\nabla F(c_{(0)})$ represents a gradient of $F(c)$ at $c = c_{(0)}$ as

$$
\nabla F(c_{(0)}) = \left[\nabla ||HTc||^2\right]_{c=c_{(0)}} = 2TH^HTc_{(0)}.
$$

Then, it follows that $F(c_{(1)}) \geq F(c_{(0)})$ with

$$
c_{(1)} = c_{(0)} + \gamma \nabla F(c_{(0)})
$$

for a small enough positive real number $\gamma$. In general, to achieve a near-optimal performance with $w_1 = TC_{(1)}$, the optimal $\gamma$ should be chosen judiciously [9]. We model $\gamma$ as

$$
\delta = \frac{\nabla ||F(c_{(0)})||^2}{\nabla ||F(c_{(0)})||^2}.
$$

so as to extract the advantage of the normalized least-mean-square (NLMS) algorithm [10], where $\delta$ is a positive real number. The proposed near-optimal method with the new coefficient $c = c_{(1)}$ is referred to as ”G-TMRC Type-II”. The algorithm of the proposed single beamforming system is summarized as follows:

**Initialization:**

$T = H^T$

**Main Body:**

$$
c = \begin{cases}
\frac{1}{||\hat{c}_1||} & \text{(Type-I)} \\
\frac{1}{||\hat{c}_1||} + \frac{\|T^TH^HTc\|}{||\hat{c}_1||} & \text{(Type-II)}
\end{cases}
$$

$$
w_1 = \frac{TC}{||TC||}.
$$

Figure 1 compares the cumulative distribution functions (CDF) of the MRC output SNR for the G-TMRC and SVD-UP for 2-by-2, 4-by-4 and 6-by-6 systems with $M = 1$, where the SNR of the G-TMRC Type-II is evaluated with $\delta = 150$. Although the continuous change of $\delta$ may provide an enhanced performance gain, we simply choose $\delta$ as a fixed value to minimize the processing complexity. The performance of the G-TMRC Type-I is less than 1 dB away from the optimum SVD scheme with much reduced complexity. In contrast, the proposed G-TMRC Type-II provides the performance very close to the SVD-UP for all antenna configurations. Note that the proposed schemes are much simpler than the SVD-UP for these configurations. Considering complexity and performance, the G-TMRC Type-II can be one of the most efficient technique. Also, we observe that the proposed G-TMRC Type-I shows a slight performance loss compared to the optimal selection criterion in [8].

**B. Multiple Beamforming ($M > 1$)**

From now on, we generalize the proposed G-TMRC schemes to the multiple beamforming system where more than one data stream is supported to increase the throughput. We now revisit the SVD-UP system. As mentioned earlier, the precoding matrix of the SVD-UP is set to $W = V_M = [v_1 \cdots v_M]$. Here, the right singular vectors $v_1, \ldots, v_M$ have the following property [11], which is a key basis for establishing our proposed G-TMRC schemes:

$$
v_n = \arg \max_{v \in S_n} ||Hv||^2 \quad (n = 1, \ldots, M) \quad (7)
$$

where $S_1$ is a set of all unit vectors of length $M_1$ and $S_n (n = 2, \ldots, M)$ indicates the set of all unit vectors orthogonal to $v_1, \ldots, v_{n-1}$. This means that $v = v_1$ maximizes $||Hv||^2$ among all $M_1$-dimensional unit vectors, while $v = v_n (n = 2, \ldots, M)$ maximizes $||Hv||^2$ within the vector space orthogonal to $v_1, \ldots, v_{n-1}$.

Considering the relations (7), we try to compute the beamforming vector $w_m$ ($m = 1, \ldots, M$) for the $m$-th data stream without relying on the SVD operation. We express all beamforming vectors in (1) as a linear combination of the TMRC vectors, that is,

$$
w_m = T^m c_m, \quad m = 1, \ldots, M
$$

where the superscript $m$ is introduced to distinguish multiple beamforming vectors. We start with initialization $T^1 = H^T$. The first beamforming vector $w_1$ can be written as $w_1 = T^1 c^1$ where $c^1$ is optimized using the single beamforming
solution, that is,  
\[ c_1^1 = \begin{cases} \frac{1}{||t_i^1||} \\ \rho_1 \left( \frac{1}{||t_i^1||} + \delta \frac{T_i^1 H_i^1 H_i^1}{||T_i^1 H_i^1 H_i^1||} \right) \end{cases} \text{ (Type-I)} \]
\[ c_2^1 = \frac{1}{||t_2^2||} \]
where \( \rho_1 \) is chosen to satisfy \( ||w_1||^2 = ||T^1 c_1^1||^2 = 1 \).

Recognizing the relation (7), we want to obtain \( w_2 \) which is orthogonal to \( w_1 \), and this can be achieved by applying orthogonal projection. To this end, we compute \( T^2 = T^1 - w_1 w_1^T T^1 \), which is equivalent to
\[ t_i^m = t_i^1 - w_1 w_1^T t_i^1, \quad i = 1, \ldots, M_r \quad (8) \]
where \( t_i^m \) denotes the i-th column of \( T^m \). As the second term in the right-hand side of (8) represents the orthogonal projection of \( t_i^1 \) onto \( w_1 \), it is obvious that \( w_1 \perp t_i^m \) for \( i = 1, \ldots, M_r \). This means that the second beamforming vector \( w_2 \) is optimized in the subspace orthogonal to \( w_1 \) since \( w_2 \) is a linear combination of \( t_i^m \).

Now, for computing \( w_2 = T^2 c_2^1 \), we have to optimize the weighting vector \( c_2^1 \). For the case of the G-TMRC Type-I, \( c_2^1 \) is prefixed as
\[ c_2^1 = \frac{1}{||t_2^2||} \]
Note that we choose \( w_2 \) matched to the second largest row vector \( h_2^T \) rather than the largest one \( h_1^T \), because a large fraction of a gain from \( h_1^T \) has already been delivered to the first stream. As in the case of the single beamforming system, we can further improve the performance by utilizing the idea of the gradient ascent algorithm. Thus, the G-TMRC Type-II chooses \( c_2^1 \) as
\[ c_2^2 = \rho_2 \left( \frac{1}{||t_2^2||} + \frac{T^2 H_1^T H_2^2}{||T^2 H_1^T H_2^2||} \right) \]
where \( \rho_2 \) is determined such that \( ||w_2||^2 = ||T^2 c_2^2||^2 = 1 \).

At the subsequent steps, we determine \( w_m \) \( (m = 3, \ldots, M) \) with the same way. The proposed G-TMRC schemes with \( M \) greater than one is described below:

\[ \text{Initialization:} \]
\[ T^1 = H^† \]
\[ \text{Main Body:} \]
for \( m = 1 : M \)
\[ c_m^m = \begin{cases} \frac{1}{||t_m^m||} \\ \rho_1 \left( \frac{1}{||t_m^m||} + \delta \frac{T_m^1 H_m^1 H_m^1}{||T_m^1 H_m^1 H_m^1||} \right) \end{cases} \text{ (Type-I)} \]
\[ w_m = \frac{T_m^1 c_m^m}{||T_m^1 c_m^m||} \]
\[ T^{m+1} = T^m - w_m w_m^† T^m \]
end

We notice that for the G-TMRC Type-I, \( w_m \) can be calculated using only first \( m \) row vectors out of full channel matrix \( H \).

Now, we make a comment on the difference of the receiver structure of the SVD-UP and our proposed schemes. The received signal vector of the SVD-UP is given as
\[ y_k = [\lambda_1(H)u_1 \cdots \lambda_M(H)u_M]s_k + n_k \]
where \( \lambda_m(A) \) denotes the \( m \)-th largest singular value of a matrix \( A \). Since the effective channel \( H^\dagger M \) has orthogonal columns, linear detection with the matched filter \( G = U^\dagger M \) is the optimum maximum-likelihood (ML) detection.

For the case of the G-TMRC, we assume the minimum mean squared error (MMSE) linear receiver
\[ G = \left( W^\dagger H W + \frac{\sigma_n^2}{E_s} I_M \right)^{-1} W^\dagger H. \]  
(9)
In general, because the receiver \( G \) in (3) introduces correlation of the noise components in \( \tilde{n} \) as shown in (3), symbol-by-symbol detection after the linear equalization is suboptimal. This is true as long as the columns of the effective channel \( HW \) are not fully orthogonal. For the proposed G-TMRC schemes, the columns of \( HW \) are not completely orthogonal but the mutual influence among the transmitted signals is small since the effective channel columns are made roughly orthogonal. Thus, we can obtain the near-ML performance with the linear MMSE detection.

IV. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the efficacy of the proposed scheme in flat fading channels. Except Figure 2 which shows the uncoded bit error rate (BER) performance, a rate 1/2 binary convolutional code with polynomial \((133, 171)\) in octal notation is applied throughout the simulations. We use the interleaver optimized for the SVD-UP suggested in [5] for a fair comparison. The frame length is set to 64 for all simulations. The performance of the G-TMRC Type-II scheme is experimented with \( \delta = 150 \) which is selected in an ad hoc manner instead of in a principled way.

First, we investigate the effect of the receiver structure on the the proposed G-TMRC scheme and open loop systems where precoding is not applied. Let BERMMSE denote the BER when the receiver employs the MMSE equalizer followed by symbol-by-symbol detection. Similarly, BERML represents the BER with ML detection at the receiver. In Figure 2, the ratio of BERMMSE to BERML is depicted as a function of the average SNR for 2-by-2 and 4-by-4 SM systems.
In this paper, we have presented a new unitary precoding scheme for MIMO systems which approaches the link performance of the SVD-UP in a simpler way. To optimize the unitary precoder with low complexity, we have generalized conventional TMRC to multiple beamforming systems in MIMO channels. In the optimization process, we have incorporated the idea of the gradient ascent algorithm. In order to obtain the near-optimal performance while avoiding iterative property of the gradient ascent, a new initialized beamforming vector has been proposed based on the observations made in [8]. Simulation results demonstrate that the proposed G-TMRC schemes yield the performance almost identical to that of the optimal SVD-UP regardless of system configurations with reduced complexity.

V. CONCLUSION

Figure 3 shows the coded FER comparison for single beamforming systems with 16QAM constellation. We can see that the G-TMRC Type-II achieves almost the same performance as the SVD-UP system for all antenna configurations. In Figure 4, the coded FER performance for multiple beamforming systems with \( M = 4 \) and 4QAM modulation is compared for the SVD-UP and the proposed G-TMRC scheme. As in the case of the single beamforming, the G-TMRC Type-I shows a performance loss of less than 1 dB, which almost disappears with the G-TMRC Type-II for any configurations. It should be emphasized that the complexity of the proposed schemes is substantially smaller than that of the SVD-UP.

From these simulation results, we observe that the proposed G-TMRC Type-II can achieve almost the same FER performance as the SVD-UP in all simulation environments with reduced complexity.

Here, BER\(_{ML}\) and BER\(_{MMSE}\) are measured via Monte-Carlo simulation with 4QAM modulation in uncoded systems. From this figure, we know that the performance loss of the MMSE receiver compared to the ML detector increases exponentially with SNR for open loop systems. In contrast, for the proposed G-TMRC Type-I, the performance gap between two receiver structures is substantially reduced. This can be contributed to the increased level of orthogonality induced by the proposed precoding as predicted in Section III. By adopting the proposed G-TMRC Type-II, two receiver structures exhibit almost the same performance regardless of SNR. This indicates that as the proposed G-TMRC Type-II creates more orthogonal spatial subchannels, a simple MMSE receiver is sufficient to attain the optimum ML detection performance. Thus, in all simulations, an MMSE receiver is used for the proposed G-TMRC schemes except for the single beamforming case where the MRC combining is applied.